A TRANSFORMATION FROM *G*-FUNCTIONS TO *H*-FUNCTIONS OF TWO VARIABLES

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In the present paper an attempt has been made to obtain a transformation from *G*-function to *H*-function of two variables. Some results involving *H*-function of two variables from the known results involving *G*-function of two variables have also been deduced.

1. INTRODUCTION

Recently Agrawal (1965, p. 537) has given a generalization of Meijer's *G* function (*see* Meijer 1946) in two arguments by means of a double Mellin-Barnes contour integral in the form†

$$G^{n,v_1,v_2,m_1,m_2}_{p,[t;t'],s,[q;q']}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(\varepsilon_p):\ (\gamma_t);\ (\gamma'_{t'})\\(\delta_s):\ (\beta_q);\ (\beta'_{q'})\end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\phi(\xi+\eta)\,\psi(\xi,\eta)\,x^{\xi}\,y^{\eta}\,d\xi d\eta \qquad \ldots(1.1)$$

where

$$\phi(\xi+\eta) = \frac{\prod_{j=1}^{n}\Gamma(1-\varepsilon_j+\xi+\eta)}{\prod_{j=n+1}^{p}\Gamma(\varepsilon_j-\xi-\eta)\prod_{j=1}^{s}\Gamma(\delta_j+\xi+\eta)}$$

and

$$\psi(\xi,\eta) = \frac{\prod_{j=1}^{m_1}\Gamma(\beta_j-\xi)\prod_{j=1}^{\gamma_1}\Gamma(\gamma_j+\xi)\prod_{j=1}^{m_2}\Gamma(\beta'_j-\eta)\prod_{j=1}^{v_2}\Gamma(\gamma'_j+\eta)}{\prod_{j=m_1+1}^{q}\Gamma(1-\beta_j+\xi)\prod_{j=v_1+1}^{t}\Gamma(1-\gamma_j-\xi)\prod_{j=m_2+1}^{q'}\Gamma(1-\beta'_j+\eta)\times}$$

$$\times\prod_{j=v_2+1}^{t'}\Gamma(1-\gamma'_j-\eta)$$

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† A slight variant notation for *G*-function of two variables have been used, though in essence the function remains the same. A similar notation for the *H*-function of two variables will be used in the present paper.

$$0 \leqslant m_1 \leqslant q, 0 \leqslant m_2 \leqslant q', \quad 0 \leqslant v_1 \leqslant t, \quad 0 \leqslant v_2 \leqslant t', \quad 0 \leqslant n \leqslant p,$$

$$p + q + s + t < 2(m_1 + v_1 + n)$$

$$p + q' + s + t' < 2(m_2 + v_2 + n)$$

$$|\arg(x)| < \pi[m_1 + v_1 + n - \tfrac{1}{2}(p + q + s + t)]$$

$$|\arg(y)| < \pi[m_2 + v_2 + n - \tfrac{1}{2}(p + q' + s + t')].$$

Also Mathur (1969, p. 215) has given a generalization of Fox's (1961) H-function in two arguments in the form

$$H^{n, v_1, v_2, m_1, m_2}_{p, [t; t'], s, [q; q']}\left[\begin{matrix} x \\ y \end{matrix}\middle|\begin{matrix} \{(\alpha_p, e_p)\} : \{(\gamma_t, c_t)\} ; \{(\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\} . \{(\beta_q, b_q)\} ; \{(\beta'_{q'}, b'_{q'})\} \end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^2} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} \phi(\xi + \eta)\, \psi(\xi, \eta)\, x^{\xi} y^{\eta}\, d\xi\, d\eta \qquad \ldots(1.2)$$

where

$$\phi(\xi + \eta) = \frac{\prod_{j=1}^{n} \Gamma(1 - \alpha_j + e_j\xi + e_j\eta)}{\prod_{j=n+1}^{p} \Gamma(\alpha_j - e_j\xi - e_j\eta) \prod_{j=1}^{s} \Gamma(\delta_j + d_j\xi + d_j\eta)}$$

