ON POINT EXPLOSION IN AN INHOMOGENEOUS MEDIUM

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An attempt has been made to study the character of the shock propagation into an inhomogeneous medium and to determine the geometric shape of the shock without solving the fluid dynamic equations.

1. INTRODUCTION

The problem of the propagation of shock waves induced by explosions is of great physical interest and is important in astrophysical as well as terrestrial contexts. Bhatnagar and Lal (1965) in their study of the propagation of a spherical shock wave have shown that the initial energy of the system and the mode of release of the explosion energy, i.e. whether it is instantaneous or time dependent, influence the shock propagation significantly. The problem of strong point explosion in a homogeneous medium admits similarity solution. Sedov (1946, 1959) obtained analytic solution to the equations of self-similar motion while Taylor (1950) and Stanyukovich (1960) obtained numerical solutions. But the gas dynamic process associated with the strong point explosion in an inhomogeneous medium is not self-similar and analytic solution, in general, cannot be obtained. The numerical computation of the equations associated with even the axial symmetric shock waves is quite tedious. So far as the geometric shape of the axial symmetric strong shock propagating into a non-homogeneous medium is concerned, Schindler (1965) has shown that it can be predetermined without the integration of the fluid dynamic equations if the flow just behind the shock is assumed to be irrotational. This constraint seems to be too restrictive and consequently, in Section 2 of this paper, we have shown that the shape of the axial symmetric strong shock can be predetermined on the assumption that the flow just behind the shock front is rotational. The result of Schindler (1965, example 4) follows as a particular case when the parameter K_3 is taken equal to zero. We find that when the density distribution of the medium is symmetric with respect to the point of explosion, the geometric shape of the shock is spherically

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symmetric with respect to the same point. In this case, following Chernyi (1957), we have obtained the speed of the shock front and the thickness of the thin layer into which the whole mass of the gas encompassed by the explosion wave is assumed to be concentrated. We find that the shock front moves faster in the rarer medium. For exponential density distribution in the atmosphere, Kompaneets (1960) has thrown light on the character of the shock propagation and on the geometric shape of the shock. In Section 3 we have studied the above aspects of the strong point explosion problem when the density distribution of the ambient medium follows the power law behaviour.

2. Choice of Coordinate System and Nature of the Wave Boundary

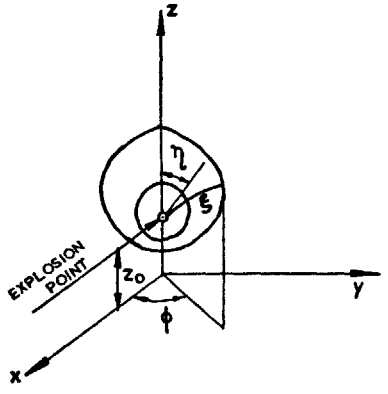
In order to represent the boundary of the wave front by a simple equation of one parameter family of surfaces, we choose orthogonal curvilinear coordinates (ξ, η, ϕ) defined by

$$\left.\begin{aligned} x &= f(\xi, \eta) \cos \phi \\ y &= f(\xi, \eta) \sin \phi \\ z &= g(\xi, \eta) \end{aligned}\right\} \quad .. \quad (2.1)$$

where

f = distance of a point in space from the z-axis

g = distance of a point in space from the plane $z = 0$.



The line element ds is given by

$$ds^2 = Ed\xi^2 + Gd\eta^2 + f^2 d\phi^2 \quad .. \quad .. \quad .. \quad (2.2)$$

where

$$E(\xi, \eta) = f_\xi^2(\xi, \eta) + g_\xi^2(\xi, \eta) \quad .. \quad .. \quad .. \quad .. \quad (2.3)$$

$$G(\xi, \eta) = f_\eta^2(\xi, \eta) + g_\eta^2(\xi, \eta). \quad .. \quad .. \quad .. \quad .. \quad (2.4)$$

Taking z-axis as the axis of symmetry we can represent any axial symmetric function in terms of f and g. Since f and g are arbitrary differentiable functions, we can choose them in such a way that the surfaces ξ (= constant) represent a one parameter family of wave fronts. The explosion point at $z = z_0$ is obviously represented by $\xi = 0$.

