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TWO LAYERED MAGNETOHYDRODYNAMIC FLOW THROUGH PARALLEL PLATES WITH APPLICATIONS

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A two-layered magnetohydrodynamic model for parallel plate hemodialysers, under the influence of a uniform transverse magnetic field has been investigated. Analytical expressions for velocity profiles in the core region and peripheral plasma layer alongwith flow rate, effective viscosity and effective Reynolds number have been obtained. Some interesting limitng cases have also been discussed. It may be noticed that the flow of suspensions has the tendency to leave a particle free layer near the wall, thereby decreasing the effective viscosity. This may be used in reducing the dialysis time. Because of the movement of red cells towards the centre, plasma exposure area to the membrane increases. Further, due to reduced blockage of membrane pores by the red cells, the membrane exposure area further increases. These two factors further help in reducing the cell injury and the dialysis time.

Key Words : Dialysis; Blood Flow; Fluid Mechanics; Magnetohydrodynamics; Two-layered Model

1. INTRODUCTION

Many a time, due to malfunctioning of kidney, external help is required to remove metabolic wastes and other unwanted chemicals from human blood. The device used for such purpose is known as dialyser (artificial kidney). There are many types of dialysers, e.g., flat plate, tubular and helical. In the present paper, the fluid mechanical aspects of flat plant type dialyser has been dealt with. Lightfoot[1] has represented the flow of blood is suspension of cells (red blood cells, white blood cells etc.) in plasma, the flow of blood tends to be two-layered[2-6, 7&8], in particular, when the distance between the plates is very small. It is also observed that after some time, the efficiency of the dialyser goes down due to deposit of red blood cells and clots in the pores of membranes of the dialysers. Further, red blood cells get damaged due to contact with membrane. It is therefore, of interest to study a model which takes less time for dialysis and red cell injury is also reduced.

It has been shown by Woodcock[9] that red cells (which constitute about 50% of total volume of blood) have negative electric charge. Hence, blood can be considered as an electrically conducting fluid. One-layered magnetohydrodynamic (MHD) models for the flow of blood have been studied by many researchers[10 & 11]. From MHD studies, it is well known that magnetic field could be used to control the movement of charged particles[12]. Therefore, it may be possible to device a magnetic field configuration such that red blood cells are pushed away from the walls in a controlled manner like pinch effect[12]. This may, in turn, increase the exposure area of plasma to dialysate and prevent blocking of the membrane pores by red blod cells which will help in reducing the dialysis time. Further, because red cells are away from the membrane, the red cell's injury is reduced.

Hence, it is of interest to study a two-layered model of the flow of blood between two infinite parallel plates under the influence of magnetic field.

2. FORMULATION

Consider a steady flow of blood between two impermeable stationary plates. It is assumed that the length and width of the plates are very large in comparison to the distance between them, so that the problem may be approximated as one-dimensional flow between two infinite parallel plates at $y = \pm h$ (plane Poiseuille flow). Further, blood is assumed to be a Newtonian fluid. The flow is along x-direction and a constant uniform magnetic field is applied in the y-direction (Fig. 1). Since blood is an electrically conducting field, a magnetic force will act on the system. Since, blood is a suspension of red cells in plasma, the cells have a tendency to move away from the walls (boundary) and this forms a peripheral plasma layer (PPL) near the walls and a core-region consisting of cells and plasma. Since the plasma is an electrically non-conducting fluid, the flow in PPL is not affected by the magnetic force, however, due to electric charge on red cells, the flow in the core-region is influenced by the magnetic field. The governing equations of a two-layered MHD flow[12] of a Newtonian fluid are,

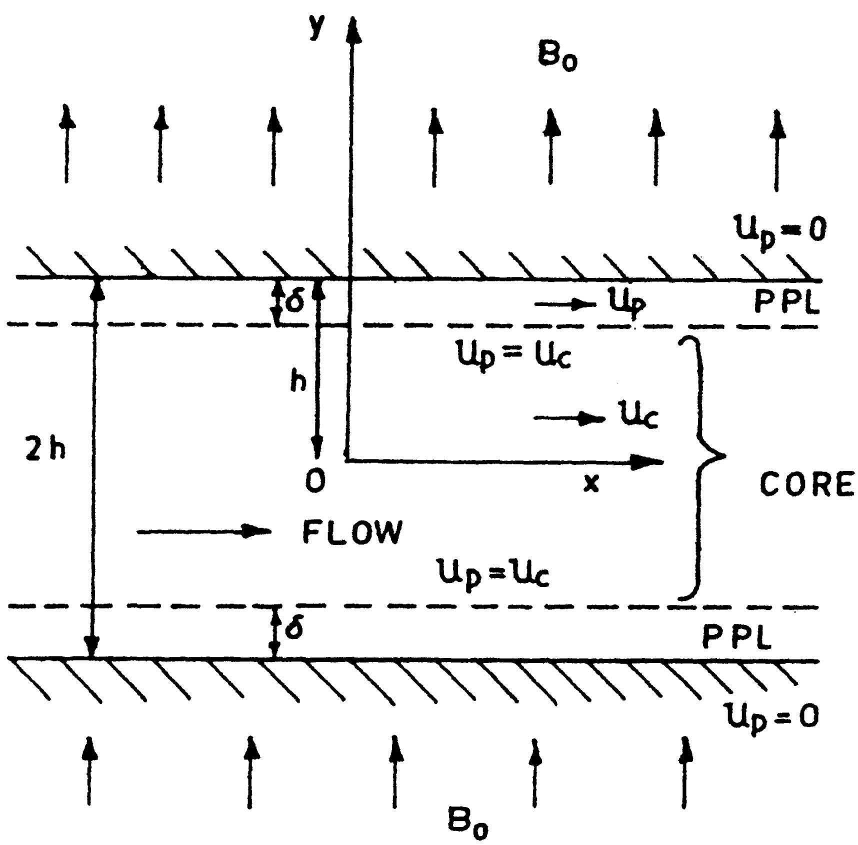

FIG. 1. Geometry of the problem

$$\frac{d^2 u_c}{dy^2} - \frac{\sigma B_0^2}{\eta_c} u_c = -\frac{P_0}{\eta_c}, -h-\delta) \le y \le h-\delta, \qquad \ldots (1)$$

and

$$\eta_p \frac{d^2 u_p}{dy^2} = -P_0, -h \le y \le -(h-\delta) \text{ and } h-\delta \le y \le h, \qquad \ldots (2)$$

where u_c and u_p are velocities of fluid in the core-region and PPL respectively, σ is electrical conductivity of blood, B_0 is applied constant uniform magnetic field (Fig. 1), η_c and η_p are viscosities of the fluid in the core-region and PPL respectively, $P_0\,(= -\partial p/\partial x)$ is pressure gradient in the x-direction and δ is the thickness of PPL.