and

$$\psi(\xi, \eta) = \frac{\prod_{j=1}^{m_1} \Gamma(\beta_j - b_j\xi) \prod_{j=1}^{v_1} \Gamma(\gamma_j + c_j\xi) \prod_{j=1}^{m_2} \Gamma(\beta'_j - b'_j\eta) \times \prod_{j=1}^{v_2} \Gamma(\gamma'_j + c'_j\eta)}{\prod_{j=m_1+1}^{q} \Gamma(1 - \beta_j + b_j\xi) \prod_{j=v_1+1}^{t} \Gamma(1 - \gamma_j - c_j\xi) \times \prod_{j=m_2+1}^{q'} \Gamma(1 - \beta'_j + b'_j\eta) \prod_{j=v_2+1}^{t'} \Gamma(1 - \gamma'_j - c'_j\eta)}$$

$$0 \leqslant m_1 \leqslant q, 0 \leqslant m_2 \leqslant q', \quad 0 \leqslant v_1 \leqslant t, \quad 0 \leqslant v_2 \leqslant t', \quad 0 \leqslant n \leqslant p.$$

The object of this paper is to obtain a transformation from G-function to H-function of two variables and to deduce some results involving H-function of two variables from the known results involving G-function of two variables.

2. We define $\{\Delta_p(a_i, \alpha_i) ; r_i\}$ as

$$\{\Delta(a_1, \alpha_1) ; r_1\}, \quad \{\Delta(a_2, \alpha_2) ; r_2\}, \ldots, \{\Delta(a_p, \alpha_p) ; r_p\}$$

where

$$\Delta(a_1, \alpha_1) = a_1/\alpha_1, \quad (a_1 + 1)/\alpha_1, \ldots, (a_1 + \alpha_1 - 1)/\alpha_1.$$

Now we have from (1.1)

$$G\left[\begin{matrix}\tau_1 x\\ \tau_2 y\end{matrix}\right] = G^{(e_n),(c_{v_1}),(c'_{v_2}),(b_{m_1}),(b'_{m_2})}_{(e_p),[(e_t):(c'_{t'})],(d_s),[(b_q):(b'_{q'})]}$$

$$\times \left[\begin{matrix}\tau_1 x \\ \tau_2 y\end{matrix}\middle|\begin{matrix}\{\underset{p}{\Delta}(\alpha_i, e_i);\ 1\}: \{\underset{t}{\Delta}(\gamma_i, c_i);\ 1\};\ \{\underset{t'}{\Delta}(\gamma'_i, c'_i);\ 1\} \\ \{\underset{s}{\Delta}(\delta_i, d_i);\ 1\}: \{\underset{q}{\Delta}(\beta_i, b_i);\ 1\};\ \{\underset{q'}{\Delta}(\beta'_i, b'_i;\ 1\}\end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^2} \int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty} \frac{\prod_{i=1}^{n}\prod_{j=1}^{e_i}\Gamma\left(1-\frac{\alpha_i+j-1}{e_i}+\xi+\eta\right)}{\prod_{i=n+1}^{p}\prod_{j=1}^{e_i}\Gamma\left(\frac{\alpha_i+j-1}{e_i}-\xi-\eta\right)\times} \times \prod_{j=1}^{s}\prod_{j=1}^{d_i}\Gamma\left(\frac{\delta_i+j-1}{d_i}+\xi+\eta\right)$$

$$\times \frac{\prod_{i=1}^{m_1}\prod_{j=1}^{b_i}\Gamma\left(\frac{\beta_i+j-1}{b_i}-\xi\right)\prod_{i=1}^{v_1}\prod_{j=1}^{c_i}\Gamma\left(\frac{\gamma_i+j-1}{c_i}+\xi\right) \times \prod_{i=1}^{m_2}\prod_{j=1}^{b'_i}\Gamma\left(\frac{\beta'_i+j-1}{b'_i}-\eta\right)}{\prod_{i=m_1+1}^{q}\prod_{j=1}^{b_i}\Gamma\left(1-\frac{\beta_i+j-1}{b_i}+\xi\right)\prod_{i=v_1+1}^{t}\prod_{j=1}^{e_i}\Gamma\left(1-\frac{\gamma_i+j-1}{c_i}-\xi\right)\times \times \prod_{i=m_2+1}^{q'}\prod_{j=1}^{b'_i}\Gamma\left(1-\frac{\beta'_i+j-1}{b'_i}+\eta\right)}$$