Since the coordinate system (ξ, η, ϕ) is taken as orthogonal, we have the relation

$$f_\xi f_\eta + g_\xi g_\eta = 0. \quad .. \quad .. \quad .. \quad .. \quad (2.5)$$

The conditions to be satisfied by $f(\xi, \eta)$ and $g(\xi, \eta)$ are

$$\left.\begin{aligned} &f(0, \eta) = f(\xi, 0) = f(\xi, \pi) = 0 \\ &g(0, \eta) = Z_0, \quad g_\eta(\xi, 0) = g_\eta(\xi, \pi) = 0 \end{aligned}\right\} . \qquad .. \qquad .. \quad (2.6)$$

The shock front at time t can be located by setting

$$\xi = h(t)$$

where $h(t)$ is some monotonically increasing and differentiable function of t with $h(0) = 0$ and $\frac{dh(0)}{dt} \neq 0$ as initial values at the time of burst $t = 0$. If we use the transformation

$$\bar{\xi} = h^{-1}(\xi), \quad \bar{\eta} = \eta \qquad .. \qquad .. \qquad .. \qquad .. \quad (2.7)$$

the shock front at time t, on dropping the bar sign, can be localized by setting

$$\xi = t. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \quad (2.8)$$

We note that the shock surfaces $\xi =$ (constant) can be determined if we know the functions $f(\xi, \eta)$ and $g(\xi, \eta)$. For this (2.5) provides one of the relations and the other relation is obtained by imposing a constraint on the flow just behind the shock front.

The Rankine-Hugoniot conditions for a polytropic gas with constant adiabatic exponent γ are

$$\left.\begin{aligned} u_1 &= \frac{2}{\gamma+1}\left[\left(D^2 - \frac{\gamma p_0}{\rho_0}\right)\Big/ D\right], \quad u_0 = 0, \quad v_1 = v_0 = 0 \\ p_1 &= \frac{1}{\gamma+1}\left[2D^2 - \frac{(\gamma-1)p_0}{\rho_0}\right]\rho_0 \\ \rho_1 &= (\gamma+1)\rho_0 \frac{D^2}{(\gamma-1)D^2 + 2g\frac{p_0}{\rho_0}} \end{aligned}\right\} \qquad .. \quad (2.9)$$

where u and v are components of the gas velocity along increasing ξ and η directions respectively and p is the pressure, ρ the density, D the normal speed of the shock front and the suffixes 0, 1 denote the variables ahead of and behind the shock front respectively. The normal speed D of the shock front, using (2.2), is expressed as

$$D = \left[\frac{ds(\xi)}{d\xi}\right]_{\xi = t} = [\sqrt{E(\xi, \eta)}]_{\xi = t} \qquad .. \qquad .. \qquad .. \quad (2.10)$$

s being the arc length along the ξ-line.

We note that if D is known from (2.9) on imposing some suitable constraint on the flow just behind the shock front, then (2.10) together with (2.3), (2.5) and (2.6) will determine $f(\xi, \eta)$ and $g(\xi, \eta)$ and thus the geometric

shape of the shock front can be determined by setting $\xi = t$ constant, p_0 and ρ_0 being given functions, i.e.

$$\left.\begin{aligned} p_0 &= [p_0(f, g)]_{\xi=t} \\ \rho_0 &= [\rho_0(f, g)]_{\xi=t} \end{aligned}\right\}. \qquad .. \quad .. \quad .. \quad (2.11)$$

To illustrate the procedure, we assume that one of the variables u, p or ρ is continuous across the wave front and the temperature T_0 of the ambient medium is a function of the distance from the point of explosion so that

$$T_0 = K\{f^2+(g-Z_0)^2\}^{\frac{n}{n+1}} \qquad .. \quad .. \quad .. \quad (2.12)$$

where K is a constant.

Under these conditions we obtain from (2.3), (2.9), (2.10) and (2.12)

$$D^2 = \frac{\gamma p_0}{\rho_0} = f_\xi^2+g_\xi^2 = K_1^2\{f^2+(g-Z_0)^2\}^{\frac{n}{n+1}} \qquad .. \quad .. \quad (2.13)$$

where $K_1^2 = \gamma RK$ = constant, R being the gas constant.

