ON GENERALIZED MHD COUETTE FLOW WITH HEAT TRANSFER BETWEEN CONDUCTING WALLS

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The fully developed generalized MHD Couette flow under transverse magnetic field has been studied between two finitely electrically conducting plates. Closed form solutions for the velocity and temperature field are obtained. It is observed that the separation at the stationary plate may be avoided by choosing the upper plate of large electrical conductance ratio compared to that of the lower plate. In the case of walls of equal electrical conductance ratios, at large M, there occurs a transition from laminar to turbulent flow. For finite value of ϕ_u, ϕ_l the viscous drag, mass flow rate, the force to move the upper plate are not affected by the pressure-induced flow superimposed on the simple shear flow when M is large (greater than 6).

Notations

C_2, C_3 = as defined on page 618
c_p = specific heat at constant pressure
D = as defined on page 612
D_1 = as defined on page 612
E = Eckert number
$H \equiv (H_x, H_y = H_0, 0)$ = magnetic field vector
h_0, h_1 = thickness of the lower and upper plates respectively
J = current density
K = thermal conductivity
L = separation between the plates
M = Hartmann number
N_u = Nusselt number
p = pressure
P = dimensionless pressure
P_1 = as defined on page 611
P_r = Prandtl number
Q = mass flow rate
$Q/\rho UL$ = dimensionless mass flow rate
R = Reynolds number
R_m = magnetic Reynolds number

T = temperature
T_1 = temperature of lower plate
T_2 = temperature of upper plate
U_1 = velocity component
U_0 = velocity of the upper plate
U = dimensionless velocity
$\bar{U}$ = velocity vector
x = coordinate in the direction of flow
y = coordinate normal to flow direction
μ = viscosity
μ_c = magnetic permeability
σ = electrical conductivity of the fluid
η = dimensionless distance normal to the flow direction
ϕ = electrical conductance ratio
ξ = dimensionless distance in the direction of flow
τ_v = viscous drag
θ = dimensionless temperature

Subscripts

u = upper plate
l = lower plate
w = wall

1. Introduction

One of the earliest exact solutions of the Navier-Stokes equations for one-dimensional flow is described in every book on hydrodynamics and is known as Couette flow. Schlichting (1960) discussed this problem in the case of the generalized Couette flow in which the pressure-induced flow is supposed to be superimposed on simple shear flow. It was observed that, when pressure decreases in the direction of the flow, the velocity is positive over the whole width of the channel, whereas in the case of pressure increasing in the direction of flow the velocity may become negative, i.e. the back flow may occur near the stationary wall.

In recent years, the plane MHD Couette flow has been discussed by Pai (1962) for incompressible fluids between non-conducting walls. Lehnert (1952) has also studied this problem without taking into consideration the electric field in the fluid. All these studies were confined to the plane Couette flow between non-conducting walls. The generalized MHD Couette flow was attempted by Agarwal (1965) for velocity field. Agarwal analysed this problem by giving the conditions under which the back flow at the stationary wall may be avoided. A detailed investigation of this problem was carried out by Soundalgekar (1967) by considering the effects of both the electric and the magnetic field on the flow and temperature field. Soundalgekar (1966) also discussed the temperature field problem in the case of the generalized Couette flow under the action of transverse magnetic field and observed that the rate of heat transfer at the stationary plate is affected by the magnetic field.

Recently, Chang and Yen (1962) and Yen and Chang (1964) discussed the effects of electrically conducting walls on the velocity field in the channel and the Couette flow respectively. The heat transfer aspect of the channel flow between conducting walls was presented by Yen (1963) and that of the Couette flow by Soundalgekar (1969*a*). Soundalgekar (1969*b*) also discussed the effects of the crossed fields on the heat transfer in a conducting wall channel flow. To the knowledge of the authors the problem of the generalized MHD Couette flow with heat transfer between conducting walls has not been attempted. What is the effect of the electrically conducting walls on the back flow at the stationary plate? This indeed is the motivation for the present investigation.

In section 2, the problem is posed under proper boundary conditions and the closed form solutions are obtained for velocity, current density, skin friction, the velocity gradient at the upper plate, the rate of mass flow, temperature and the Nusselt number. The asymptotic forms for the skin friction, the velocity gradient at the upper plate, the rate of mass flow, for large ϕ_u, ϕ_l and M are presented. The velocity profiles, magnetic field and current density are shown graphically, whereas the numerical values of the others are entered in tables. In section 3, important features of the flow are discussed.