$$\times \frac{\prod_{i=1}^{v_2}\prod_{j=1}^{c'_i}\Gamma\left(\frac{\gamma'_i+j-1}{c'_i}+\eta\right)}{\prod_{i=v_2+1}^{t'}\prod_{j=1}^{c'_i}\Gamma\left(1-\frac{\gamma'_i+j-1}{c'_i}-\eta\right)} (\tau_1 x)^{\xi}(\tau_2 y)^{\eta}\, d\xi d\eta$$

where (e_k) denote $\sum_{i=1}^{k} e_i$ and the similar interpretations for (c_{v_1}), (c_{v_2}), (b_{m_1}), (b_{m_2}), etc., and where

$$\tau_1 = \prod_{i=1}^{s}\left(\frac{D_i}{d_i}\right)^{d_i}\prod_{i=1}^{q}\left(\frac{B_i}{b_i}\right)^{b_i}\prod_{i=1}^{p}\left(\frac{e_i}{A_i}\right)^{e_i}\prod_{i=1}^{t}\left(\frac{c_i}{C_i}\right)^{e_i}$$

and

$$\tau_2 = \prod_{i=1}^{s} \left(\frac{D_i}{d_i}\right)^{d_i} \prod_{i=1}^{q'} \left(\frac{B_i'}{b_i'}\right)^{b'_i} \prod_{i=1}^{p} \left(\frac{e_i}{A_i}\right)^{e_i} \prod_{i=1}^{t'} \left(\frac{c_i'}{C_i'}\right)^{c'_i}.$$

Now by using Gauss's multiplication theorem (cf., Whittaker and Watson 1969, p. 240); we have

$$G\left[\begin{matrix}\tau_1 x \\ \tau_2 y\end{matrix}\right] = A \cdot \frac{1}{(2\pi i)^2} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} \frac{\prod_{i=1}^{n} \Gamma(1-\alpha_i+e_i\xi+e_i\eta)}{\prod_{i=n+1}^{p} \Gamma(\alpha_i-e_i\xi-e_i\eta) \prod_{i=1}^{s} \Gamma(\delta_i+d_i\xi+d_i\eta)}$$

$$\times \frac{\prod_{i=1}^{m_1} \Gamma(\beta_i-b_i\xi) \prod_{i=1}^{v_1} \Gamma(\gamma_i+c_i\xi) \prod_{i=1}^{m_2} \Gamma(\beta_i'-b_i'\eta) \times \prod_{i=1}^{v_2} \Gamma(\gamma_i'+c_i'\eta) (\tau_3 x)^{\xi} (\tau_4 y)^{\eta} \, d\xi d\eta}{\prod_{i=m_1+1}^{q} \Gamma(1-\beta_i+b_i\xi) \prod_{i=v_1+1}^{t} \Gamma(1-\gamma_i-c_i\xi) \times \prod_{i=m_2+1}^{q'} \Gamma(1-\beta_i'+b_i'\eta) \prod_{i=v_2+1}^{t'} \Gamma(1-\gamma_i'-c_i'\eta)}$$

where

$$\tau_3 = \prod_{i=1}^{s} (D_i)^{d_i} \prod_{i=1}^{q} (B_i)^{b_i} \prod_{i=1}^{p} (A_i)^{-e_i} \prod_{i=1}^{t} (C_i)^{-e_i},$$

$$\tau_4 = \prod_{i=1}^{s} (D_i)^{d_i} \prod_{i=1}^{q'} (B_i')^{b'_i} \prod_{i=1}^{p} (A_i)^{-e_i} \prod_{i=1}^{t'} (C_i')^{c'_i}$$

and A is a multiple, independent of variables.

Now by again applying Gauss's multiplication theorem, we get

$$G\left[\begin{matrix}\tau_1 x \\ \tau_2 y\end{matrix}\right] = K \cdot \frac{1}{(2\pi_i)^2} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} \frac{\prod_{i=1}^{n} \prod_{j=1}^{A_i} \Gamma\left(1-\frac{\alpha_i+j-1}{A_i}+\frac{e_i\xi+e_i\eta}{A_i}\right)}{\prod_{i=n+1}^{p} \prod_{j=1}^{A_i} \Gamma\left(\frac{\alpha_i+j-1}{A_i}-\frac{e_i\xi+e_i\eta}{A_i}\right)}$$