The boundary conditions are,

$$u_c = u_p \text{ at } y = \pm (h - \delta)$$

and

$$u_p = 0 \text{ at } y = \pm h. \qquad \text{... (3)}$$

3. SOLUTION

Equations (1) and (2) are solved subject to the boundary conditions (3). Velocity in the core-region is given by,

$$u_c = \frac{P_0}{\sigma B_0^2}\left[1 - \frac{\cosh M\bar{y}}{\cosh M\left(1 - \frac{\delta}{h}\right)}\right] - \frac{P_0 h^2}{2\eta_p}\left(\frac{\delta}{h}\right)\left(\frac{\delta}{h} - 2\right)\frac{\cosh M\bar{y}}{\cosh M\left(1 - \frac{\delta}{h}\right)}, \qquad \text{... (4)}$$

where

$$\bar{y} = \frac{y}{h}, \text{ and } M = B_0 h \sqrt{\frac{\sigma}{\eta_c}}, \qquad \text{... (5)}$$

M is a non-dimensional number, called Hartmann number.

Velocity in the PPL is given by,

$$u_p = -\frac{P_0 h^2}{2\eta_p}(\bar{y}^2 - 1). \qquad \text{... (6)}$$

The induced magnetic field in x-direction B_x, can be obtained from the Maxwell's equation,

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j}, \qquad \text{... (7)}$$

and the Ohm's law,

$$\boldsymbol{J} = \sigma(\boldsymbol{u} \times \boldsymbol{B}), \qquad \text{... (8)}$$

where μ_0 is magnetic permeability of the fluid, $\boldsymbol{j}$ is current density, $\boldsymbol{u}$ is velocity and $\boldsymbol{B}$ is magnetic flux and is given by,

$$\boldsymbol{B_x} = \frac{P_0 \mu_0 h}{B_0 M}\tanh M\bar{y} - \frac{P_0 \mu_0 h}{B_0}\bar{y} + \frac{P_0 h^3 \sigma \mu_0 B_0}{2\eta_p M}\left(\frac{\delta}{h}\right)\left(\frac{\delta}{h} - 2\right)\tanh M\bar{y}. \qquad \text{... (9)}$$

The total pressure, p on the system is obtained by the equation of motion in y-direction and is given by,

$$p = \frac{1}{2} B_x^2 - P_0 x + \text{const.} \qquad \text{... (10)}$$

which is obtained from the equation of motion in y-direction, i.e., by integrating,

$$\frac{\partial p}{\partial y} = \frac{1}{2} \frac{d}{dy} (B_x^2). \qquad \text{... (11)}$$

Eq. (11) shows that the magnetic force ($\boldsymbol{j} \times \boldsymbol{B}$) in the y-direction acts to push the red cells towards the axis until the magnetic pressure is balanced by the fluid pressure.

The flow rate Q is defined as,

$$Q = \int_{-h}^{h} u dy = \int_{-h}^{-(h-\delta)} u_p dy + \int_{-(h-\delta)}^{0} u_c dy + \int_{0}^{h-\delta} u_c dy + \int_{h-\delta}^{h} u_p dy \qquad \text{... (12)}$$

which on substituting for u_c and u_p from eqs. (4) and (6) gives,

$$Q = -\frac{P_0 h^3}{\eta_p} \left(\frac{\delta}{h}\right)^2 \left[\frac{1}{3}\left(\frac{\delta}{h}\right) - 1\right] - \frac{P_0 h^3}{\eta_p} \left(\frac{\delta}{h}\right)\left(\frac{\delta}{h} - 2\right)$$
$$\times \frac{\tanh M\left(1 - \frac{\delta}{h}\right)}{M} + \frac{2P_0 h^3}{\eta_c M^2}\left[1 - \frac{\delta}{h} - \frac{\tanh M\left(1 - \frac{\delta}{h}\right)}{M}\right] \qquad \text{... (13)}$$

4. Results and Discussion

The effect of PPL thickness and Hartmann number on physiologically important fluid dynamic quantities (velocity, flow rate, effective viscosity, effective Reynolds number and induced magnetic field) may be used in designing the dialysers. A quantitative analysis is being provided in the following text with parameters: $h = 5 \times 10^{-4} m$, σ = 1.4mho.m^{-1}, $\eta_p = 1.2 \times 10^{-3}$kg m^{-1} sec^{-1}, $P_0 = 5 \times 10^4$ N.m^{-3}. The values of η_c are given in Table I.

