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STABILITY OF TWO SUPERPOSED VISCOELASTIC FLUIDS IN A HORIZONTAL MAGNETIC FIELD

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The stability of the plane interface separating two superposed viscoelastic fluids through porous medium has been investigated in a uniform two dimensional horizontal magnetic field. By applying the normal mode technique, the dispersion relation has been derived and solved numerically. It is found that elasticity has a stabilizing influence, the permeability of porous medium and viscosity have destabilizing influence on the growth rate of unstable mode of disturbance.

Key Words : Viscous Conducting Fluid; Porous Medium; Magnetic Field; Stability

INTRODUCTION

The instability which arises at the plane interface between two incompressible, invscid and perfectly conducting fluids of different densities when the lighter is accelerated into the heavier has been studied by several authors in the past. A comprehensive account of these investigations has been given by Chandrasekhar[1]. Roberts[2] extended this stability problem to include the effects of finite kinematic viscosity and magnetic resistivity but he considered only the case wherein these quantities are constant, equal to each other, and same in both the fluids. Jukes[3] also investigated the Rayleigh-Taylor instability problem in MHD with finite conductivity. His observations were based on the case of two inviscid fluids of different densities, one supported on the other in a uniform horizontal magnetic field and subjected to vertical gravitational force. He concluded that finite conductivity introduces new and unexpected solutions.

For more realistic physical situations in Astrophysics and Geophysics the case of viscous fluids should be considered. Bhatia[4] studied the problem of the Rayleigh-Taylor instability of two superposed viscous conducting fluids. Bhatia and Chhonkar[5] have studied the instability of the plane interface separating two superposed viscous conducting fluids. While Hooper and Grimshah[6] have examined the nonlinear instability of the plane interface between two viscous fluids. D'Anglo and Song[7-8] have studied the Kelvin-Helmholtz instability problem in superposed dusty plasma. Gupta and Bhatia[9] have studied the stability of the plane interface between two viscous superposed partially ionized plasma of uniform densities in a uniform two dimensional horizontal magnetic field. Srivastava and Khare[10] have investigated the Rayleigh-Taylor instability of two viscous superposed conducting fluids on a vertical magnetic field. Daval Osorozco[11] has studied the Rayleigh-Taylor instability of a two fluid layer under a general rotation field and a horizontal magnetic field. Elgowaing and Ashgriz[12] have investigated the Rayleigh-Taylor instability of viscous fluid layers. More recently, Bhatia and Sharma[13] have studid the instability of the plane interface between two viscous fluids through porous medium under a uniform vertical magnetic field.

Wooding[14] studied to instability of a thermal boundary flow through a porous medium. Since then several researchers (e.g., Rudraiah and Srimani[15], Sharma and Misra[16], Sharma and Rani[17]) have studied the effects of permeability of the porous medium on the different instability problems in view of the importance of such studies in rocks and heavy oil recovery. The physical properties of the comets, Meteroites and inter-planetary dust strongly suggest the significance of the effects of porosity in astrophysical context (McDonnel[18]).

Kumar and Singh[19] have investigated MHD Hele-Shaw flow of an elasticoviscous fluid through porous medium. More recently, Yadav and Ray[20] studied unsteady flow of n-immiscible visco-elastic fluids through a porous medium between two parallel plates in the presence of a transverse magnetic field. These studies have been carried for inviscid fluids. Ali and Bhatia[21] have studied the Rayleigh-Taylor instabilities of a stratified conducting fluid through porous medium in a 2-D magnetic field. El. Sayeed[22] has recently investigated the electro-hydrodynamics instability of two superposed viscous streaming fluid through porous media. Since viscoelastic fluids play an important role in industrial applications, it would be of interest to investigate the stability of two superposed viscoelastic fluids through porous medium. This aspect forms the basis of this paper wherein the fluids are permeated by uniform two-dimensional horizontal magnetic field. Earlier Sharma and Kumar[23] have studied the same problem for viscoelastic fluids in one dimensional horizontal magnetic field.