2. Mathematical Analysis

Velocity Field

Here a fully developed generalized MHD Couette flow is considered between two electrically conducting plates. The x-axis is chosen along the lower stationary plate in the direction of the flow and y-axis is chosen normal to it. The transverse magnetic field of constant strength H_0 is supposed to be applied in a direction parallel to the y-axis. Then the velocity field and magnetic field components are given for one-dimensional flow as follows:

$$\overline{V} \equiv (U_1, 0, 0), \quad \overline{H} \equiv (H_x, H_0, 0). \qquad \text{(1)}$$

In view of (1), the following are the governing equations obtained from Maxwell's equations, modified Navier-Stokes equations and Ohm's law in Gaussian system of units:

$$\mu \frac{d^2U_1}{dy^2} + \frac{\mu_e H_0}{4\pi} \frac{dH_x}{dy} = \frac{\partial p}{\partial x} \qquad \text{(2)}$$

$$H_0 \frac{dU_1}{dy} + \frac{1}{4\pi\mu_e\sigma} \frac{d^2H_x}{dy^2} = 0. \qquad \text{(3)}$$

The boundary conditions are

$$\left.\begin{aligned} U_1 &= 0 \quad \text{at } y = 0 \\ U_1 &= U_0 \quad \text{at } y = L \end{aligned}\right\} \qquad \ldots \quad (4)$$

and following Shercliff (1956),

$$\left.\begin{aligned} \frac{dH_x}{dy} - \frac{\sigma}{\sigma_w h_0} H_x &= 0 \quad \text{at } y = 0 \\ \frac{dH_x}{dy} + \frac{\sigma}{\sigma_w h_1} H_x &= 0 \quad \text{at } y = L \end{aligned}\right\} \qquad \ldots \quad (5)$$

Equations (2)–(5) reduce to the following non-dimensional form:

$$R_m \frac{d^2U}{d\eta^2} + M^2 \frac{dU}{d\eta} = P_1 \qquad \ldots \quad (6)$$

$$R_m \frac{dU}{d\eta} + \frac{d^2H}{d\eta^2} = 0 \qquad \ldots \quad (7)$$

$$U = 0, 1 \quad \text{at } \eta = 0, 1 \qquad \ldots \quad (8)$$

and

$$\left.\begin{aligned} \frac{dH}{d\eta} - \frac{1}{\phi_l} H &= 0 \quad \text{at } \eta = 0 \\ \frac{dH}{d\eta} + \frac{1}{\phi_u} H &= 0 \quad \text{at } \eta = 1 \end{aligned}\right\} \qquad \ldots \quad (9)$$

where

$$\left.\begin{aligned} &\xi = x/L, \quad \eta = y/L, \quad H = H_x/H_0 \\ &U = U_1/U_0, \quad P = p/\rho U_0^2, \quad P_1 = RR_M \frac{dP}{d\xi} \\ &R_m = 4\pi\mu_c \sigma L U_0, \quad M = \mu_c H_0 L(\sigma/\mu)^{1/2} \\ &\frac{\sigma L}{\sigma_w h_0} = \frac{1}{\phi_l}, \quad \frac{\sigma L}{\sigma_w h_1} = \frac{1}{\phi_u}. \end{aligned}\right\} \qquad \ldots \quad (10)$$

Here ϕ_u and ϕ_l are the electrical conductance ratios for the upper and the lower plates with thickness h_1 and h_0 respectively. The solutions of eqns. (6) and (7) in virtue of eqns. (8) and (9) are

$$U = \frac{\left\{ \begin{aligned} &[1+\cosh M(1-\eta) - \cosh M\eta - \cosh M] + M[\phi_u \sinh M(1-\eta) - \phi_l \sinh M\eta \\ &-\phi_u \sinh M] + \frac{P_1}{MR_M}(\phi_u+\phi_l+1)[\sinh M - \sinh M\eta - \sinh M(1-\eta)] \end{aligned} \right\}}{2(1-\cosh M) - M(\phi_u+\phi_l)\sinh M} \qquad \ldots \quad (11)$$

$$\begin{aligned} H = \frac{P_1}{2M^2 D}\{&(1-\cosh M)(2\eta - 1 + \phi_l - \phi_u) - [(\phi_u+\phi_l)\eta - 1]M \sinh M \\ &-2(\phi_u+\phi_l+1)[\cosh M(1-\eta) - \cosh M]\} + \frac{R_M}{2MD}\{2[\sinh M(1-\eta) \\ &+\sinh M\eta - \sinh M] + 2M[\phi_u \cosh M(1-\eta) + \phi_l(\cosh M\eta - \cosh M) - \phi_u] \\ &+2M\phi_l(\phi_l - \phi_u)\sinh M\} \end{aligned} \qquad \ldots \quad (12)$$

where $D = 2(1-\cosh M)-M(\phi_u+\phi_l)\sinh M$.

From eqns. (11) and (12), it is evident that the velocity and the magnetic field are affected by the individual electrical conductance ratio of each plate.

When $\phi_u = \phi_l = \phi$, we have

$$H_\phi = \frac{P_1}{2MD_1}\{(2\eta-1)(1-\cosh M-\phi M\sinh M)-2(2\phi+1)[\cosh M(1-\eta)-\cosh M]\}$$
$$+\frac{R_m}{MD_1}\{\sinh M(1-\eta)+\sinh M\eta-\sinh M$$
$$+M\phi[\cosh M(1-\eta)+\cosh M\eta-\cosh M-1]\} \quad \ldots \quad (13)$$

where $D_1 = 2(1-\cosh M)-2M\phi\sinh M$.

U and H are plotted in Figs. 1–8 under different conditions.