The system of equations (2.5) and (2.13) with conditions (2.6) admit the following solutions:

$$\left.\begin{aligned} f(\xi, \eta) &= K_2\xi^{n+1} \sin \eta \\ g(\xi, \eta) &= Z_0+K_2\xi^{n+1} \cos \eta \end{aligned}\right\}. \qquad .. \quad .. \quad .. \quad (2.14)$$

where

$$K_2 = \left(\frac{K_1}{n+1}\right)^{n+1}$$

The case of isothermal medium of example 1 of Schindler (1965) is recovered by putting $n = 0$ in (2.14). For $n = 1$ we get the solution for the case for which the temperature of the ambient medium is a linear function of the distance from the point of explosion and $n = -\frac{1}{2}$ corresponds to the situation when the temperature of the ambient is proportional to the reciprocal of the square of the distance from the explosion centre. Obviously the sonic surfaces $\xi =$ constant are spherical surfaces concentric to the point of explosion. Further, we notice that the solution does not hold for $n = -1$ which corresponds to the infinite temperature of the ambient medium.

Vorticity Constraint

Now we shall proceed to examine the nature of shock surfaces on the assumption that the flow just behind the shock front is rotational so that

$$\left[\frac{\partial}{\partial\xi}(\sqrt{G}v)-\frac{\partial}{\partial\eta}(\sqrt{E}u)\right]_{\xi=t} = \left[K_3\frac{\partial E}{\partial\eta}\right]_{\xi=t} \qquad .. \quad .. \quad (2.15)$$

where K_3 is a constant, the choice of which has been indicated in what follows.

Further, we consider the situation when the strength of the shock is sufficiently large so that $p_1 \gg p_0$ and in this case the Rankine-Hugoniot

conditions (2.9) reduce to

$$\left.\begin{aligned} u_1 &= \frac{2}{\gamma+1} D, \quad v_1 = 0 \\ p_1 &= \frac{2}{\gamma+1} D^2 \rho_0, \quad \rho_1 = \frac{\gamma+1}{\gamma-1} \rho_0 \end{aligned}\right\} \quad .. \quad .. \quad (2.16)$$

The choice of our curvilinear coordinates ξ, η enables us to have the relations

$$\left.\begin{aligned} \left[\frac{\partial u}{\partial \eta}\right]_{\xi=t} &= \frac{\partial u_1}{\partial \eta}, \quad \left[\frac{\partial v}{\partial \eta}\right]_{\xi=t} = 0 \\ \left[\left(\frac{\partial v}{\partial \eta}+\frac{\partial v}{\partial \xi}\right)\right]_{\xi=t} &= 0 \end{aligned}\right\} \quad .. \quad .. \quad (2.17)$$

Since the ambient medium is taken to be at rest we have

$$\left.\begin{aligned} \frac{1}{\sqrt{E}} \left\{\frac{1}{\rho_0} \frac{\partial p_0}{\partial \xi}+\frac{\partial \Phi}{\partial \xi}\right\} &= 0 \\ \frac{1}{\sqrt{G}} \left\{\frac{1}{\rho_0} \frac{\partial p_0}{\partial \eta}+\frac{\partial \Phi}{\partial \eta}\right\} &= 0 \end{aligned}\right\} \quad .. \quad .. \quad (2.18)$$

where Φ is the potential of the external force per unit mass. The momentum equation

$$\sqrt{G} \frac{\partial v}{\partial t}+\frac{u}{\sqrt{E}} \left[\frac{\partial}{\partial \xi}(\sqrt{G} v)-\frac{\partial}{\partial \eta}(\sqrt{E} u)\right]+\frac{1}{2} \frac{\partial}{\partial \eta}(u^2+v^2)+\frac{1}{\rho} \frac{\partial p}{\partial \eta}+\frac{\partial \Phi}{\partial \eta} = 0$$

with the help of (2.15) to (2.18) reduces to

$$\frac{1}{E} \frac{\partial E}{\partial \eta} = \frac{2(\gamma-1)}{2+K_3(\gamma^2-1)} \cdot \frac{1}{\rho_0} \frac{\partial \rho_0}{\partial \eta} \quad .. \quad .. \quad .. \quad (2.19)$$

which on integrating yields

$$E = \omega^2(\xi) \rho_0 \frac{2(\gamma-1)}{2+K_3(\gamma^2-1)} \quad .. \quad .. \quad .. \quad (2.20)$$

where $w(\xi)$ is an arbitrary positive function of ξ. Using the transformation

$$\xi^* = \int_0^{\xi} \omega(\tau)\, d\tau, \quad \eta^* = \eta$$

and dropping the star (*) we write (2.20), using (2.10) in the form

$$D^2 = f_\xi^2+g_\xi^2 = E = \rho_0 \frac{2(\gamma-1)}{2+K_3(\gamma^2-1)}. \quad .. \quad .. \quad (2.21)$$

Equation (2.5) together with (2.21) and (2.6) determine the geometric shape of the shock, the density distribution of the ambient medium being given.