PERTURBATION EQUATIONS

We consider the motion of an incompressible, infinitely conducting viscoelastic fluid (of variable viscosity $\mu(z)$) through a porous medium. The fluid is assumed to be immersed in a uniform two dimensional horizontal magnetic field

$$\boldsymbol{H} = (H_x, H_y, 0).$$

The constitutive equations for Oldroyd viscoelastic fluid are given by

$$T_{ij} = -p\delta_{ij} + \tau_{ij} \quad \text{... (1)}$$

and

$$\left(1 + \lambda \frac{\partial}{\partial t}\right)\tau_{ij} = 2\,\mu\left(1 + \lambda_0 \frac{\partial}{\partial t}\right)e_{ij}, \quad \text{... (2)}$$

where

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \quad \text{... (3)}$$

Here e_{ij} is the rate-of-strain tensor, τ_{ij} is the shear stress tensor, δ_{ij} is Kronecker tensor, μ is the coefficient of viscosity, p is the pressure, λ_0 and $\lambda\,(\lambda_0 < \lambda)$ are strain retardation time and stress relaxation time respectively.

The equations of motion of an electrically conducting viscoelastic Oldroydian fluid through a porous medium in a uniform magnetic field are

$$\frac{\rho}{\varepsilon}\frac{Du_i}{Dt} = -g\rho\lambda_i + \frac{\partial T_{ij}}{\partial x_j}\mu_e\,\varepsilon_{ijk}\,\varepsilon_{jlm}\frac{\partial H_{1n}}{\partial x_j}H_k - \frac{\mu}{k_1}u_i, \quad \text{... (4)}$$

where $$\frac{D}{dt}=\frac{\partial}{\partial t}+u_j\frac{\partial}{\partial x}$$

is the mobile operator, H_1 is magnetic field, ρ is the density of the fluid, g is acceleration due to gravity, k_i is the permeability of the porous medium, ε is the medium porosity, u_i is the Darcian (filter) velocity of the fluid and $\lambda_i=(0, 0, 1)$ is a unit vector along the vertical.

The hydromagnetic equations of motion of an Oldryodian viscoelastic fluid through a porous medium become, on using the constitutive equation (1) for the viscoelastic fluid in conjunction with relations (2) and (3), become

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{D\boldsymbol{u}}{Dt}=\left(1+\lambda\frac{\partial}{\partial t}\right)[-\nabla p+\mu_e(\nabla\times\boldsymbol{H})\times\boldsymbol{H}+\boldsymbol{q}\rho]$$
$$+\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left[\frac{\mu}{\varepsilon}\nabla^2\boldsymbol{u}+\frac{1}{\varepsilon}\{(\nabla\boldsymbol{u})\cdot\nabla\mu+(\nabla\mu)\cdot\nabla\boldsymbol{u}\}-\frac{\mu}{k_1}\boldsymbol{u}\right], \quad \text{... (5)}$$

where $\boldsymbol{g}=(0,\ 0,\ -g)$. The relevant equations governing the motion of an incompressible ideally conducting Oldroydian viscoelastic fluid through a porous medium in a uniform magnetic field are, therefore, eq. (5) and

$$\varepsilon\frac{\partial\rho}{\partial t}+(\boldsymbol{u}\cdot\nabla)\,\rho=0, \quad \text{... (6)}$$

$$\varepsilon\frac{\partial\boldsymbol{H}}{\partial t}=\nabla\times(\boldsymbol{u}\times\boldsymbol{H}) \quad \text{... (7)}$$

and $$\nabla\cdot\boldsymbol{u}=0,\ \nabla\cdot\boldsymbol{H}=0. \quad \text{... (8)}$$

Let $\delta\rho$, δp and $\boldsymbol{h}=(h_x, h_y, h_z)$ denote respectively the perturbation in density ρ, pressure p and the magnetic field $\boldsymbol{H}$ due to small disturbance given to the system which produces the velocity field $\boldsymbol{u}=(u, v, w)$ in the system. Retaining only the linear terms in the perturbed quantities, we obtain the linearized perturbation equations