TABLE I
Viscosity of blood in the core for different PPL thickness

δ/h	η_c (x10^{-3} kgm^{-1} s^{-1})	δ/h	η_c (x10^{-3} kgm^{-1} s^{-1})
0.02	5.25	0.10	5.82
0.04	5.37	0.12	5.99
0.06	5.51	0.14	6.18
0.08	5.66	0.16	6.39

4.1. Velocity Profile

It is of interest to note that the velocity profiles for the two-layered MHD flow models (Fig. 3) are much different from the one-layered MHD flow models (Fig. 2) and the two layered hydrodynamic (non-magnetic) flow models (Fig. 3, curve for M = 0). The velocity profiles of a

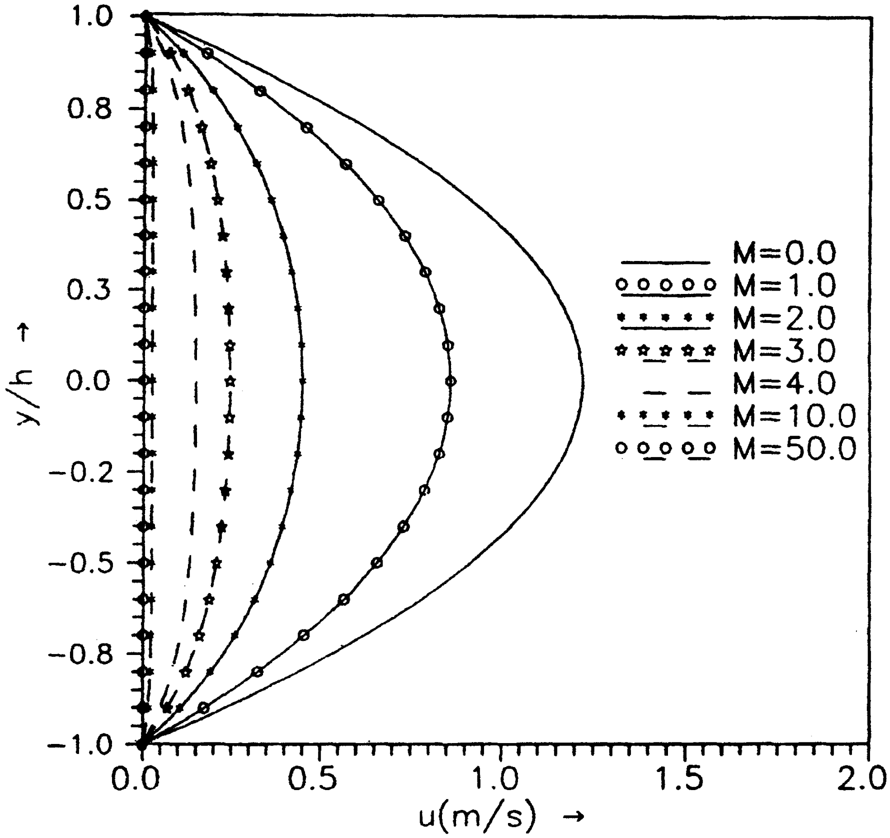

FIG. 2. Velocity profile for one-layered model (δ/h = 0.0)

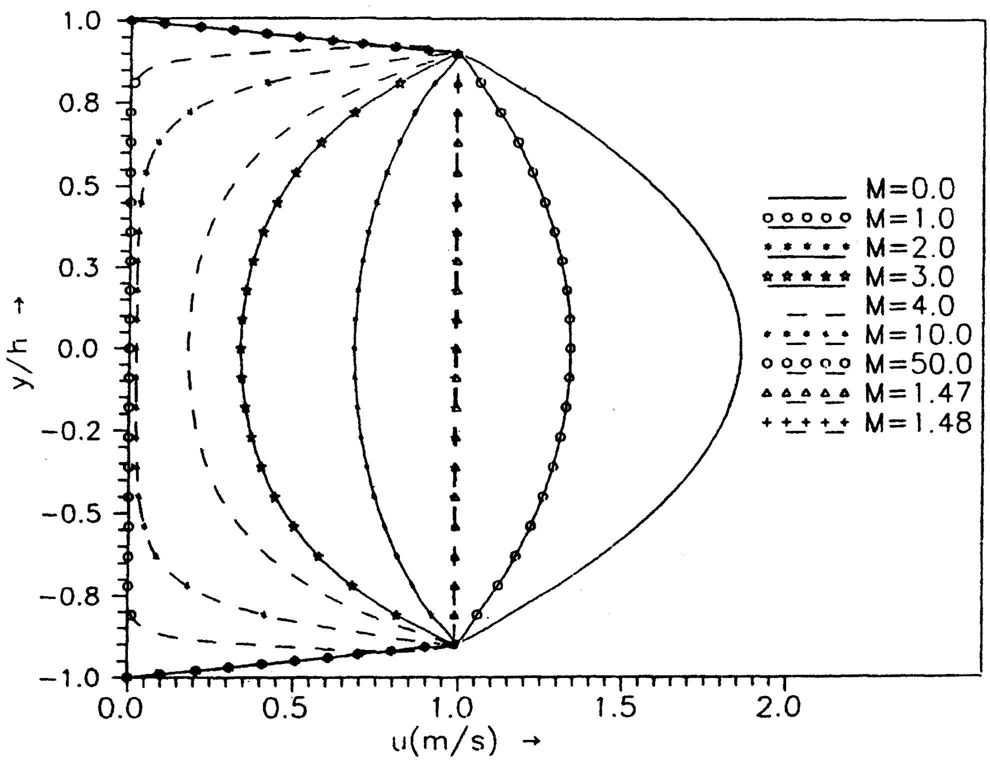

FIG. 3. Velocity profile for two-layered model (δ/h = 0.0)

one-layered MHD flow can be obtained as a special case of the present model by substituting $\delta = 0$ in eq. (4). The effect of Hartmann number on this profile is shown in Fig. 2. It is observed that an increase in hartmann number M leads to flattening of the profiles. For very high values of $M\,(>\cong 50)$, the flow is almost stagnant. Further, the one-layered hydrodynamic flow (parallel plate Poiseuille flow) can be obtained as a limiting case of the present problem by substituting $\delta = 0$ in eq. (4) and taking the limit as $M \to 0$.