We find that for $\gamma = 1$ or $\rho_0 \doteq$ constant, the shape of the shock is spherically symmetric and the general case of varying ρ_0 leads to non-spherical shock waves. This conclusion has also been arrived at in example 4 of Schindler (1965) on the assumption that the flow just behind the shock front is irrotational which follows from our treatment by putting $K_3 = 0$. However,

we may point out that the constant K_3 occurring in (2.21) cannot be chosen arbitrarily. The choice of K_3 ought to have bearing on the nature of density distribution in the ambient medium.

We make the choice as follows :

By making use of (2.21) the expression for p_1 given in (2.16) takes the form

$$p_1 = \frac{2}{\gamma+1}\rho_0 \frac{2\gamma+K_3(\gamma^2-1)}{2+K_3(\gamma^2-1)}. \quad .. \quad .. \quad .. \quad (2.22)$$

We may determine K_3 by comparing (2.22) with

$$p_1 = \frac{(\gamma-1)\lambda E'}{\Omega} \quad .. \quad .. \quad .. \quad .. \quad (2.23)$$

of Kompaneets (1960), where E' is the explosion energy, Ω the volume enclosed by the shock front and $\lambda(\gamma)$ a numerical coefficient which can be estimated from the solution to the explosion problem in a homogeneous medium and $\Omega(t)$ can be determined analytically for small time as we have done in Section 3 of this paper for a medium whose density distribution is given by a power law.

Symmetric Density Distribution with Respect to Explosion Centre

If ρ_0 is a function of ξ only, it is obvious from (2.21) that the geometric shape of the shock front $\xi =$ constant is spherically symmetric with respect to the explosion centre. Following Chernyi (1957) we assume that the entire mass of gas encompassed by the explosion wave is concentrated into a thin layer Δr just behind the shock front and the density inside the layer is constant and is equal to the density just behind the shock front given by (2.16). We assume that the density of the ambient medium is given by

$$\rho_0 = K_4 r^\delta \quad .. \quad .. \quad .. \quad .. \quad (2.24)$$

where r is the distance from the centre of explosion and K_4 is a constant.

For $\delta = 1$, the thickness of the layer Δr is determined from the conservation of mass as follows:

$$4\pi R^2 \Delta r \rho_1 = 4\pi \int_0^R r^2 . K_4 r \, dr$$

i.e.

$$\frac{\Delta r}{R} = \frac{1}{4}\frac{\gamma-1}{\gamma+1} = 0{\cdot}0326 \text{ (for } \gamma = 1{\cdot}3) \quad .. \quad .. \quad (2.25)$$

where R is the radius of the shock front.

For

$$\delta = -1, \quad \frac{\Delta r}{R} = 0{\cdot}0652 \text{ (for } \gamma = 1{\cdot}3). \quad .. \quad .. \quad (2.26)$$

For

$$\delta = 0, \quad \frac{\Delta r}{R} = 0{\cdot}0435 \text{ (for } \gamma = 1{\cdot}3), \qquad .. \qquad .. \quad (2.27)$$

which corresponds to the homogeneous medium.

A comparison of (2.25), (2.26) and (2.27) shows that the compressed layer behind the shock front in a rarer medium is thicker than that in a denser medium on account of the strong point explosion. Since the layer is assumed to be thin, the gas velocity inside it remains approximately constant and this can be taken equal to the gas velocity u_1 just behind the shock front.

For $\delta = 1$, the mass of the layer approximately equal to the mass M contained initially in the space of radius R is

$$M = 4\pi K_4 \int_0^R r^3\, dr = K_4 \pi R^4. \qquad .. \qquad .. \qquad .. \quad (2.28)$$

We have according to Newton's law for the mass M

$$\frac{d}{dt}(Mu_1) = 4\pi R^2 p_c = 4\pi R^2 \alpha p_1 \qquad .. \qquad .. \qquad .. \quad (2.29)$$

where $p_c = \alpha p_1$ = pressure at the inner side of the layer, α being a positive fraction.