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\rho}{\varepsilon}\frac{\partial\boldsymbol{u}}{\partial t}=\left(1+\lambda\frac{\partial}{\partial t}\right)[-\nabla\delta p+\boldsymbol{g}\,\delta\rho+(\nabla\times\boldsymbol{h})\times\boldsymbol{H}]$$
$$+\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left[\frac{\mu}{\varepsilon}\nabla^2\boldsymbol{u}+\frac{1}{\varepsilon}\{(\nabla\mu\cdot\nabla)\,\boldsymbol{u}+\nabla\boldsymbol{u}\cdot\nabla\mu\}-\frac{\mu}{k_1}\boldsymbol{u}\right], \quad \text{... (9)}$$

$$\varepsilon\frac{\partial\boldsymbol{h}}{\partial t}=\nabla\times(\boldsymbol{u}\times\boldsymbol{H}), \quad \text{... (10)}$$

$$\varepsilon\frac{\partial}{\partial t}\delta\rho=-(\boldsymbol{u}\cdot\nabla)\,\rho \quad \text{... (11)}$$

and $$\nabla\cdot\boldsymbol{u}=0 \text{ and } \nabla\cdot\boldsymbol{h}=0. \quad \text{... (12)}$$

Analysing the disturbances in terms of normal modes we assume that the perturbed quantities vary in space $(x,\ y,\ z)$ and time (t) as

$$F(z) \exp [ik_x x + ik_y y + nt], \qquad \text{... (13)}$$

where $F(z)$ is some function of z, k_x and k_y are the horizontal wave numbers $(k^2 = k_x^2 + k_y^2)$ and n (may be complex) dentoes the rate at which the system departs away from equilibrium.

On using expression (13) in eqs. (9) to (12) and eliminating some of the variables we finally obtain an equation in w, writing $D = \frac{d}{dz}$

$$\frac{n}{\varepsilon} [k^2 \rho w - D \{\rho (Dw)\}] - \frac{gk^2 (D\rho)q}{n \varepsilon} - \frac{1}{n \varepsilon} (k_x H_x + k_y H_y)^2 (D^2 - k^2) w$$
$$+ \left(\frac{1 + \lambda_0 n}{1 + \lambda_n} \right) \left[\frac{\mu}{\varepsilon} (D^2 - k^2)^2 w + \frac{2}{\varepsilon} (D^2 - k^2) Dw (D\mu) \right.$$
$$\left. + \frac{1}{\varepsilon} D^2 \mu (D^2 + k^2) w + \frac{1}{k_1} \left\{ \mu w k^2 - D (\mu D w) \right\} \right] = 0 \qquad \text{... (14)}$$

Two Viscoelastic Superposed Fluids of Uniform Densities

We now consider the case when two superposed conducting viscoelastic fluids of uniform densities ρ_1 and ρ_2 and uniform viscosities μ_1 and μ_2 occupy the regions $z < 0$ and $z > 0$ and are separated by a horizontal boundary at $z = 0$. For both the fluids, eq. (14) reduces to

$$(D^2 - k^2)(D^2 - M^2) w = 0, \qquad \text{... (15)}$$

where

$$M^2 = k^2 + \left(\frac{1 + \lambda n}{1 + \lambda_0 n} \right) \left[\frac{\varepsilon}{k_1} + \frac{n}{\nu} \left\{ 1 + \frac{1}{n^2 \rho} (k_x H_x + k_y H_y)^2 \right\} \right] \qquad \text{... (16)}$$

and $\nu = \frac{\mu}{\rho}$ is the coefficient of kinematic viscosity.

Since w must vanish both when $z \to -\infty$ (in the lower fluid) and $z \to +\infty$ (in the upper fluid), the solutions of eq. (15) appropriate to the two regions are

$$W_1 = P_1 \exp (+kz) + Q_1 \exp (M_1 z), (z < 0) \qquad \text{... (17)}$$

and

$$W_2 = P_2 \exp (-kz) + Q_2 \exp (-M_2 z), (z > 0), \qquad \text{... (18)}$$

where P_1, Q_1, P_2, Q_2 are constants and M_1, M_2 are the positive square roots of (16) for the two regions. In writing the solutions (17) and (18) it is assumed that M_1 and M_2 are so defined that either real parts are positive.

