PRIMES IN AN ANGLE OF AN ELLIPSE

by STEPHEN I. GENDLER* *and* JOYCE GENDLER*, *Department of Mathematics, Pennsylvania State University, University Park, Pennsylvania*

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An example is given to investigate the conjective that in an ellipse equal areas subtended by two radii vectors contain asymptotically equal number of prime lattice points.

Let x and y be integers. Hecke (1920) has shown that the number of primes of the form $p = F(x, y)$, with (x, y) inside a noneuclidean angle is directly proportional to the size of the angle and that equal angles have asymptotically equal numbers of primes. In the case of an ellipse this result implies that angles of an ellipse which sweep out equal areas contain asymptotically equal numbers of primes (Gendler 1969).

In the case of an ellipse

$$N = F(x, y) = x^2+2y^2 \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

the first quadrant of the ellipse was divided into two regions

$$\left.\begin{aligned} A &= x \geqslant y \\ B &= x < y \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

and the number of primes

$$p = x^2+2y^2 \leqslant N \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

inside of the regions A and B were calculated for various $N \leqslant 5000$ to yield quantities $P_A(N)$ and $P_B(N)$. The ratio

$$R_N = \frac{P_A(N)}{P_B(N)} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

could then be compared with the ratio R:

$$R = \frac{\text{area } B}{\text{area } A} \cong 1{\cdot}57 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

to determine how closely R_N approaches the asymptotic value R.

Noting that only primes $p \equiv 1 \pmod{8}$ and $p \equiv 3 \pmod{8}$ are expressible in the form

$$p = F(x, y) = x^2+2y^2 \qquad \ldots \quad \ldots \quad \ldots \quad (6)$$

* *Change of address*: **Dept. of Mathematics, Clarion State College, Clarion, Pennsylvania.**

and that they are expressible uniquely, one may form a table of p, x and y. Between 3000 and 3100, for instance, one obtains:

No.	x	y	$x > y$?
3001	43	24	*
3011	33	31	*
3019	29	33	–
3041	27	34	–
3049	49	18	*
3067	5	39	–
3083	45	23	*
3089	39	28	*

One finds that $R_{500} = 1{\cdot}63$ and that for $500 \leqslant N \leqslant 5000$

$$1{\cdot}45 < R_N < 1{\cdot}64$$

with

$$1{\cdot}55 < R_N < 1{\cdot}64$$

for $N > 2000$. $R_{4457} = 1{\cdot}59$, approximately $1{\cdot}01$ of the expected value $R = 1{\cdot}57$. In view of the fact that the ratio of the primes $p \equiv 1 \pmod{8}$ to the number of primes $p \equiv 3 \pmod{8}$ is $R^* = 0{\cdot}98$, it appears that the convergence of the ratio R_N is comparable.

It is proposed to continue this study for larger N using digital computer techniques.

References

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Hecke, E. (1920). Eine Neue Art von Zetafunktion. *Math. Z.*, 11–51.