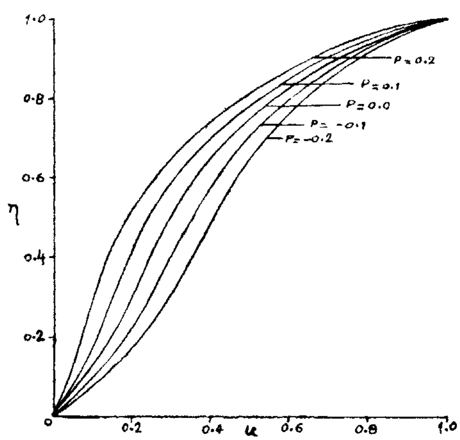

FIG. 1. Velocity profiles, $\phi_l > \phi_u$ ($M = 4$, $\phi_l = 2$, $\phi_u = 0{\cdot}4$).

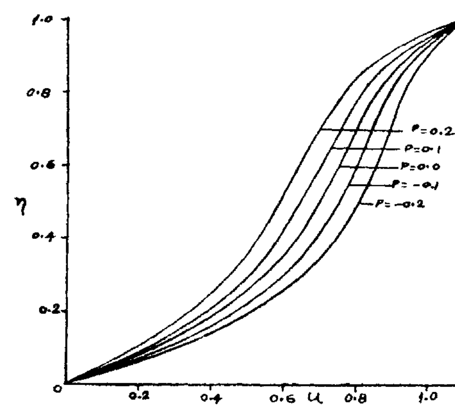

FIG. 2. Velocity profiles, $\phi_l < \phi_u$ ($M = 4$, $\phi_l = 0{\cdot}4$, $\phi_u = 2$).

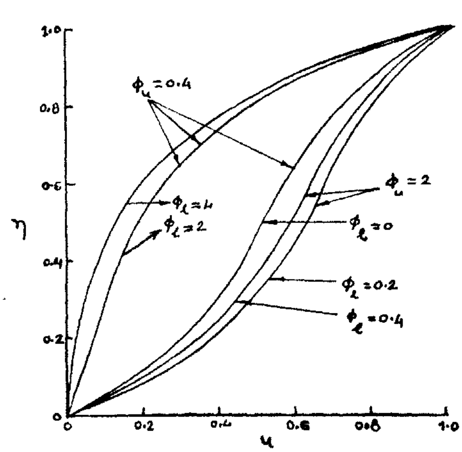

FIG. 3. Velocity profiles (variation of ϕ_l, $\phi_u = 0{\cdot}4$, $M = 4$, $P = 0{\cdot}2$).

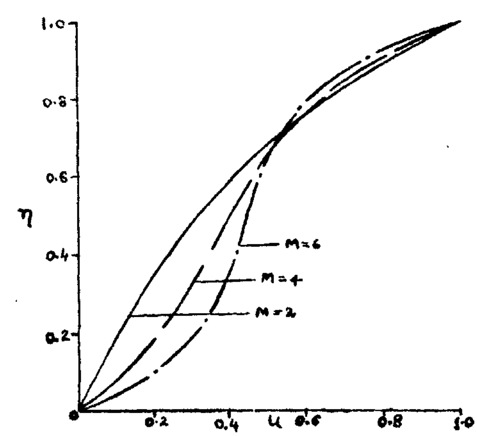

FIG. 4. Velocity profiles (variation of ϕ_u, $\phi_l = 0{\cdot}4$, $M = 4$, $P = 0{\cdot}2$).

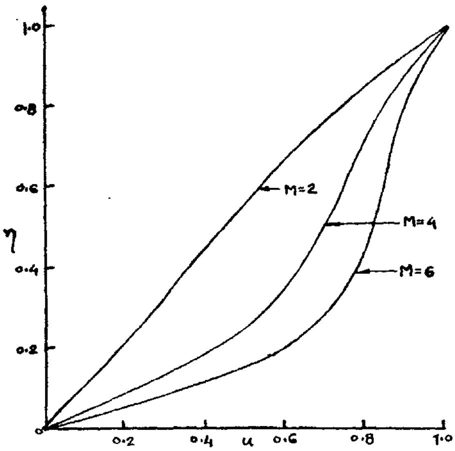

FIG. 5. Velocity profiles, $\phi_l < \phi_u$ ($\phi_l = 0{\cdot}2$, $\phi_u = 4$, $P = 0{\cdot}2$).

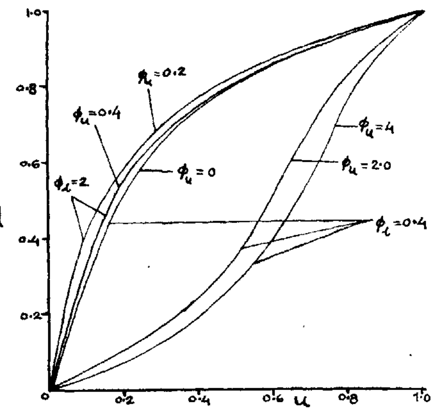

FIG. 6. Velocity profiles ($\phi_l = \phi_u = 2$, $P = 0{\cdot}2$).