$$\times \frac{\prod_{i=1}^{m_1}\prod_{j=1}^{B_i}\Gamma\left(\frac{\beta_i+j-1}{B_i}-\frac{b_i\xi}{B_i}\right)\prod_{i=1}^{v_1}\prod_{j=1}^{C_i}\Gamma\left(\frac{\gamma_i+j-1}{C_i}-\frac{c_i\xi}{C_i}\right)\times\prod_{i=1}^{m_2}\prod_{j=1}^{B'_i}\Gamma\left(\frac{\beta'_i+j-1}{B'_i}-\frac{b'_i\eta}{B'_i}\right)}{\prod_{i=1}^{s}\prod_{j=1}^{D_i}\Gamma\left(\frac{\delta_i+j-1}{D_i}+\frac{d_i\xi+d_i\eta}{D_i}\right)\times}$$

$$\times\prod_{i=m_1+1}^{q}\prod_{j=1}^{B_i}\Gamma\left(1-\frac{\beta_i+j-1}{B_i}+\frac{b_i\xi}{B_i}\right)\times$$

$$\times\prod_{i=v_1+1}^{t}\prod_{j=1}^{C_i}\Gamma\left(1-\frac{\gamma_i+j-1}{C_i}-\frac{c_i\xi}{C_i}\right)$$

$$\times\frac{\prod_{i=1}^{v_2}\prod_{j=1}^{C'_i}\Gamma\left(\frac{\gamma'_i+j-1}{C'_i}+\frac{c'_i\eta}{C'_i}\right)x^{\xi}\,y^{\eta}\,d\xi d\eta}{\prod_{i=m_2+1}^{q'}\prod_{j=1}^{B'_i}\Gamma\left(1-\frac{\beta'_i+j-1}{B'_i}+\frac{b'_i\eta}{B'_i}\right)\times}$$

$$\times\prod_{i=v_2+1}^{t'}\prod_{j=1}^{C'_i}\Gamma\left(1-\frac{\gamma'_i+j-1}{C'_i}-\frac{c'_i\eta}{C'_i}\right)$$

$$= K\,.\,H^{(A_n),\,(C_{v_1}),(C'_{v_2}),\,(B_{m_1}),(B'_{m_2})}_{(A_p),\,[(C_t);\,(C'_{t'})],\,(D_s),\,[(B_q);\,(B'_{q'})]}$$

$$\times\left[\begin{matrix}x\\ y\end{matrix}\middle|\begin{matrix}\left\{\Delta_p(\alpha_i,A_i);\ \frac{e_i}{A_i}\right\}:\left\{\Delta_t(\gamma_i,C_i);\ \frac{c_i}{C_i}\right\};\left\{\Delta_{t'}(\gamma'_i,C'_i);\ \frac{c'_i}{C'_i}\right\}\\ \left\{\Delta_s(\delta_i,D_i);\ \frac{d_i}{D_i}\right\}:\left\{\Delta_q(\beta_i,B_i);\ \frac{b_i}{B_i}\right\};\left\{\Delta_{q'}(\beta'_i,B'_i);\ \frac{b'_i}{B'_i}\right\}\end{matrix}\right] \qquad \ldots(2.1)$$

where K is a multiple independent of variables and

$$\sum_1^p e_i+\sum_1^q(b_i)+\sum_1^s d_i+\sum_1^t c_i<2\,(\sum_1^{m_1}b_i+\sum_1^{v_1}c_i+\sum_1^n e_i)$$

$$\sum_1^p e_i+\sum_1^{q'}(b'_i)+\sum_1^s d_i+\sum_1^{t'}c'_i<2\,(\sum_1^{m_2}b'_i+\sum_1^{v_2}c'_i+\sum_1^n e_i)$$

$$|\arg x|<\pi\,[\sum_1^{m_1}b_i+\sum_1^{v_1}c_i+\sum_1^n e_i-\tfrac{1}{2}\,(\sum_1^p e_i+\sum_1^q b_i+\sum_1^s d_i+\sum_1 c_i)]$$

$$|\arg y|<\pi\,[\sum_1^{m_2}b'_i+\sum_1^{v_2}c'_i+\sum_1^n e_i-\tfrac{1}{2}\,(\sum_1^p e_i+\sum_1^{q'}b'+\sum_1^s d_i+\sum_1^{t'}c'_i)].$$

This transformation provides a generalization to the transformation obtained by Agrawal and Singhal (*in press*) and Soni (1971).