The variation of velocity profiles shape for a two-layered hydrodynamic (non-magnetic) flow between parallel plates is shown through Fig. 3, (curve for $M = 0$). The mathematical expression for such profiles can be obtained as a limiting case, $M \to 0$, of the eq. (4) and is given by :

$$u_c = -\frac{P_0 h^2}{2\eta_c}\left[\bar{y}^2 - \left(1-\frac{\delta}{h}\right)^2\right] - \frac{P_0 h^2}{2\eta_c}\left(\frac{\delta}{h}\right)\left(\frac{\delta}{h}-2\right),\ -\left(1-\frac{\delta}{h}\right) \le \bar{y} \le \left(1-\frac{\delta}{h}\right) \quad \dots (14a)$$

$$u_p = -\frac{P_0 h^2}{2\eta_p}(\bar{y}^2 - 1),\ -1 \le \bar{y} \le (1-\delta/h) \text{ and } (1-\delta/h) \le \bar{y} \le 1. \quad \dots (14b)$$

It is of interest to note that the velocity profiles of one-layered MHD models (Fig. 2) and two-layered hydrodynamic models (Fig. 3, curve for $M = 0$) have maximum velocity at the central plane, i.e., parabolic/blunted parabolic profiles are opening towards y-axis. However, in case of two-layered. MHD models (Fig. 3), there are two possibilities: (*a*) parabolic velocity profiles opening towards y-axis (as in the usual hydrodynamic case), and (*b*) there are magnetic field strengths for which the parabolic profiles open in the opposite direction, which is a new feature of this model.

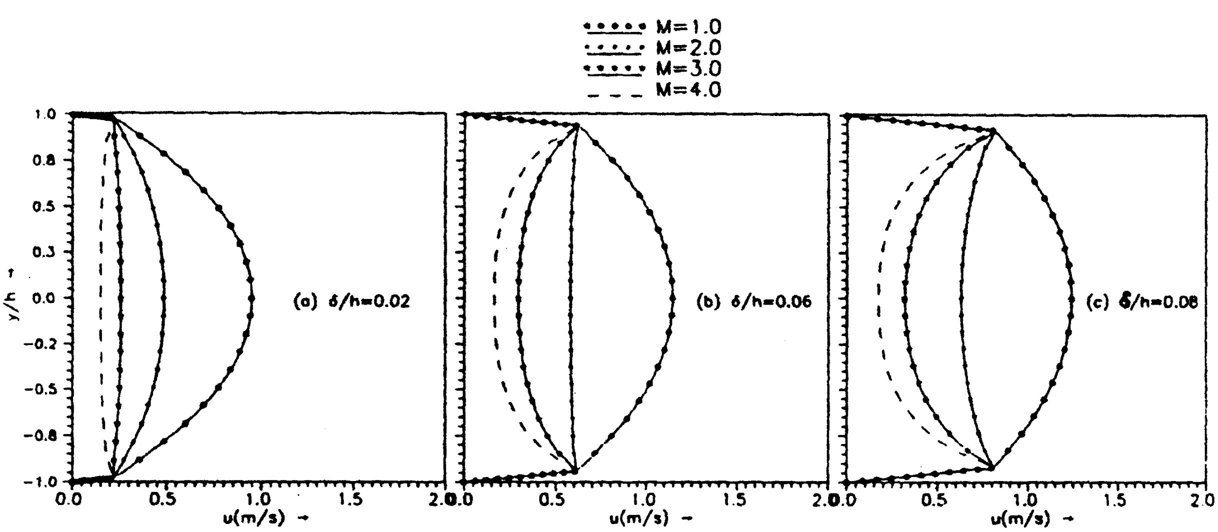

FIG. 4. Velocity profile for two-layered MHD model showing variation of critical values of Hartmann numbers with PPL thickness

It seems, there exists a critical value of M, say M_{cr}- such that, when $M < M_{cr}$, the parabolic profiles open towards y-axis and when $M > M_{cr}$, the profiles open in the opposite direction. It may be noticed that for one-layered MHD models, there is no such M_{cr}. For the two-layered MHD models, M_{cr} depends on PPL thickness, i.e., a decrease in the PPL thickness leads to an increase in M_{cr} (Fig. 4 and Table II).

TABLE II
Critical values of Hartmann number for different PPL thickness

δ/h	M_{cr} (approx.)
0.02	3.30
0.06	1.95
0.08	1.70
0.10	1.475

A comparison of velocities in case of one-layered and two-layered MHD models is shown in Fig. 5. It is observed that in case of two-layered models, velocities are higher than that of the corresponding one-layered models.