For determining the four constants P_1, P_2, Q_1, Q_2 we require the four boundary conditions. The three conditions require continuity of

$$w, \ Dw \ \text{and} \ \mu(D^2 + k^2) w \qquad \text{... (19, 20 \& 21)}$$

across the interface $z = 0$. Integrating eq. (14), we obtain the fourth condition

$$\left[\left\{\rho_2 - \frac{\mu_2}{n}(D^2 - k^2) + \frac{\mu_2 \varepsilon}{nk_1} + \frac{1}{n^2}(\boldsymbol{k}\cdot\boldsymbol{H})^2\right\} Dw_2\right]_{z=0}$$

$$-\left[\left\{\rho_1 - \frac{\mu_1}{n}(D^2 - k^2) + \frac{\mu_1 \varepsilon}{nk_1} + \frac{1}{n^2}(\boldsymbol{k}\cdot\boldsymbol{H})^2\right\} Dw_1\right]_{z=0}$$

$$= -\frac{k^2}{n^2} g(\rho_2 - \rho_1) w_0 - \frac{2k^2}{n}(\mu_2 - \mu_1)(Dw)_0. \qquad \text{... (22)}$$

where w_0 and $(Dw)_0$ are the unique values of these quantities at $z = 0$.

On applying conditions (19) to (22) to the solution (17)-(18), we get

$$P_1 + Q_1 = P_2 + Q_2, \qquad \text{... (23)}$$

$$kP_1 + M_1 Q_1 = -kP_2 - M_2 Q_2, \qquad \text{... (24)}$$

$$\mu_1 \{2k^2 p_1 + (M_1^2 + k) Q_1\} = \mu_2 \{2k^2 p_2 + (M_2^2 + k) Q_2\} \qquad \text{... (25)}$$

$$- k(\rho_2 p_2 + \rho_1 p_1) = M_1 Q_1 \frac{(\boldsymbol{k}\cdot\boldsymbol{H})^2}{n^2}$$

$$= \frac{gk^2}{2n^2}(\rho_2 - \rho_1)(P_1 + Q_1 + P_2 + Q_2)$$

$$+ \frac{k^2}{n}(\mu_1 - \mu_2)(kp_1 + M_1 Q_1 - kP_2 - M_2 Q_2). \qquad \text{... (26)}$$

Eliminating constants P_1, P_2, Q_1, Q_2 from eqs. (23)-(26), we obtain the characteristic equations

$$(M_1 - k)\left[(R-1)\left\{\alpha_2 n + \frac{(\boldsymbol{k}\cdot\boldsymbol{V}_A)^2}{n}\right\} + 2k^2(\alpha_1 \nu_1 - \alpha_2 \nu_2)\right.$$

$$\left.\left\{\alpha_2 + \frac{C}{k}(M_2 - k) + \frac{M_2}{n^2 k}(\boldsymbol{k}\cdot\boldsymbol{V}_A)^2\right\}\right] - 2k\left[\left\{\alpha_1 n + \frac{(\boldsymbol{k}\cdot\boldsymbol{V}_A)^2}{n}\right\}\right.$$

$$\left.\left\{\alpha_2 + \frac{C}{k}(M_2 - k) + \frac{M_2}{n^2 k}(\boldsymbol{k}\cdot\boldsymbol{V}_A)^2\right\}\right] + \left\{\alpha_2 n + \frac{(\boldsymbol{k}\cdot\boldsymbol{V}_A)^2}{n}\right\}$$

$$\left.\left\{\alpha_1 - \frac{C}{k}(M_1 - k) + \frac{M_1}{n^2 k}(\boldsymbol{k}\cdot\boldsymbol{V}_A)^2\right\}\right]$$

$$+ (M_2 - k)\left[(R-1)\left\{ \alpha_1 n + \frac{(k \cdot V_A)^2}{n} \right\} - 2k^2 (\alpha_1 \nu_1 - \alpha_2 \nu_2) \right.$$

$$\left. \left\{ \alpha_1 - \frac{C}{k}(M_1 - k) + \frac{M_1}{n^2 k}(k \cdot V_A)^2 \right\} \right], \quad \text{... (27)}$$

where

$$\alpha_1 = \frac{\rho_1}{\rho_1 + \rho_2},\ \alpha_2 = \frac{\rho_2}{\rho_1 + \rho_2},\ (\alpha_1 + \alpha_2 = 1), \quad \text{... (28)}$$

$$R = -\frac{gk}{n^2}(\alpha_1 - \alpha_2), \quad \text{... (29)}$$

$$C = \frac{k^2}{n}\left(\frac{\mu_2 - \mu_1}{\rho_1 + \rho_2} \right) = -\frac{k^2}{n}(\alpha_1 \nu_1 - \alpha_2 \nu_2) \quad \text{... (30)}$$

and

$$(k \cdot V_A)^2 = \frac{(k \cdot H)^2}{\rho_1 + \rho_2} = \frac{(k_x H_x + k_y H_y)^2}{\rho_1 + \rho_2}. \quad \text{... (31)}$$