3. INTEGRALS

In this section we shall obtain some integrals involving H-function of two variables with the help of the above transformation.

We have (Srivastava and Joshi 1969, p. 473)

$$\int_0^\infty t^{2\gamma-1} W_{\lambda,\mu}(zt)\, W_{-\lambda,\mu}(zt)\, G^{0,q,k,r,l}_{A,[C:E],B,[D,F]}\left[\begin{matrix} xt^{2n} \\ yt^{2n}\end{matrix}\middle|\begin{matrix}(a):(c);(e)\\(b):(d);(f)\end{matrix}\right] dt$$

$$= \alpha[n,\gamma,z]\; G^{4n,q,k,r,l}_{A+4n,[C:E],B+2n,[D:F]}$$

$$\times\left[\begin{matrix} x(z/2n)^{-2n} \\ y(z/2n)^{-2n}\end{matrix}\middle|\begin{matrix}\nabla[2n,2\gamma+1],\ \nabla[n,\gamma\pm\mu+1],\ (a):(c);(e)\\ (b),\ \Delta[n,\ \gamma\pm\lambda+1]:(d);(f)\end{matrix}\right], \qquad \ldots(3.1)$$

provided

$$2(q+r) > A+B+C+D$$

$$2(k+l) > A+B+E+F$$

$$|\arg(x)| < \pi[q+r-\tfrac{1}{2}(A+B+C+D)]$$

$$|\arg(y)| < \pi[k+l-\tfrac{1}{2}(A+B+E+F)], \quad |\arg(z)| < \tfrac{1}{2}\pi,$$

$$R(z) > 0, \quad R[\gamma + n(d_j+f_h)+\tfrac{1}{2}] > R(\mu)$$

where $j = 1, 2, \ldots, r$; $h = 1, 2, \ldots, l$; and for the sake of brevity

$$\alpha[n,\gamma,z] = \frac{(2\pi)^{\frac{1}{2}-n}\,(2n)^{2\gamma-\frac{1}{2}}}{z^{2\gamma}}$$

and where $\Delta[k,\alpha]$ and $\nabla[k,\alpha]$ stands for the sets

$$\frac{\alpha}{k},\ \frac{\alpha+1}{k},\ \ldots,\ \frac{\alpha+k-1}{k},$$

and

$$1-\frac{\alpha}{k},\ 1-\frac{\alpha+1}{k},\ \ldots,\ 1-\frac{\alpha+k-1}{k},\ \text{respectively.}$$

Also (a) is taken to denote the sequence of A parameters

$$a_1, a_2, \ldots, a_j, \ldots, a_A.$$

The above integral can be put in the following form with appropriate changes in the parameters:

$$k^{-1}\int_0^\infty t^{2\gamma-1}\ W_{\lambda,\mu}(zt)\,W_{-\lambda,\mu}(zt)\ G^{0,(av_1),(a'v_2),(bm_1),(b'm_2)}_{(e_p),[(ct);(c't')],(d_s),[(bq);(b'q')]}$$

$$\times\left[\begin{matrix}\tau_1 x t^{2n}\\ \tau_2 y t^{2n}\end{matrix}\middle|\begin{matrix}\{\underset{p}{\Delta}(\alpha_i,e_i);1\}:\{\underset{t}{\Delta}(\gamma_i,c_i);1\};\{\underset{t'}{\Delta}(\gamma_i',c_i');1\}\\ \{\underset{s}{\Delta}(\delta_i,d_i);1\}:\{\underset{q}{\Delta}(\beta_i,b_i);1\};\{\underset{q'}{\Delta}(\beta_i',b_i');1\}\end{matrix}\right]dt$$

$$=K^{-1}\,.\,\alpha[n,\gamma,z]\ G^{4n,(av_1),(a'v_2),(bm_1),(b'm_2)}_{(e_p+4n),[(ct);(c't')],(d_s)+2n,[(bq);(b'q')]}$$