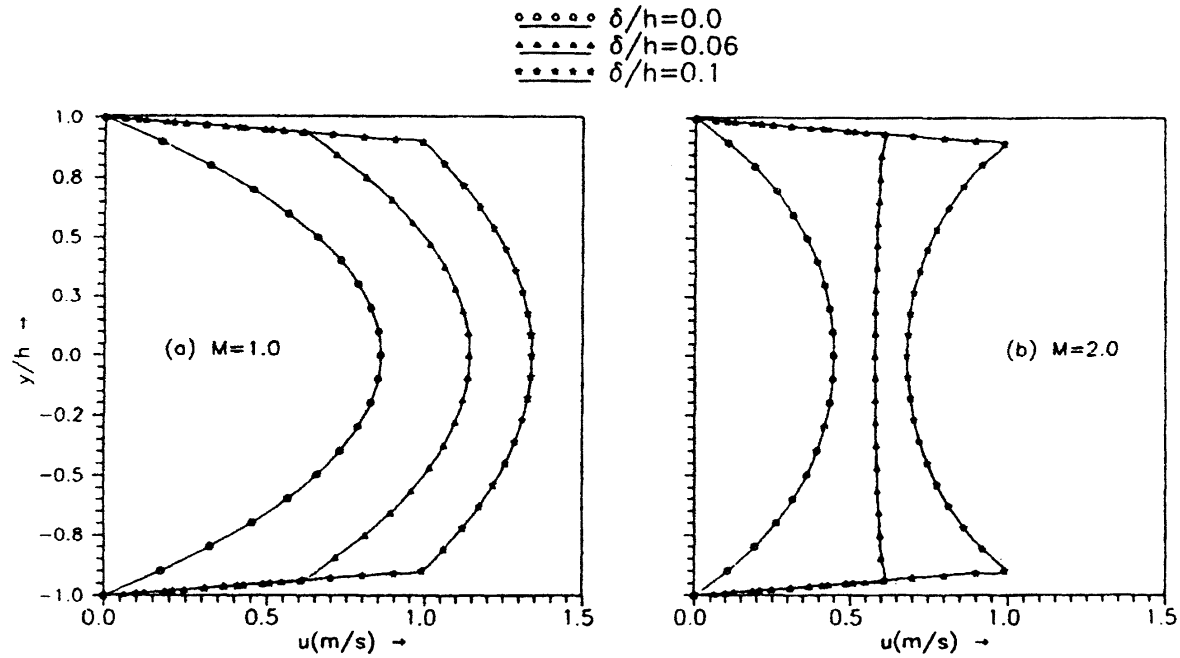

FIG. 5. Variation of velocity for different values of PPL thickness

It is of interest to note that in case of two-layered flow, we can have a transfer flow with faster removal of metabolic wastes by increasing the exposure area of plasma to dialysate. Another advantage is that in a two-layered model red cells will move out from the dialyser more quickly than in case of one-layered model which in turn will reduce their clotting.

4.2 *Flow Rate*

The expression of flow rate of a two-layered MHD model is given by eq. (13). In case of the two-layered hydrodynamic model, the flow-rate is obtained from eq. (13) by taking the limit $M \to 0$ and is given by,

$$Q = -\frac{P_0 h^3}{\eta_p}\left(\frac{\delta}{h}\right)\left[\left(\frac{\delta}{h}-2\right)\left(1-\frac{\delta}{h}\right)+\frac{\delta}{h}\left\{\frac{1}{3}\left(\frac{\delta}{h}\right)-1\right\}\right]+\frac{2}{3}\frac{P_0 h^3}{\eta_c}\left(1-\frac{\delta}{h}\right)^3 \quad \text{... (15)}$$

which for one-layered model ($\delta = 0$) reduces to,

$$Q = \frac{2P_0 h^3}{3\eta_c}. \qquad \text{... (16)}$$

Variation of flow rate with PPL thickness for different values of Hartmann number is shown in Fig. 6. It may be noticed that the curves are almost linear. This may be justified in view of the

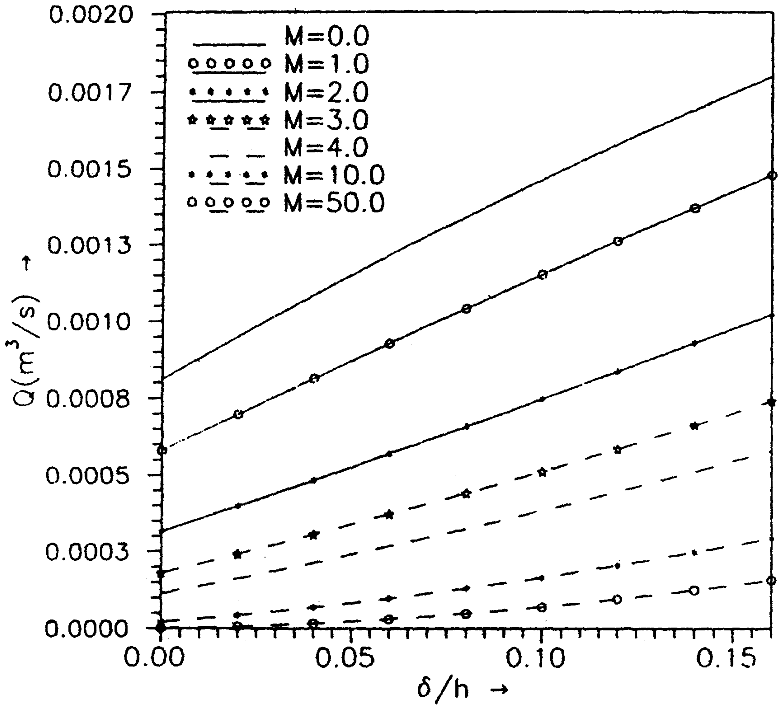

FIG. 6. Variation of flow rate with PPL thickness

following linear analysis of the expression of flow rate (eq. 13). Expanding the right hand side of eq. (13) in powers of δ/h and neglecting higher order terms (since $\delta/h << 1$) the flow rate, of a two layered MHD model can be written as,

$$Q = A + C\frac{\delta}{h}, \qquad \text{... (17)}$$

where

$$A = -\frac{2P_0 h^3}{\eta_c M^2}\left[\frac{\tanh M}{M} - 1\right] \qquad \text{... (18)}$$

and

$$C = -\frac{2P_0 h^3}{\eta_p} \cdot \frac{\tanh M}{M} - \frac{2P_0 h^3}{\eta_c M^3}\left[1 - \frac{1}{M} + \frac{(\tan M)^2}{M}\right]. \qquad \text{... (19)}$$

Here, A is the intercept on the y-axis and C is its slope (Fig. 6). Further, the flow rate decreases with an increase in the Hartmann number and this increase becomes very significant as the PPL thickness increases. For very large values of M ($\geq \cong 50.0$), the flow rate is almost zero for one-layered models.

It may also be noticed from Fig. 6 that the flow rate is increasing with the increase in PPL thickness. Thus, in dialysers, to have more exposure of plasma to the dialysate, it may be helpful to increase the PPI thickness which, in turn, will reduce the dialysis time.