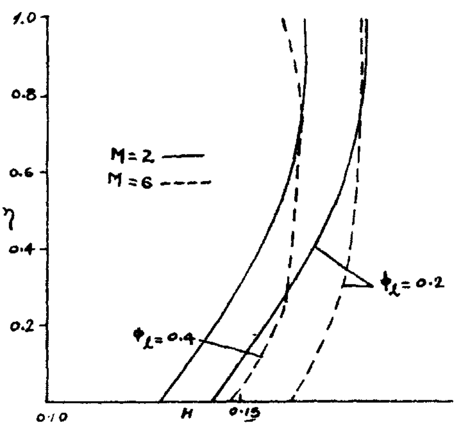

FIG. 7. Magnetic field lines, $\phi_l < \phi_u$ ($\phi_u = 4$, $P = 0{\cdot}2$).

The current density is given by

$$J = -\frac{1}{R_M}\frac{dH}{d\eta} \qquad (14)$$

J is shown in Figs. 9–11 under different conditions.

The viscous drag coefficient at the lower plate is given by

$$R\,.\,\tau_v = \left(\frac{du}{d\eta}\right)_{\eta=0}$$

$$= \frac{\frac{P_1}{R_m}(\cosh M-1)(\phi_u+\phi_l+1)-M(\sinh M+M\phi_u\cosh M+M\phi_l)}{2(1-\cosh M)-M(\phi_u+\phi_l)\sinh M}. \qquad (15)$$

The magnetic drag coefficient can be obtained.

The numerical values of $R\tau_v$ are given in Table I under different conditions.

From eqn. (15) we have the following limiting values of $R\tau_v$:

(i) for $\phi_l \to \infty$

$$R\,.\,\tau_v \approx \frac{P_1}{MR_M}(\operatorname{cosech} M - \coth M) + M \operatorname{cosech} M \qquad \ldots \quad \ldots \quad (16)$$

(ii) for $\phi_u \to \infty$

$$R\,.\,\tau_v \approx \frac{P_1}{MR_m}(\operatorname{cosech} M - \coth M) + M \coth M \qquad \ldots \quad \ldots \quad (17)$$

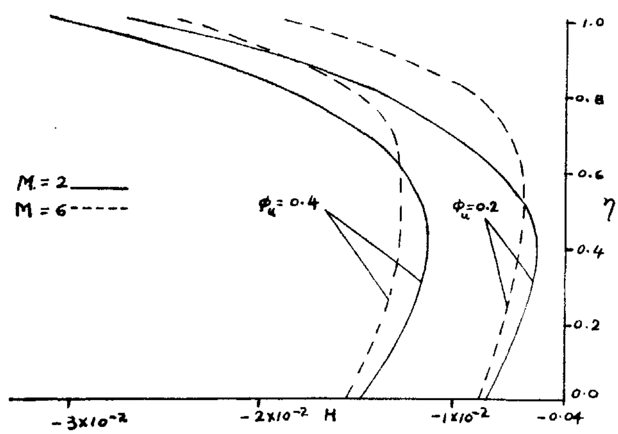

FIG. 8. Magnetic field lines, $\phi_l > \phi_u$ ($\phi_l = 4$, $P = 0{\cdot}2$).

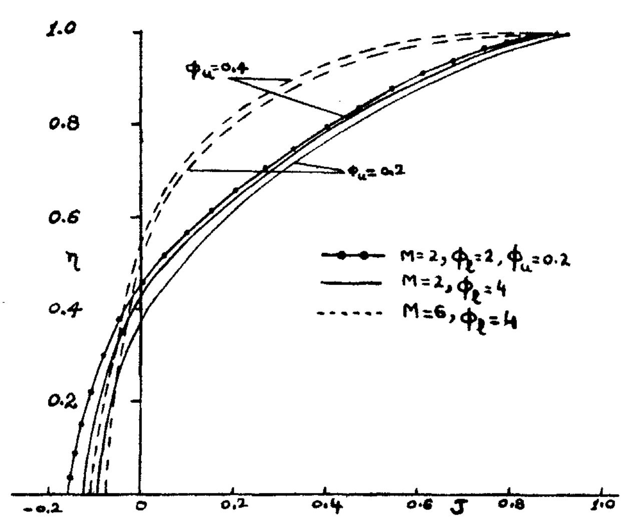

FIG. 9. Current density, $\phi_l > \phi_u$ ($P = 0{\cdot}2$).

(iii) for large M

$$R\,.\,\tau_v \approx \frac{M(1+M\phi_u)-\dfrac{P_1}{R_M}(\phi_u+\phi_l+1)}{M(\phi_u+\phi_l)+2}+O(1/M^3) \qquad (18)$$

$$\approx \frac{1+M\phi_u}{\phi_u+\phi_l}+O(1/M^2) \qquad (18a)$$

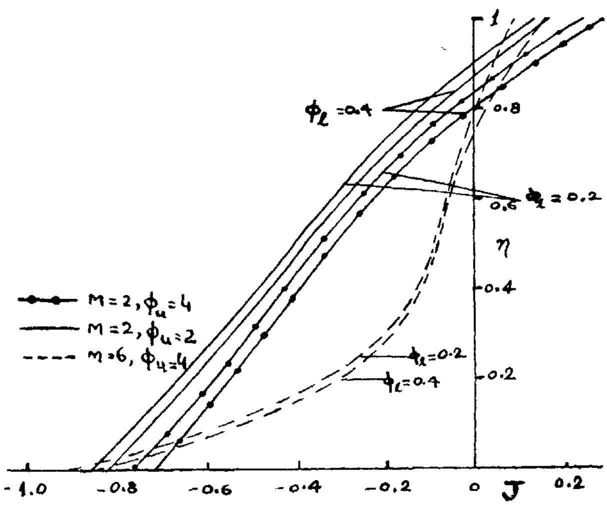

FIG. 10. Current density, $\phi_l \leqslant \phi_u$ ($P = 0{\cdot}2$).