$$\times\left[\begin{matrix}\tau_1 x\,(z/2n)^{-2n}\\ \tau_2 y\,(z/2n)^{-2n}\end{matrix}\middle|\begin{matrix}\nabla[2n,2\gamma+1],\ \nabla[n,\gamma\pm\mu+1],\ \{\underset{p}{\Delta}(\alpha_i,c_i);1\}:\\ \{\underset{t}{\Delta}(\gamma_i,c_i);1\};\{\underset{t'}{\Delta}(\gamma_i',c_i');1\}\\ \{\underset{s}{\Delta}(\delta_i,d_i);1\},\ \Delta[n,\gamma\pm\lambda+1]:\{\underset{q}{\Delta}(\beta_i,b_i);1\};\\ \{\underset{q'}{\Delta}(\beta_i',b_i');1\}\end{matrix}\right]$$

with corresponding changes in the condition of convergence. Now use of the transformation (2.1) followed by a little change in parameters assumes the form

$$\int_0^1 t^{2\gamma-1}\ W_{\lambda,\mu}(zt)\ W_{-\lambda,\mu}(zt)\ H^{0,v_1,v_2,m_1,m_2}_{p,[t;t'],s,[q;q']}$$

$$\times\left[\begin{matrix}xt^{2n}\\ yt^{2n}\end{matrix}\middle|\begin{matrix}\{(\alpha_p,e_p)\}:\{(\gamma_t,c_t)\};\{(\gamma'_{t'},c'_{t'})\}\\ \{(\delta_s,d_s)\}:\{(\beta_q,b_q)\};\{(\beta'_{q'},b'_{q'})\}\end{matrix}\right]dt$$

$$=\alpha[n,\gamma,z]\ H^{4n,v_1,v_2,m_1,m_2}_{p+4n,[t;t'],s+2n,[q,q']}\left[\begin{matrix}x\,(z/2n)^{-2n}\\ y\,(z/2n)^{-2n}\end{matrix}\middle|\right.$$

$$\left.\times\begin{matrix}(\nabla[2n,2\gamma+1],1),(\nabla[n,\gamma\pm\mu+1],1),\{(\alpha_p,e_p)\}:\{(\gamma_t,c_t)\};\{(\gamma'_{t'},c'_{t'})\}\\ \{(\delta_s,d_s)\},(\Delta[n,\gamma\pm n+1],1):\{(\beta_q,b_q)\};\{(\beta'_{q'},b'_{q'})\}\end{matrix}\right]$$

provided

$$p+q+s+t<2(m_1+v_1)$$

$$p+q'+s+t'<2(m_2+v_2)$$

$$|\arg(x)|<\pi[m_1+v_1-\tfrac{1}{2}(p+q+s+t)]$$

$$|\arg(y)|<\pi[m_2+v_2-\tfrac{1}{2}(p+q'+s+t')],\quad|\arg(z)|<\tfrac{1}{2}\pi,$$

and

$$R(z)>0,\quad R\left[\gamma+n\left\{\left(\frac{\beta_j}{b_j}\right)+\left(\frac{\beta'_h}{b'_h}\right)\right\}+\frac{1}{2}\right]>|R(\mu)|,$$

where

$$j=1,\ 2,\ldots,m_1\ ;h=1,\ 2,\ldots,m_2.$$

In a similar manner, from (2.3) of Srivastava and Joshi (1969) it follows that for positive integral values of n

$$\int_0^\infty t^{2\lambda-1} K_{2\mu}(zt) K_{2\nu}(zt) H^{0,\,v_1,\,v_2,\,m_1,\,m_2}_{p,\,[t;\,t'],\,s,\,[q;\,q']} \times \left[\begin{matrix} xt^{2n} \\ yt^{2n}\end{matrix}\middle|\begin{matrix}\{(\alpha_p, e_p)\} : \{(\gamma_t, c_t)\} ; \{(\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\} : \{(\beta_q, b_q)\} ; \{(\beta'_{q'}, b'_{q'})\}\end{matrix}\right]$$

$$= \beta[n, \lambda, z]\, H^{4n,\,v_1,\,v_2,\,m_1,\,m_2}_{p+4n,\,[t;\,t'],\,s+2n,\,[q;\,q']} \left[\begin{matrix} x(z/n)^{-2n} \\ y(z/n)^{-2n}\end{matrix}\middle|\right.$$

$$\left.\times\begin{matrix}(\nabla[n, \lambda\pm\mu+\nu], 1), (\nabla[n, \lambda\pm\mu-\nu], 1), \{(\alpha_p, e_p)\} : \{(\gamma_t, c_t)\} ; \{(\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\}, (\Delta[2n, 2\lambda], 1). \{(\beta_q, b_q)\} ; \{(\beta'_{q'}, b'_{q'})\}\end{matrix}\right]$$