4.3 Effective Viscosity

The effective viscosity of the one-layered non-magnetic flow is defined by,

$$\mu_{eff} = \frac{2P_0 h^3}{3Q}. \quad \text{... (20)}$$

Substituting for Q from eq. (13), the expression for the effective viscosity of a two-layered MHD model is given by,

$$\mu_{eff} = \frac{2}{3}\left[-\frac{1}{\eta_p}\left(\frac{\delta}{h}\right)^2 \left\{\frac{1}{3}\left(\frac{\delta}{h}\right) - 1\right\} - \frac{1}{\eta_p}\left(\frac{\delta}{h}\right)\left(\frac{\delta}{h} - 2\right)\frac{\tanh M(1-\delta/h)}{M} - \frac{2}{\eta_c}\left\{\frac{\tanh M(1-\delta/h)}{M^3} - \frac{1}{M^2}\left(1 - \frac{\delta}{h}\right)\right\}\right]^{-1}. \quad \text{... (21)}$$

It may be observed that the effective viscosity of a non-magnetic two-layered model can be obtained from the eq. (21) as its limiting case $(M \to 0)$ and is given by,

$$\mu_{eff} = \frac{2}{3}\left[\frac{2}{3\eta_c}\left(1 - \frac{\delta}{h}\right)^3 - \frac{1}{\eta_p}\left(\frac{\delta}{h}\right)\left\{\left(\frac{\delta}{h} - 2\right)\left(1 - \frac{\delta}{h}\right) + \left(\frac{\delta}{h}\right)\left(\frac{\delta}{h} - 1\right)\right\}\right]^{-1} \quad \text{... (22)}$$

Further, the expression for the effective viscosity of one layered MHD flow can be obtained from eq. (21), by substituting $\delta = 0$ and is given by,

$$\mu_{eff} = \frac{\eta_c M^2}{3}\left[1 - \frac{\tanh M}{M}\right]^{-1}. \quad \text{... (23)}$$

It may be noticed that, in case of one-layered non-magnetic case, μ_{eff} is same as the viscosity of the blood, (i.e. $\mu_{eff} = \eta_c$) which can be obtained by taking the limit $M \to 0$ of eq. (23).

Fig. 7 shows the variation of effective viscosity with PPL thickness for different values of Hartmann number. It is observed that for a one-layered MHD flow, the effective viscosity is maximum. As the layer thickness increases, effective viscosity decreases. In case of non-magnetic (hydrodynamic) model, the variation of effective viscosiy with PPL thickness is almost linear. A two-layered MHD model shows deviation from linearity and at higher values of M, the curves are highly nonlinear, i.e., the effective viscosity of blood increases with an increase in Hartmann number and this increase is more pronounced at lower values of δ/h.

Thus, the effective viscosity of blood can be controlled by varying the strength of magnetic field (Hartmann number) and PPL thickness. The effective viscosity can be decreased by increasing the PPL thickness and it can be increased by increasing the hartmann number. Further, the flow of suspensions have the tendency to have a particle free layer near the boundary, in particular, in narrow

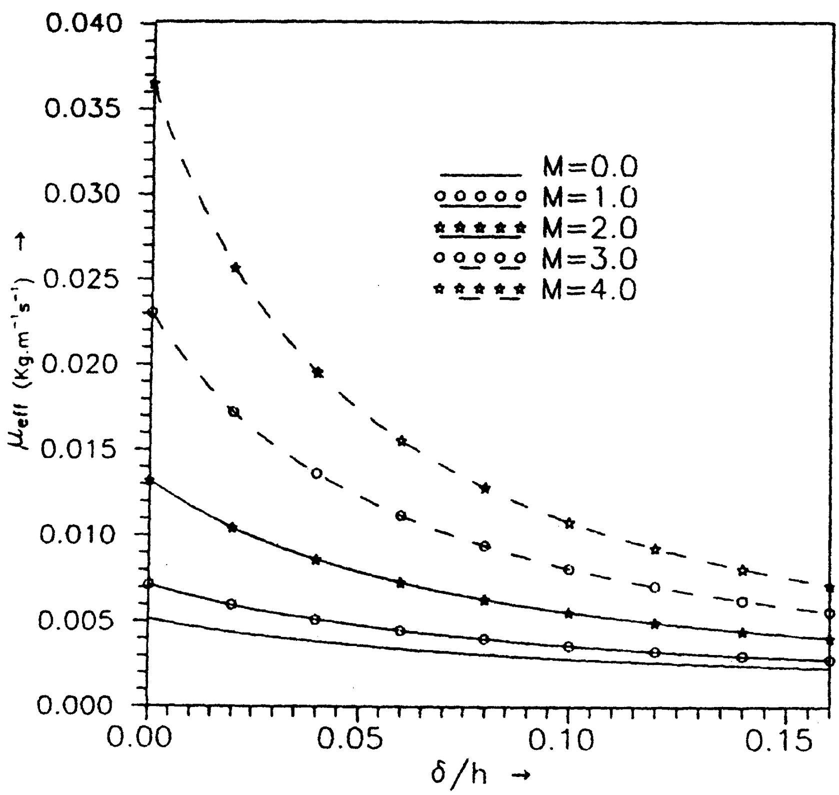

FIG. 7. Variation of effective viscosity with PPL thickness

channels[2-5]. Hence, a two-layered model seems to be more appropriate to be considered for a dialyser.

4.4 Effective Reynolds Number

The effective Reynolds number, N_{Re} for plane Poiseuille flow may be defined as,

$$N_{Re} = \frac{3\rho Q^2}{2h^3 P_0} \qquad \text{... (24)}$$

Fig. 8 shows variation of effective Reynolds number with PPL thickness. It has been observed that there is a significant increase in N_{Re} with the increase in the PPL thickness and this increase is more pronounced at lower values of Hartmann number. It is also observed that an increase in the Hartmann number leads to a decrease in N_{Re} . In case of one-layered models, the decrease in N_{Re} with increase in M is less significant as compared to two-layered models (Fig. 8).

4.5 Induced Magnetic Field

The expression of induced magnetic field is given by eq. (9) and is shown in (Fig. 9) for one-layered. It may be noticed that the moving fluid (blood) pulls the magnetic field lines in the direction of flow. It is also observed that for $0.2 \leq M \leq 2$, the magnitude of the induced magnetic field increases as the Hartmann number increases and after a critical value of M, ($\cong 2.0$), an increase in M leads to a decrease in its magnitude. The reasons for the existence of such critical values of M are not clear.