V_A is the Alfvén velocity vector.

The dispersion relation (27) is quite complex, particularly as M_1 and M_2 involve square roots. We, therefore, carry out the stability analysis for highly viscous, conducting superposed fluids. Then we can write

$$M_1 = k + \left(\frac{1 + \lambda_n}{1 + \lambda_0 n} \right)\left[\frac{\varepsilon}{2kk_1} + \frac{n}{2k \nu_1} + \frac{(k \cdot V_A)^2}{2nk \nu_1 \alpha_1} \right], \quad \text{... (32)}$$

$$M_2 = k + \left(\frac{1 + \lambda_n}{1 + \lambda_0 n} \right)\left[\frac{\varepsilon}{2kk_1} + \frac{n}{2k\nu_2} + \frac{(k \cdot V_A)^2}{2nk \nu_2 \alpha_2} \right], \quad \text{... (33)}$$

neglecting square and higher order terms in $\frac{1}{\nu_{1,2}}$.

Substituting the values of M_1 and M_2 in eq. (27) and putting $\nu_1 = \nu_2 = \nu$ (the case of equal kinematic viscosities), we obtain the dispersion relation in the dimensionless form as

$$\sum_{i=0}^{8} B_i Y^i = 0, \quad \text{... (34)}$$

where

$$B_8 = TT_0 \alpha_1 \alpha_2, \quad \text{... (35)}$$

$$B_7 = \alpha_1 \alpha_2 [8k^2 N \alpha_1 \alpha_2 (T - T_0)^2 + (T + T_0) + NTT_0 (P + 2k^2) - 2k^2 NT (T - T_0)]. \quad \text{... (36)}$$

$$B_6 = \alpha_1 \alpha_2 [1 + 4k^2 NT_0 + KTT_0 4k (V_1 \cos\theta + V_2 \sin\theta)^2 + (\alpha_1 - \alpha_2)$$

$$+ NP (T + T_0) + 2k^2 N^2 T (1 - 4\alpha_1 \alpha_2)(2kT - PT_0)]$$

$$+ TT_0 (V_1 \cos\theta + V_2 \sin\theta)^2 (1 - 2\alpha_1 \alpha_2), \quad \dots (37)$$

$$B_5 = \alpha_1 \alpha_2 [N(1 + 2k^2) + k(T + T_0) \{2k (V_1 \cos\theta + V_2 \sin\theta)^2 + (\alpha_1 - \alpha_2)\}$$

$$- 2k^2 PTN \{TN^2 (1 - 4\alpha_1 \alpha_2) + 2T_0 (V_1 \cos\theta + B_2 \sin\theta)^2 \alpha_1 \alpha_2\}]$$

$$+ k^2 (V_1 \cos\theta + V_2 \sin\theta^2 [(T + T_0) + 2k^2 NT_0 \{T(\alpha_1 - \alpha_2) + 4\alpha_1 \alpha_2\}]$$

$$+ kTT_0 (\alpha_1 - \alpha_2) - 2k^2 N (1 - 4\alpha_1 \alpha_2)(T - T_0) \{2NP + k^2 (V_1 \cos\theta + V_2 \sin\theta)^2 T\}, \quad \dots (38)$$

$$B_4 = k^2 (V_1 \cos\theta + V_2 \sin\theta)^2 [\{1 + 3k^2 TT_0 (V_1 \cos\theta + V_2 \sin\theta)^2 (1 - 2\alpha_1 \alpha_2)$$

$$+ 2\alpha_1 \alpha_2 \{2 + 4k^2 NT_0 + N(T + T_0)(1 + P)\} + k(\alpha_1 - \alpha_2) \{2kN (T + T_0)$$