which holds when

$$p + q + s + t < 2(m_1 + v_1)$$

$$p + q' + s + t' < 2(m_2 + v_2)$$

$$|\arg(x)| < \pi[m_1 + v_1 - \tfrac{1}{2}(p + q + s + t)]$$

$$|\arg(y)| < \pi[m_2 + v_2 - \tfrac{1}{2}(p + q' + s + t')], \quad |\arg(z)| < \tfrac{1}{2}\pi$$

and

$$R(z) > 0, \quad R\left[\lambda + n\left\{\left(\frac{\beta_j}{b_j}\right) + \left(\frac{\beta'_h}{b'_h}\right)\right\}\right] > |R(\mu)| + |R(\nu)|,$$

where

$$j = 1, 2, \ldots, m_1 ; \quad h = 1, 2, \ldots, m_2$$

and

$$\beta[n, \lambda, z] = \frac{\pi^{3/2-n}\, n^{2\lambda-3/2}}{2^{n+1}\, z^{2\lambda}}.$$

Also from (4.2) of Srivastava and Joshi (1969) it follows that, when n is a positive integer

$$\int_0^\infty t^{\rho-1}\phi(t)\, H^{0,\,v_1,\,v_2,\,m_1,\,m_2}_{p,\,[t;\,t'],\,s,\,[q;\,q']}\left[\begin{matrix} xt^n \\ yt^n\end{matrix}\middle|\begin{matrix}\{(\alpha_p, e_p)\} : \{(\gamma_t, c_t)\} ; \{(\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\} : \{(\beta_q, b_q)\} ; \{(\beta'_{q'}, b'_{q'})\}\end{matrix}\right] dt$$

$$= (2\pi)^{\frac{1}{2}(n-1)(B'+C'-A')}\, n^{\sum_{j=1}^{A'} a'_j - \sum_{j=1}^{B'} b'_j - \sum_{j=1}^{C'} c'_j + \rho(A'-B'+C') - \frac{1}{2}(A'-B'-C')}$$

$$\times H^{nA',\,v_1,\,v_2,\,m_1,\,m_2}_{p+n(A'+C'),\,[t;\,t'],\,s+nB',\,[q;\,q']}\left[\begin{matrix} xn^{(A'-B'+C')n} \\ yn^{(A'-B'+C')n}\end{matrix}\middle|\right.$$

$$\left.\times\begin{matrix}(\nabla[n, (a')+\rho], 1), (\Delta[n, (c')-\rho], 1), \{(\alpha_p, e_p)\} : \{(\gamma_t, c_t)\} ; \{(\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\}, (\Delta[n, (b')+\rho], 1)\} : \{(\beta_q, b_q)\} ; \{(\beta'_{q'}, b'_{q'})\}\end{matrix}\right]$$

provided that the integral converges, and where

$$\phi(t) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{\Gamma[(a') + s]}{\Gamma[(b') + s]\ \Gamma[(c') - s]}\ t^{-s}\ ds, \qquad (A' + C' \geqslant B')$$

and $\Gamma[(a) + \lambda]$ stands for the gamma product

$$\prod_{j=1}^{A} \Gamma[a_j + \lambda].$$

Similarly other integrals can be obtained with the help of known integrals involving G-function of two variables.

4. An Infinite Series

We have (Singhal 1974)

$$\sum_{M,N=0}^{\infty} \frac{(c)_M\ (c')_N}{M!\ N!}\ G^{n,\ v_1+2,\ v_2+2,\ m_1,\ m_2}_{p,\ [t+2;\ t'+2],\ s+2,\ [q,\ q']}$$

$$\times \begin{bmatrix} x \\ y \end{bmatrix} \begin{matrix} (\varepsilon_p):\ \sigma+M,\ r,\ (\gamma_t);\ \sigma+N,\ \mu,\ (\gamma'_{t'}) \\ (\delta_s),\ \sigma+M+\mu,\ \sigma+N+r:\ (\beta_q);\ (\beta'_{q'}) \end{matrix}$$

$$= G^{n,\ v_1+2,\ v_2+2,\ m_1,\ m_2}_{p,\ [t+2;\ t'+2],\ s+2,\ [q;\ q']} \begin{bmatrix} x \\ y \end{bmatrix} \begin{matrix} (\varepsilon_p) & \sigma,\ r-c',\ (\gamma_t)\ ;\ \sigma,\ \mu-c,\ (\gamma'_{t'}) \\ (\delta_s), & \sigma+\mu-c,\ \sigma+r-c':\ (\beta_p);\ (\beta'_{p'}) \end{matrix}.$$