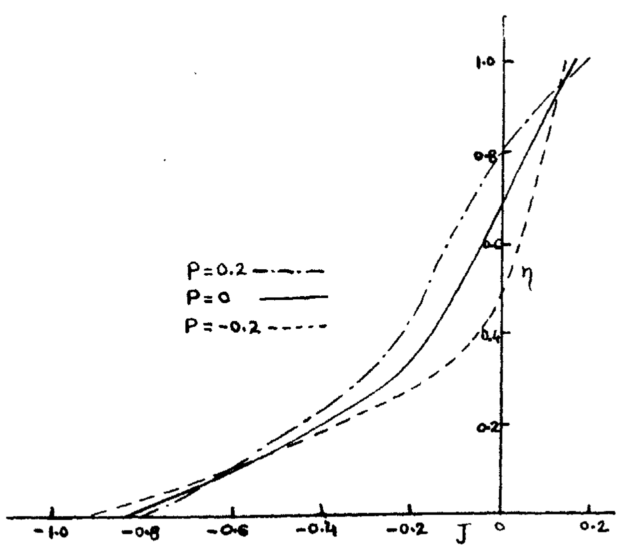

FIG. 11. Current density, $\phi_l < \phi_u$ ($\phi_l = 0{\cdot}2$, $\phi_u = 2{\cdot}0$, $M = 4$).

TABLE I

Values of $R\tau_v$

ϕ_l	ϕ_u \ M	2	4	6
		$\phi_u > \phi_l$, $P_1 = 0{\cdot}2$		
	0·0	0·313	1·075	2·015
0	2·0	1·037	3·046	5·149
	4·0	1·153	3·258	5·389
0·2	2·0	0·953	2·793	4·718
	4·0	1·098	3·104	5·137
0·4	2·0	0·879	2·576	4·351
	4·0	1·047	2·963	4·908
		$\phi_u = \phi_l$, $P_1 = 0{\cdot}2$		
0·0	0·0	0·313	1·075	2·015
1·0	1·0	0·486	1·492	2·588
2·0	2·0	0·513	1·537	2·632
3·0	3·0	0·525	1·554	2·648
		$\phi_u < \phi_l$, $P_1 = 0{\cdot}2$		
2·0	0·0	−0·066	−0·062	0·028
	0·2	0·027	0·206	0·473
	0·4	0·108	0·436	0·854
4·0	0·0	−0·126	−0·134	−0·124
	0·2	−0·068	−0·026	0·131
	0·4	−0·015	0·120	0·365

$M = 4$

ϕ_l	ϕ_u \ P_1	−0·2	−0·1	0·0	0·1	0·2
				$M = 4$		
0·0	0·0	3·075	2·574	2·075	1·575	1·075
0·2	2·0	3·944	3·658	3·369	3·081	2·794
2·0	2·0	2·612	2·343	2·075	1·806	1·537
2·0	0·2	1·356	1·068	0·781	0·493	0·206

The velocity gradient at the upper plate is given by

$$\left(\frac{du}{d\eta}\right)_{\eta=1} = \frac{\dfrac{P_1}{R_m}(\cosh M-1)(\phi_u+\phi_l+1)-\eta(\sinh M+M\phi_u+M\phi_l\cosh M)}{2\,(\cosh M-1)+M(\phi_u+\phi_l)\sinh M}. \tag{19}$$

The numerical values of $\left(\frac{du}{d\eta}\right)_{\eta=1}$ are given in Table II,

TABLE II

Values of $\left(\frac{du}{d\eta}\right)_{\eta=1}$

ϕ_l	ϕ_u \ M	2	4	6
		$\phi_u > \phi_l$, $P_1 = 0{\cdot}2$		
	0·0	2·313	3·074	4·015
0·0	2·0	1·589	1·104	0·881
	4·0	1·473	0·892	0·641
0·2	2·0	1·673	1·356	1·312
	4·0	1·528	1·045	0·892
0·4	2·0	1·747	1·573	1·679
	4·0	1·579	1·186	1·122
		$\phi_l = \phi_u$, $P_1 = 0{\cdot}2$		
1·0	1·0	2·140	2·657	3·442
2·0	2·0	2·113	2·612	3·398
3·0	3·0	2·101	2·595	3·382
		$\phi_u < \phi_l$, $P_1 = 0{\cdot}2$		
	0·0	2·691	4·211	6·002
2·0	0·2	2·599	3·944	5·577
	0·4	2·577	3·714	5·176
	0·0	2·753	4·333	6·154
4·0	0·2	2·694	4·175	5·899
	0·4	2·641	4·029	5·665