Substituting for u_1 and p_1 in (2.29) from (2.16) we readily get

$$\frac{d}{dt}(R^4 \,.\, D) = 4\alpha D^2 R^3. \qquad .. \qquad .. \qquad .. \qquad .. \quad (2.30)$$

Making use of

$$\frac{d}{dt} = \frac{dR}{dt} \cdot \frac{d}{dR} = D \frac{d}{dR} \qquad .. \qquad .. \qquad .. \qquad .. \quad (2.31)$$

(2.30) reduces to

$$R \frac{dD}{dR} = -4D(1-\alpha)$$

i.e.

$$\frac{dD}{D} = -4(1-\alpha)\frac{dR}{R} \qquad .. \qquad .. \qquad .. \qquad .. \quad (2.32)$$

which on integrating yields

$$D = a_1 R^{-4(1-\alpha)} \qquad .. \qquad .. \qquad .. \qquad .. \quad (2.33)$$

where a_1 is a constant of integration. Using the principle of conservation of energy we determine the quantities a_1 and α as follows.

The internal energy concentrated in the 'cavity' bounded by the infinitesimally thin layer is given by

$$E_T = \frac{1}{\gamma - 1} \cdot \frac{4}{3} \pi R^3 p_c \qquad .. \qquad .. \qquad .. \qquad .. \quad (2.34)$$

and the kinetic energy of the gas is given by

$$E_K = \frac{M u_1^2}{2}. \qquad .. \qquad .. \qquad .. \qquad .. \quad (2.35)$$

The sum of (2.34) and (2.35) gives the explosion energy

$$E' = E_T + E_K = \frac{1}{\gamma-1} \cdot \frac{4}{3} \pi R^3 p_c + \tfrac{1}{2} M u_1^2. \quad .. \quad .. \quad (2.36)$$

Equation (2.36) with the help of (2.16) and (2.33) reduces to

$$E' = 2\pi K_4 a_1^2 \left[\frac{4\alpha}{3(\gamma^2-1)} + \frac{1}{(\gamma+1)^2} \right] R^{4-8(1-\alpha)}. \quad .. \quad .. \quad (2.37)$$

Since E' is constant, the exponent of the variable R must vanish, i.e.

$$4-8(1-\alpha) = 0$$

which gives $\alpha = \frac{1}{2}$.

It may be noted that $\alpha = \frac{1}{2}$ for homogeneous medium also.

Putting $\alpha = \frac{1}{2}$ in (2.37) we get

$$a_1 = \left[\frac{3(\gamma-1)(\gamma+1)^2}{2\pi(5\gamma-1)} \right]^{\frac{1}{2}} \left(\frac{E'}{K_4} \right)^{\frac{1}{2}}. \quad .. \quad .. \quad .. \quad (2.38)$$

Substituting $\alpha = \frac{1}{2}$ in (2.33) we get

$$D = a_1 R^{-2}. \quad .. \quad .. \quad .. \quad .. \quad (2.39)$$

For $\delta = -1$, we find that

$$D = a_1 R^{-1} \quad .. \quad .. \quad .. \quad .. \quad (2.40)$$

and for homogeneous medium of density ρ_{00}

$$D = a R^{-\frac{3}{2}} \quad .. \quad .. \quad .. \quad .. \quad (2.41)$$

where

$$a = \left[\frac{3}{4\pi} \frac{(\gamma-1)(\gamma+1)^2}{(3\gamma-1)} \right]^{\frac{1}{2}} \left(\frac{E'}{\rho_{00}} \right)^{\frac{1}{2}}. \quad .. \quad .. \quad .. \quad (2.42)$$

Comparing (2.39) and (2.40) we find that the shock front moves faster in a rarer medium as it should.

3. Determination of the Geometric Shape of the Shock Front Under Power-law Density Distribution

In this Section we proceed to determine the geometric shape of the strong shock front when the density of the ambient medium is proportional to the reciprocal of the distance from the (x, y) plane. The approach is based on the work of Kompaneets (1960) in which it is assumed that the pressure is almost constant throughout the entire volume bounded by the explosion wave and is thus constant just behind the shock front and it is given by (2.23).

As in Kompaneets (1960) we assume λ to be a constant and that the equation of the surface of the wave front is given by

$$f = f(g, t). \quad .. \quad .. \quad .. \quad .. \quad (3.1)$$

Further, we assume that the density distribution of the ambient medium is given by

$$\rho_0 = \frac{A}{g} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3.2)$$

where A is a known constant.