$$+ TT_0 (1 - 3\alpha_1 \alpha_2)\}] - k^2 NT (1 - 4 \alpha_1 \alpha_2) \{k^2 (V_1 \cos\theta + V_2 \sin\theta)^2 (1 + 2PT)$$

$$+ 4NP \alpha_1, \alpha_2\} + k\alpha_1 \alpha_2 (\alpha_1 - \alpha_2) \{1 + N (T_0 + PT)\}, \quad \dots (39)$$

$$B_3 = k^4 (V_1 \cos\theta + V_2 \sin\theta)^4 [(1 + 2\alpha_1 \alpha_2)(T + T_0) + 8N\alpha_1 \alpha_2 (1 + k^2 T_0^2)$$

$$+ 2T_0 \{1 + (PNT + 2)\alpha_1 \alpha_2\} - 2T (1 - 4\alpha_1 \alpha_2) \{1 + KNT_0 - Nk^2 (T + T_0)\}]$$

$$+ k^2 (V_1 \cos\theta + V_2 \sin\theta)^2 [K(T + T_0)(\alpha_1 - \alpha_2) + \{k(\alpha_1 - \alpha_2)(k + PTT_0 \alpha_1 \alpha_2)$$

$$+ 2P\alpha_1 \alpha_2\} N] - 2k^2 N^2 P (1 - 4\alpha_1 \alpha_2)(2k^2 T + N\alpha_1 \alpha_2), \quad \dots (40)$$

$$B_2 = k^4 (V_1 \cos\theta + V_2 \sin\theta)^4 [kTT_0 (\alpha_1 - \alpha_2) + (3 - 2 \alpha_1 \alpha_2) - 4k^2 NT$$

$$+ (T + T_0) \{16 k^2 N\alpha_1 \alpha_2 + 2kN (1 - 4\alpha_1 \alpha_2)(1 - k)\}$$

$$+ 2k^2 (V_1 \cos\theta + V_2 \sin\theta)^2 TT_0] + k^3 (V_1 \cos\theta + V_2 \sin\theta)^2$$

$$\cdot [(\alpha_1 - \alpha_2) + 2 (T + T_0) NP\alpha_1 \alpha_2 \{(\alpha_1 - \alpha_2) + k (V_1 \cos\theta + V_2 \sin\theta)^2]$$

$$- 2PkN^2 (1 - 4 \alpha_1 \alpha_2)], \quad \dots (41)$$

$$B_1 = k^5 (V_1 \cos\theta + V_2 \sin\theta)^4 [(\alpha_1 - \alpha_2)(T + T_0) + 2N(1 - 4\alpha_1\alpha_2) - 4Nk(1 - 6\alpha_1\alpha_2)$$

$$+ 2k(T + T_0)(V_1 \cos\theta + V_2 \sin\theta)^2] + 2k^3 N^2 P\alpha_1\alpha_2 (V_1 \cos\theta + V_2 \sin\theta)^2$$

$$\cdot [k(V_1 \cos\theta + V_2 \sin\theta)^2 = (\alpha_1 - \alpha_2)] \quad \ldots (42)$$

and

$$B_0 = k^4 (V_1 \sin\theta + V_2 \sin\theta)^4 [2k^2 (V_1 \cos\theta + V_2 \sin\theta)^2 + k(\alpha_1 - \alpha_2)]. \quad \ldots (43)$$

In obtaining the dimensionless form of the dispersion relation, we have written

$$Y = \frac{n}{\sqrt{g}}, N = \frac{\gamma}{\sqrt{g}}, V_1 = \frac{V_x}{\sqrt{g}}, V_2 = \frac{V_y}{\sqrt{g}}, \quad \ldots (44)$$

$$P = \frac{\varepsilon}{k_1}, T = \lambda\sqrt{g}, T_0 = \lambda_0\sqrt{g}.$$

Here V_x and V_y are the Alfvén velocities in x and y directions and θ is the angle between $\boldsymbol{k}$ and H_x.