$M = 4$

ϕ_l	ϕ_u \ P_1	$-0{\cdot}2$	0·0	0·2
0·2	2·0	0·201	0·781	1·356
2·0	2·0	1·537	2·074	2·612
2·0	0·2	2·794	3·369	3·944

The limiting values of $\left(\frac{du}{d\eta}\right)_{\eta=1}$ are given below:

(i) for $\phi_l \to \infty$

$$\left(\frac{du}{d\eta}\right)_{\eta=1} \approx \frac{P_1}{MR_M}(\coth M - \operatorname{cosech} M) + M \coth M \quad \ldots \quad \ldots \quad (20)$$

(ii) for $\phi_u \to \infty$

$$\left(\frac{du}{d\eta}\right)_{\eta=1} \approx \frac{P_1}{MR_m}(\coth M - \operatorname{cosech} M) + M \operatorname{cosech} M \quad \ldots \quad (21)$$

(iii) for large M

$$\left(\frac{du}{d\eta}\right)_{\eta=1} \approx \frac{M(1+M\phi_l)+\frac{P_1}{R_M}(\phi_u+\phi_l+1)}{M(\phi_u+\phi_l)+2}+O(1/M^3) \quad (22)$$

$$\approx \frac{1+M\phi_l}{\phi_u+\phi_l}+O(1/M^2). \quad (22a)$$

The mass flow rate is given by

$$Q/\rho UL = \int_0^1 u\, d\eta \quad (23)$$

$$Q/\rho UL = \frac{C_3(M-\sinh M)-C_2(\cosh M-1)}{R_m} \quad (24)$$

where

$$C_2 = \frac{\frac{R_m}{M}(\sinh M+M\phi_u \cosh M+M\phi_l)-\frac{P_1}{M^2}(\cosh M-1)(\phi_u+\phi_l+1)}{2(1-\cosh M)-M(\phi_u+\phi_l)\sinh M}$$

$$C_3 = \frac{\frac{P_1}{M^2}(\phi_u+\phi_l+1)\sinh M-\frac{R_m}{M}(\cosh M-1+M\phi_u \sinh M)}{2(1-\cosh M)-M(\phi_u+\phi_l)\sinh M}.$$

The numerical values of $Q/\rho UL$ are given in Table III.
The limiting values of $Q/\rho UL$ are as follows:

(i) For $\phi_l \to \infty$

$$Q/\rho UL \approx \frac{\coth M-\operatorname{cosech} M}{M}-\frac{P_1}{M^3R_m}[M+2(\operatorname{cosech} M-\coth M)] \quad (25)$$

(ii) For $\phi_u \to \infty$

$$Q/\rho UL \approx \frac{\coth M-\operatorname{cosech} M}{M}-\frac{P_1}{M^3R_m}[M+2(\operatorname{cosech} M-\coth M)]-1 \quad (25a)$$

(iii) For large M

$$Q/\rho UL \approx \frac{M\phi_u+(\phi_l-\phi_u-1)-\frac{P_1}{MR_m}(\phi_u+\phi_l+1)}{M(\phi_u+\phi_l)+2}+O\left(\frac{1}{M^3}\right) \quad (26)$$

$$\approx \frac{M\phi_u+\phi_l-\phi_u-1}{M(\phi_u+\phi_l)+2}+O\left(\frac{1}{M^2}\right). \quad (26a)$$

Integration of Energy Equation

For fully developed one-dimensional flow, the energy equation in non-dimensional form is given by

$$\frac{d^2\theta}{d\eta^2}+P_rE\left[\left(\frac{du}{d\eta}\right)^2+M^2J^2\right]=0 \quad (27)$$

TABLE III

Values of $Q/\rho UL$

ϕ_l	$M \backslash \phi_u$	2	4	6
		$\phi_u > \phi_l$, $P_1 = 0{\cdot}2$		
0·0	0·0	0·343	0·366	0·388
	2·0	0·457	0·630	0·739
	4·0	0·475	0·659	0·766
0·2	2·0	0·444	0·597	0·691
	4·0	0·466	0·638	0·738
0·4	2·0	0·432	0·567	0·650
	4·0	0·458	0·619	0·712
		$\phi_u = \phi_l$, $P_1 = 0{\cdot}2$		
2·0	0·0	0·284	0·213	0·166
	0·2	0·299	0·249	0·215
	0·4	0·311	0·280	0·258
4·0	0·0	0·275	0·197	0·149
	0·2	0·284	0·218	0·177
	0·4	0·292	0·237	0·203

ϕ_l	$\phi_u \backslash P_1$	−0·2	0·0	0·2
		$M = 4$		
0·2	2·0	0·751	0·674	0·597
2·0	2·0	0·572	0·500	0·428
2·0	0·2	0·403	0·326	0·249

where

$$\theta = \frac{T-T_1}{T_2-T_1}$$

$P_r = \mu C_p/K$, Prandtl number

$E = U_0^2/C_p(T_2-T_1)$, Eckert number

T_1, T_2 are the temperatures of the lower and upper plates respectively. The corresponding boundary conditions on θ are

$$\theta = 0,\ 1 \quad \text{at } \eta = 0,\ 1. \qquad \qquad (28)$$

Substituting for U and J from eqns. (11) and (14) in eqn. (27), simplifying and solving under the boundary conditions (28), we get

$$\theta = \eta + P_r \,.\, E \,.\left(\frac{M}{R_m}\right)^2 \left[\frac{C_2^2+C_3^2}{4}\{\eta\,(\cosh 2M-1)-\cosh 2M\eta+1\}\right.$$
$$+\frac{2P_1}{M^3}C_3\{\eta\,(\cosh M-1)-\cosh M\eta+1\}+C_2(\eta\sinh M-\sinh M\eta)$$
$$\left.+\frac{C_2C_3}{2}(\eta\sinh 2M-\sinh 2M\eta)+\frac{P_1^2}{2M^4}(\eta-\eta^2)\right] \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

where C_2, C_3 are as defined above.