The normal shock front velocity D for a strong shock wave is

$$D = \left(\frac{\gamma+1}{2}\frac{p_1}{\rho_0}\right)^{\frac{1}{2}}. \quad .. \quad .. \quad .. \quad .. \quad (3.3)$$

Differentiating (3.1), we deduce that the normal shock front velocity is

$$D = -\frac{\partial f}{\partial t}\Big/\left[1+\left(\frac{\partial f}{\partial g}\right)^2\right]^{\frac{1}{2}} \quad .. \quad .. \quad .. \quad (3.4)$$

which on equating to (3.3) and making use of (2.23) yields the differential equation

$$\left(\frac{\partial f}{\partial t}\right)^2 = \frac{(\gamma^2-1)E'}{2\Omega A}\left\{1+\left(\frac{\partial f}{\partial g}\right)^2\right\} g. \quad .. \quad .. \quad .. \quad (3.5)$$

By using the transformation

$$\mu = \int_0^t \left\{\frac{\lambda E'(\gamma^2-1)}{2\Omega A}\right\}^{\frac{1}{2}} dt \quad .. \quad .. \quad .. \quad (3.6)$$

eqn. (3.5) reduces to

$$\left(\frac{\partial f}{\partial \mu}\right)^2 = g\left\{1+\left(\frac{\partial f}{\partial g}\right)^2\right\}. \quad .. \quad .. \quad .. \quad (3.7)$$

Evidently, eqn. (3.7) is of separable type and has the solution

$$f = \beta\mu + \int_{Z_0}^{g} \left(\frac{\beta^2-g}{g}\right)^{\frac{1}{2}} dg \quad .. \quad .. \quad .. \quad (3.8)$$

where $\partial f/\partial\mu = \beta =$ an arbitrary constant and Z_0 is the distance of the point of explosion from the (x, y) plane.

Differentiating (3.8) with respect to β we get

$$\frac{\partial f}{\partial \beta} = \mu + \int_{Z_0}^{g} \frac{\beta\, dg}{\{(\beta^2-g)g\}^{\frac{1}{2}}} = F(\beta). \quad .. \quad .. \quad .. \quad (3.9)$$

Following Kompaneets (1960) we set $F(\beta)$ equal to zero for small t or μ as the wave then is nearly spherical, so that

$$\mu + \int_{Z_0}^{g} \frac{\beta\, dg}{\{(\beta^2-g)g\}^{\frac{1}{2}}} = 0 \quad .. \quad .. \quad .. \quad (3.10)$$

gives

$$\mu = \sin^{-1}\left\{\frac{\beta^2-2g}{\beta^2}\right\}$$

i.e.

$$\beta^2 = \frac{2g}{1-\sin\mu}, \quad (\sin\mu+1). \qquad .. \qquad .. \qquad .. \quad (3.11)$$

Eliminating β between (3.8) and (3.11) we get

$$f+Z_0 = \left\{\frac{2g}{1-\sin\mu}\right\}^{\frac{1}{2}}\mu+\left\{\frac{1+\sin\mu}{1-\sin\mu}\right\}^{\frac{1}{2}}g \qquad .. \qquad .. \quad (3.12)$$

taking positive value of $\beta\left(=\frac{\partial f}{\partial\mu}\right)$ as f is an increasing function of μ. (3.12) determines the shape of the shock front.

The highest and lowest points g_1, g_2 of the shock front for which $f = 0$, and taking $Z_0 = 0$, are given by

$$g_1 = \frac{2\mu^2}{1+\sin\mu} \qquad .. \qquad .. \qquad .. \qquad .. \quad (3.13)$$

$$g_2 = 0. \qquad .. \qquad .. \qquad .. \qquad .. \qquad .. \quad (3.14)$$

The height g_3 at which the shock front is at a maximum distance from the z-axis is given by $\left(\frac{\partial f}{\partial g} = 0\right)$

$$g_3 = \frac{\mu^2}{2(1+\sin\mu)}. \qquad .. \qquad .. \qquad .. \qquad .. \quad (3.15)$$

Comparing (3.13) and (3.15) we find that

$$g_1 = 4g_3 \qquad .. \qquad .. \qquad .. \qquad .. \quad (3.16)$$

that is the maximum height reached by the shock front is four times the height at which its distance from the Z-axis is maximum.

The maximum distance f_3 from the Z-axis corresponding to g_3 is

$$f_3 = \frac{3}{2}\frac{\mu^2}{\cos\mu}. \qquad .. \qquad .. \qquad .. \qquad .. \quad (3.17)$$

The volume Ω enclosed by the shock front is given by

$$\Omega = \int_0^{g_1} f^2\,dg = \frac{196\pi}{15}\cdot\frac{\mu^6}{(1-\sin\mu)(1+\sin\mu)^2}. \qquad .. \qquad .. \quad (3.18)$$

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