CONCLUSIONS

The dispersion relation (34) is quite complex. In order to study the effects of various physical parameters, we have therefore performed the numerical calculations of eq. (34) to locate the roots of Y (positive real part) against wave number x, for several vaues of parameters. The numerical calculations are presented in Table I to II, where we have taken $V_1 = V_2 = 0.5$ and $\theta = 45^0$.

From Table I we see that as N (viscosity) increases, growth rate Y increases showing destabilizing character of viscosity. Table II shows that growth rate Y decreases as T_0 (elasticity)

TABLE I :

Values of growth rate (real positive value of Y) against wave number x for N = 1.0, 2.0, 3.0, 4.0 and 5.0 when $V_1 = V_2 = 0.5$, P = 1.0, T = 1.0, T_0 = 1.0 and $\theta = 45^0$

X	Values of growth rate				
	N = 1.0	N = 2.0	N = 3.0	N = 4.0	N = 5.0
0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.2534	0.4019	0.6114	0.8646	1.1488
0.4	0.4354	1.0026	1.6925	2.4017	3.1156
0.6	0.6865	1.7925	2.8585	3.8829	4.8763
0.8	1.0711	2.5788	3.8846	5.0811	6.1974
1.0	1.5459	3.3154	4.7210	5.9190	6.9673
1.2	2.0701	3.9904	5.3345	6.3469	7.1488
1.4	2.6358	4.5958	5.7097	6.4023	6.8987
1.6	3.2486	5.1239	5.8730	6.2316	6.4892
1.8	3.9220	5.5686	5.8926	5.9914	6.0990
2.0	4.6795	5.9285	5.8435	5.7682	5.7840

TABLE II :
Values of growth rate (real positive value of Y) against wave number x for T_0 = 1.0, 2.0, 3.0, 4.0 and 5.0 when $V_1 = V_2 = 0.5$, P = 1.0, T = 1.0, N 1.0 and $\theta = 45^o$

x	Values of growth rate				
	T_0 = 1.0	T_0 = 2.0	T_0 = 3.0	T_0 = 4.0	T_0 = 5.0
0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.2534	0.2533	0.2532	0.2532	0.2531
0.4	0.4354	0.4231	0.4122	0.4024	0.3935
0.6	0.6865	0.6245	0.5735	0.5297	0.4910
0.8	1.3008	1.0805	0.9066	0.8633	0.6701
1.0	1.5459	1.2594	1.0401	1.8228	0.7561
1.2	2.0701	1.6296	1.3174	1.0112	0.9149
1.4	2.6358	2.0051	1.5937	1.2987	1.0742
1.6	3.2486	2.3806	1.8616	1.5728	1.2818
1.8	3.9220	2.7548	2.2404	1.4194	1.3020
2.0	4.6795	3.1271	2.3656	1.6699	1.5094

increases showing thereby stabilizing influence of elasticity. Table III shows that as P (permeability) increases, the growth rate Y increases showing thereby destabilizing influence of permeability of the porous medium on the unstable mode of disturbance.

TABLE III :
Values of growth rate (real positive value of Y) against wave number x for p = 1.0, 2.0, 3.0, 4.0 and 5.0 when $V_1 = V_2$ = 0.5, T = 1.0, T_0 = 1.0, N = 1.0 and θ = 45^o

x	Values of growth rate				
	P = 1.0	P = 2.0	P = 3.0	P = 4.0	P = 5.0
0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.2534	0.2989	0.3286	0.3499	0.3660
0.4	0.4354	0.6090	0.7326	0.8218	0.8880
0.6	0.6865	1.0951	1.3516	1.5267	1.6548
0.8	1.0711	1.6933	2.0688	2.3273	2.5190
1.0	1.5459	2.3429	2.8333	3.1765	3.4343
1.2	2.0701	3.0329	3.6399	4.0698	4.3962
1.4	2.6358	3.7695	4.4977	5.0168	5.4109
1.6	3.2486	4.5674	5.4243	6.0355	6.4991
1.8	3.9220	5.4505	6.4489	7.1536	7.6865
2.0	4.6795	6.4578	7.6085	8.4132	9.0130

Thus we may conclude that elasticity has stabilizing influence while permeability of porous medium and viscosity have destabilizing influence on the Rayleigh-Taylor instability of superposed viscoelastic fluids.

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