The numerical values of θ are given in Table IV.

TABLE IV

Values of θ

		M = 2				M = 6			
ϕ_l	ϕ_u \ Y	0·0	0·4	0·8	1·0	0·0	0·4	0·8	1·0
				$\phi_u > \phi_l$, $P_1 = 0{\cdot}2$					
	0·0	0·0	0·4126	0·8136	1·0	0·0	0·4127	0·8149	1·0
0·0	2·0	0·0	0·4196	0·8121	1·0	0·0	0·4278	0·8100	1·0
	4·0	0·0	0·4219	0·8127	1·0	0·0	0·4300	0·8107	1·0
0·4	2·0	0·0	0·4170	0·8116	1·0	0·0	0·4213	0·8239	1·0
	4·0	0·0	0·4198	0·8121	1·0	0·0	0·4257	0·8095	1·0
				$\phi_u = \phi_l$, $P_1 = 0{\cdot}2$					
0·0	0·0	0·0	0·4126	0·8136	1·0	0·0	0·4127	0·8149	1·0
1·0	1·0	0·0	0·4131	0·8124	1·0	0·0	0·4134	0·8121	1·0
2·0	2·0	0·0	0·4132	0·8122	1·0	0·0	0·4135	0·8119	1·0
3·0	3·0	0·0	0·4133	0·8122	1·0	0·0	0·4135	0·8119	1·0
				$\phi_u < \phi_l$, $P_1 = 0{\cdot}2$					
2·0	0·0	0·0	0·4140	0·8181	1·0	0·0	0·4172	0·8308	1·0
	0·4	0·0	0·4129	0·8157	1·0	0·0	0·4140	0·8230	1·0
4·0	0·0	0·0	0·4146	0·8190	1·0	0·0	0·4180	0·8325	1·0
	0·4	0·0	0·4136	0·8173	1·0	0·0	0·4156	0·8274	1·0

ϕ_l	ϕ_u	p_1 \ y	0·0	0·4	0·8	1·0
			M = 4			
		−0·2	0·0	0·4204	0·8076	1·0
0·2	2·0	0·0	0·0	0·4203	0·8080	1·0
		0·2	0·0	0·4229	0·8100	1·0
		−0·2	0·0	0·4138	0·8108	1·0
2·0	2·0	0·0	0·0	0·4123	0·8103	1·0
		0·2	0·0	0·4137	0·8118	1·0
		−0·2	0·0	0·4176	0·8221	1·0
2·0	0·2	0·0	0·0	0·4147	0·8214	1·0
		0·2	0·0	0·4145	0·8225	1·0

The rate of heat transfer expressed in terms of the Nusselt number is

$$Nu = \frac{L}{T_2-T_1}\left(\frac{dT}{dy}\right)_{y=0}$$
$$= \left(\frac{d\theta}{d\eta}\right)_{\eta=0}. \quad \ldots \quad (30)$$

Hence, from eqns. (29) and (30) we have

$$Nu = 1+P_rE\left(\frac{M}{R_m}\right)^2\left[\frac{C_2^2+C_3^2}{4}(\cosh 2M-1)+\frac{2P_1}{M^3}\{C_3(\cosh M-1) + C_2(\sinh M-M)\}+\frac{C_2C_3}{2}(\sinh 2M-2M)+\frac{P_1^2}{2M^4}\right]. \quad \ldots \quad (31)$$

The numerical values of Nu are given in Table V.

TABLE V

Values of Nu

ϕ_l	ϕ_u \ M	2	4	6
	$\phi_u > \phi_l$, $P_1 = 0{\cdot}2$			
0·0	0·0	1·042	1·066	1·100
	2·0	1·105	1·267	1·457
	4·0	1·121	1·301	1·498
0·2	2·0	1·094	1·230	1·389
	4·0	1·113	1·276	1·456
0·4	2·0	1·085	1·201	1·335
	4·0	1·106	1·254	1·418
	$\phi_u = \phi_l$, $P_1 = 0{\cdot}2$			
0·0	0·0	1·042	1·066	1·100
0·2	0·2	1·046	1·080	1·125
2·0	2·0	1·053	1·095	1·145
4·0	4·0	1·054	1·097	1·147
	$\phi_u < \phi_l$, $P_1 = 0{\cdot}2$			
2·0	0·0	1·037	1·042	1·045
	0·2	1·036	1·041	1·046
	0·4	1·037	1·044	1·052
4·0	0·0	1·037	1·043	1·046
	0·2	1·037	1·041	1·044
	0·4	1·036	1·041	1·045