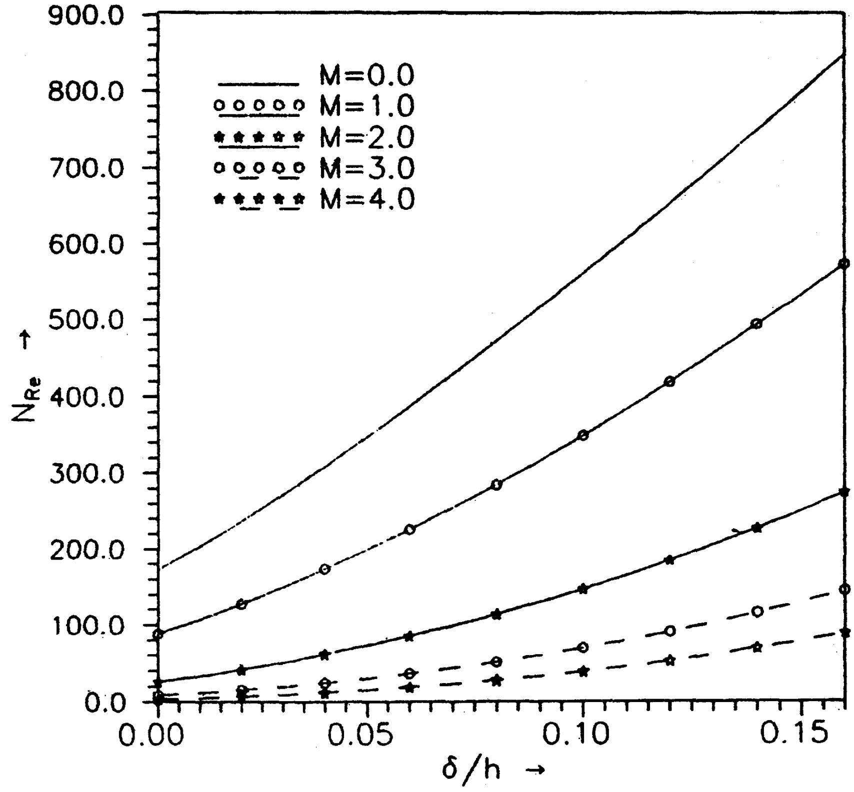

FIG. 8. Variation of effective Reynolds number with PPL thickness

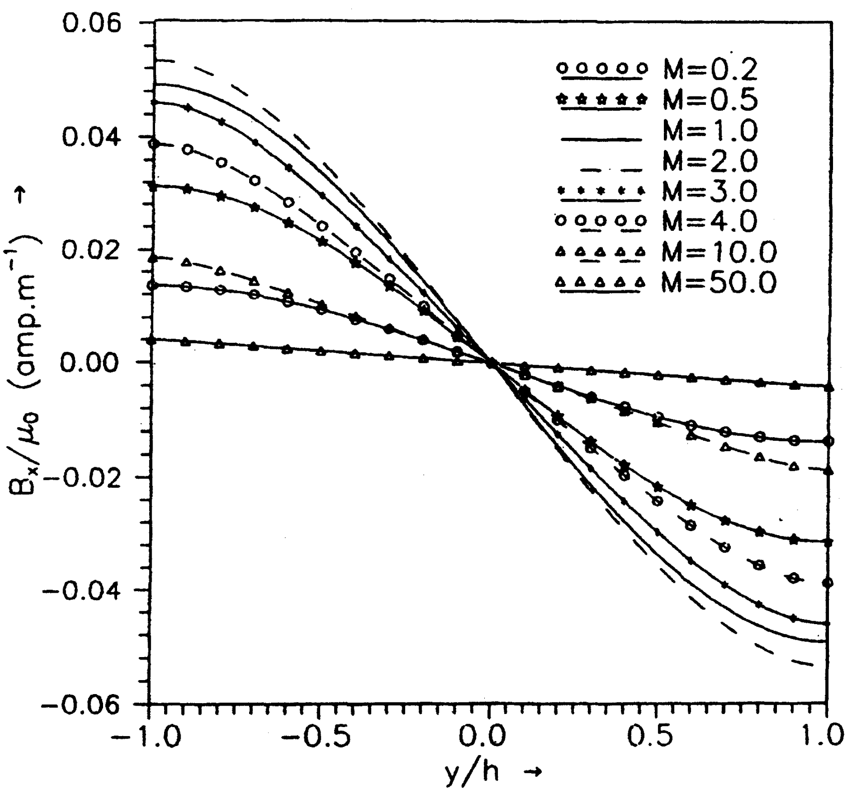

FIG. 9. Induced magnetic field lines for one-layered MHD model

Variation of induced magnetic field in case of two-layered MHD models is shown in Figs. 10(*a*) and 10(*b*) for $\delta/h = 0.06$ and 0.1 respectively. Like one-layered model, in this model also, there exists a critical value of M ($\cong 2.0$ for $\delta/h = 0.06$ and 0.1). It is observed that for $M \leq 2$, field lines are almost linear with respect to $\bar{y}$. Further, increase in M shows curved field lines and the curvature of the induced magnetic field lines increases with an increase in M. At higher values of M, there is a sharp drop in the magnitude of the induced magnetic field, near the interfaces and in the mid-region, induced magnetic field becomes constant (almost equal to zero). This pattern of induced magnetic field is similar to the one given in ref[12].