A similar application (as above) of the transformation (2.1) yields

$$\sum_{M,N=0}^{\infty} \frac{(\lambda)_M\ (\lambda')_N}{M!\ N!}\ H^{n,\ v_1+2,\ v_2+2,\ m_1,\ m_2}_{p,\ [t+2;\ t'+2],\ s+2,\ [q;\ q']}$$

$$\times \begin{bmatrix} x \\ y \end{bmatrix} \begin{matrix} \{(\alpha_p, e_p)\}:\ (\sigma+M, 1),\ (r, 1),\ \{(\gamma_t, c_t)\};\ (\sigma+N, 1),\ (\mu, 1),\ \{(\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\},\ (\sigma+M+\mu, 1),\ (\sigma+N+r, 1);\ \{(\beta_q, b_q)\};\ \{(\beta'_{q'}, b'_{q'})\} \end{matrix}$$

$$= H^{n,\ v_1+2,\ v_2+2,\ m_1,\ m_2}_{p,\ [t+2;\ t'+2],\ s+2,\ [q;\ q']}$$

$$\times \begin{bmatrix} x \\ y \end{bmatrix} \begin{matrix} \{(\alpha_p, e_p)\}:\ (\sigma, 1),\ (r-\lambda', 1),\ \{(\gamma_t, c_t)\};\ (\sigma, 1),\ (\mu-\lambda, 1),\ \{(\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\},\ (\sigma+\mu-\lambda, 1),\ (\sigma+r-\lambda', 1):\ \{(\beta_q, b_q)\};\ \{(\beta'_{q'}, b'_{q'})\} \end{matrix}$$

5. Expansion Formulae

On applying the transformation (2.1) on the result (2.1) of Shah (1971), after a little change in the parameters, we get

$$\sqrt{\pi}\ H^{n,\ v_1,\ v_2,\ m_1,\ m_2}_{p,\ [t;\ t'],\ s,\ [q,\ q']}$$

$$\times \begin{bmatrix} x\cos^2\tfrac{1}{2}\theta \\ y\cos^2\tfrac{1}{2}\theta \end{bmatrix} \begin{matrix} \{(\alpha_p, e_p)\}:\ \{(\gamma_r, c_t)\}\ ;\ \{\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\}:\ \{(\beta_q, b_q)\}\ ;\ \{(\beta'_{q'}, b'_{q'})\} \end{matrix}$$

$$= H^{n+1,\,v_1,\,v_2,\,m_1,\,m_2}_{p+3,\,[s;\,s'],\,s+1,\,[a;\,a']} \left[\begin{matrix} x \\ y \end{matrix} \middle| \begin{matrix} (\tfrac{1}{2}, 1), \{(\alpha_g, e_g)\} : \{(\gamma_t, c_t)\}; \{(\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\}, (0,\ 1); \{(\beta_g, b_g)\}; \{(\beta'_{g'}, b'_{g'})\} \end{matrix} \right]$$

$$+ 2 \sum_{\mu=1}^{\infty} \cos \mu\theta \; H^{n+2,\,v_1,\,v_2,\,m_1,\,m_2}_{p+2,\,[s;\,s'],\,s+2,\,[a;\,a']}$$

$$\times \left[\begin{matrix} x \\ y \end{matrix} \middle| \begin{matrix} (\tfrac{1}{2}, 1), (0, 1), \{(\alpha_g, e_g)\} : \{(\gamma_t, c_t)\}; \{(\gamma'_{t'}, c'_{t'})\} \\ \{(\delta_s, d_s)\}, (\mu, 1), (-\mu, 1) : \{(\beta_g, b_g)\}; \{(\beta'_{g'}, b'_{g'})\} \end{matrix} \right]$$

Similarly many other results involving H-function of two variables may be obtained with the help of known results (cf., e.g. Chandel and Agrawal 1971, pp. 88–90) involving G-function of two variables using the transformation (2.1).

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