ϕ_l	ϕ_u \ P_1	−0·2	0·0	0·2
	$M = 4$			
0·2	2·0	1·279	1·248	1·230
2·0	2·0	1·125	1·104	1·095
2·0	0·2	1·065	1·046	1·041

3. Discussion

In order to study the physical aspect of the problem, numerical calculations are carried out for $M = 2, 4, 6$; $P_1 = -0{\cdot}2, -0{\cdot}1, 0, 0{\cdot}1, 0{\cdot}2$; $\phi_u \gtreqless \phi_l$. Throughout the analysis, we have assumed the value of the magnetic Reynolds number as 0·1. On laboratory scale, this is a valid approximation. In order to study the separation at the stationary plate both positive and negative values of P_1 are considered, $P_1 > 0$ indicating the pressure increasing in the direction of the flow and $P_1 < 0$ the same decreasing in the direction of flow. It was observed by Soundalgekar (1967) that in the case of non-conducting walls an increase in P_1 (> 0) causes the back flow at the stationary wall and it can be avoided by the application of an electric field in a direction normal to the direction of flow and the magnetic field. We have the following conclusions from Figs. 1 to 6:

1. In Figs. 1 and 2, the velocity is observed to be positive for all values of P. Hence the back flow at the stationary plate may be avoided if non-conducting walls are replaced by electrically conducting walls.

2. From Figs. 3 and 4, it is evident that to avoid back flow, the electrical conductance ratio of the upper wall ϕ_u should be increased and that of the lower wall ϕ_l kept constant.

3. For $\phi_u = \phi_l$ the inflection points are observed along the velocity profiles at large values of M. It has been confirmed experimentally by Murgatroyd (1953) that a magnetic field exerts a stabilizing influence in the flow of electrically conducting fluids between non-conducting parallel walls. But from Fig. 5 one can conclude that when $\phi_u = \phi_l$, there may occur a transition from laminar to turbulent flow at large M.

Skin Friction (Table I).

(*a*) $\phi_u > \phi_l$

1. Viscous drag increases due to the conducting walls.
2. An increase in M leads to an increase in $R\tau_v$.
3. For the same ϕ_l, an increase in ϕ_u leads to an increase in $R\tau_v$, but an increase in ϕ_l leads to a decrease in $R\tau_v$ when ϕ_u is constant.

(*b*) $\phi_u < \phi_l$

1. For non-zero ϕ_l, $R\tau_v$ is negative for small values of M when $\phi_u = 0$ and $P = 0{\cdot}2$, which shows that separation may occur at small values of M if the upper wall is non-conducting. This separation may be avoided by choosing M large. This is also evident from eqn. (18*a*), which shows that at large M the skin friction at the stationary plate is not affected by the pressure-induced flow. However, as ϕ_l increases, ϕ_u also must be increased to avoid back flow at the stationary wall even at small M.

2. An increase in P_1 leads to a decrease in R_{τ_v} when M is constant. In order to get a good physical picture of the skin friction for large ϕ_u, ϕ_l and M, eqn. (15) is expanded under different conditions. The expressions (18) and (18*a*) are particularly interesting. Values of R_{τ_v}, for $M = 6$, obtained from eqns. (15) and (18), are found to differ by a negligible fraction. Hence, the skin friction is affected by the pressure-induced flow only for values of M up to 6.

3. From eqns. (16) and (17) we can conclude that, if one of the plates is perfectly conducting, the skin friction at the stationary plate is not affected by the conductivity of the other plate. This is true also for the velocity gradient at the upper plate and the rate of mass flow.

Velocity Gradient at the Upper Plate (Table II)

1. From Table II one can see that the values of $\left(\frac{du}{d\eta}\right)_{\eta=1}$ in the case of conducting walls are less than those in the case of non-conducting walls. This leads to the conclusion that the force to move the upper plate is less if the plates are electrically conducting.

2. In the case of non-conducting walls, an increase in M leads to an increase in the force to move the upper wall whereas, in the case of conducting walls, it decreases with increasing M.

3. For all values of ϕ_l, the force to move the upper wall is less as ϕ_u increases.

4. For the same value of ϕ_u, an increase in ϕ_l leads to an increase in the force to move the upper plate.

5. The force to move the upper plate is more in the case of $P_1 > 0$ than that for $P_1 < 0$.

Rate of Mass Flow (Table III)

1. The flow rate in the case of non-conducting walls is less than that in the case of conducting walls. It increases with increasing M. When ϕ_l is constant, an increase in ϕ_u leads to an increase in the flow rate. However, for constant ϕ_u and M, it decreases with increasing ϕ_l.

2. $Q/\rho UL$ is greater when $\phi_l < \phi_u$ than that in the case of $\phi_l > \phi_u$ for the same M.

Nusselt Number (Table V)

1. Nu increases with increasing M.

2. The values of Nu are found to be greater in the case of $\phi_u > \phi_l$ than those for $\phi_u < \phi_l$.

3. With ϕ_l constant, an increase in ϕ_u leads to an increase in Nu and vice versa.

4. The values of Nu are greater when $P_1 < 0$ than those in the case of $P_1 > 0$.

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