Further the variation of non-uniform induced magnetic field (Figs. 10(a) and 10(b), and eq. (11)) shows the existence of a transverse pressure gradient, on the red cells. Because of the transverse pressure gradient red cells will be pushed towards the axis. This is called pinch effect which gives

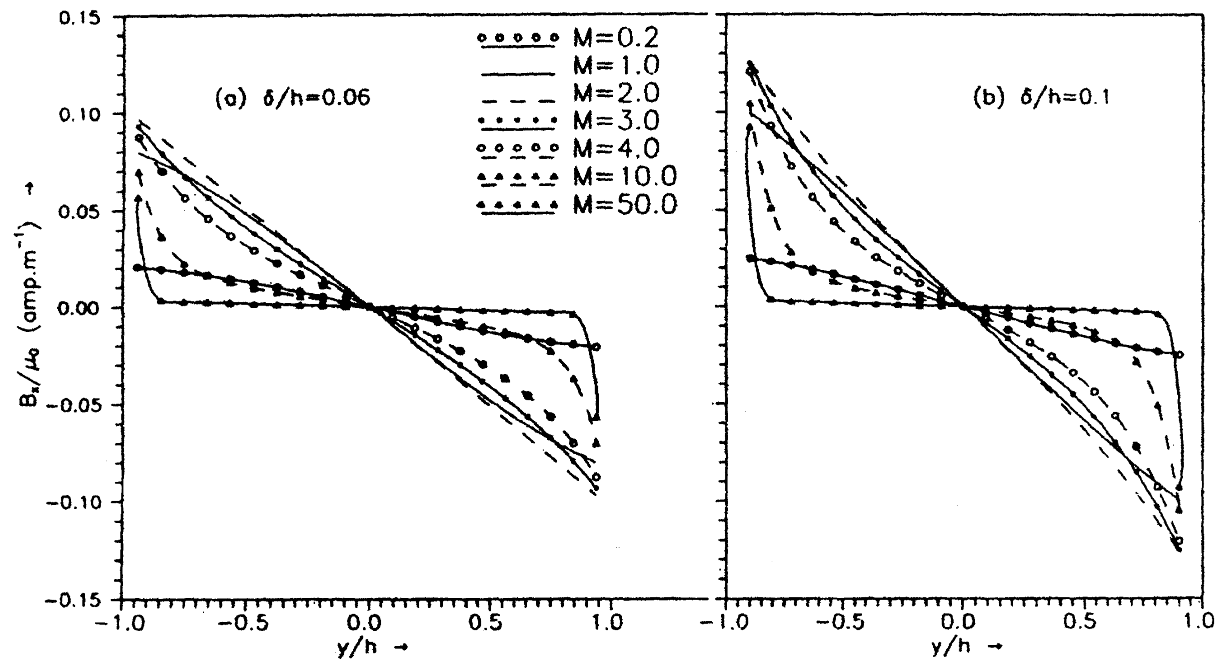

FIG. 10. Induced magnetic field lines for two-layered MHD model

PPL due to induced magnetic field. Hence, with the application of magnetic field, we can get red cells free layer near the boundary of which the thickness will vary with the strength of applied magnetic field. Experiments on the blood flow in dialysers may be conducted to show the existence of PPL with magnetic field and its variation with magnetic field strength.

5. Conclusions

In this paper, the idea of MHD dialyser has been proposed. The analytic expression for physiologically important fluid dynamic quantities (velocity, flow rate, effective Reynolds number, effective viscosity etc.) have been obtained for a two-layered MHD flow between two parallel plates. One-layered MHD flow and non-magnetic two-layered flow results are obtained as special cases of the present analysis. It is of interest to note that the velocity profiles of two-layered MHD flow have a new-feature which is not observed in one-layered MHD flow and two-layered non-magnetic flow. Here, we can get velocity profiles which have convexity of the similar type as that of one-layered and non-magnetic two-layered as well as of opposite nature. The existence of critical values of Hartmann number is predicted ($M < M_{cr}$ for one nature, $M > M_{cr}$ for the other nature). The effects of magnetic field strength and PPL layer thickness on physiologically important fluid dynamic quantities have been studied.

With uniform transverse magnetic field (as in the present case) velocity/effective viscosity of the flow decreases/increases with the increase in magnetic field intensity. The induced magnetic field is non-uniform and it gives rise to a transverse pressure gradient on the red cells which will try to move red cells towards the axis-a two-dimensional flow (briefly discussed in the latter part of the section). This phenomena, called pinch effect will generate a cell-free layer near the walls. It may be noticed that this cell-free layer is generated by the magnitude force due to induced magnetic field which will vary with the applied magnetic field. Experiments may be conducted to show the existence of PPL with magnetic field and its variation with magnetic field strength. If this is established, the PPL thickness can be controlled by the strength of the applied magnetic field.

The application of non-uniform magnetic field which increases as the magnitude of y increases, may give rise to pinch effect directly generated by the applied magnetic field, i.e. red cells will be moving from the wall towards the axis and hence, the density of red cells in the neighbourhood of the walls decreases. This gives rise to a two-dimensional flow. A two-dimensional flow with non-uniform magnetic field forms the contents of our next communication.

It is observed that in the problem considered, there is over determinancy of boundary conditions. The boundary condition, i.e., the stresses are equal at the interfaces, have not been used. Thus, in the present problem stresses are not equal at the interfaces. This may lead to a slip at the interfaces.

It is of interest to note that such two-layered dialysers will have three-fold advantages over the usual parallel plate dialyser. First, since the red blood cells are not in contact with walls (membrane), the clogging of the membrane will be considerably reduced which may lead to longer life of the membrane and faster dialysis. Second, since in dialysis metabolic wastes are removed from plasma, the absence of red cells near membrane increases the contact area of the plasma with membrane, which in turn increases the efficiency of the membrane, hence reduces the dialysis time. Third, since red cells do not come in contact with the membrane, injury to red cells/loss of red cells, by frictional force is reduced.

Looking at the advantages of two-layered MHD dialyser, experiments may be conducted to confirm the existence of PPL due to a constant applied magnetic field. Also, it is possible to have PPL with non-uniform applied magnetic field (it increases, as magnitude of y increases). Hence, it will be of interest to find a relation between the magnetic field strength and the PPL thickness.

In the present paper, impermeable walls have been considered. It will be more realistic to consider permeable walls which will lead to diffusion through membrane. Further, here blood has been considered as one-phase Newtonian fluid, it will be more appropriate to consider it as a two-phase/on-Newtonian model.

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