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STRING MODELS IN LYRA GEOMETRY

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Some Bianchi-type string cosmological models are presented here in the context of Lyra geometry.

Key words: Field Equations, Bianchi

1. INTRODUCTION

An intensive study of cosmic strings in elementary particle physics is due to gauge theories with spontaneous symmetry-breaking. It appears that after the big bang, the universe may have experienced a number of phase transitions[1] thereby producing vacuum domain structures such as domain walls, strings and monopoles[2]. Of these, cosmic strings have excited considerable interest, as they may act as gravitational lenses[3], and may give rise to density perturbations leading to the formation of galaxies[4].

Letelier[5] has initiated the study of a new model of a cloud formed by massive strings in the context of general relavity. This model has been used as a source for Bianchi type-I and Kantowski-Sachs cosmologies. The possibility that during the evolution of the universe the strings disappear leaving only particles has been examined. Krori, *et al.*[6-11] have done some exact solutions in string cosmologies. Pradhan, *et al.*[12] have studied LRS Bianchi type-I in the cosmological theory based on Lyra's geometry.

In this paper, we study some Bianchi type string cosmologies in the context of Lyra's geometry. The paper is organised as follows. We reproduce Einstein equations based on Lyra's geometry in section 2, derive the solutions in section 3 and finally end the paper with a conclusion in section 4.

2. EINSTEIN EQUATIONS BASED ON LYRA'S GEOMETRY

The field equations in normal gauge for Lyra's manifold as obtained by Sen[13] are

$$R_{\mu\gamma}-\frac{1}{2}g_{\mu\gamma}R=-T_{\mu\gamma}-\frac{3}{2}\phi_\mu\phi_\gamma+\frac{3}{4}g_{\mu\gamma}\phi_\alpha\phi^\alpha, \qquad \ldots (1)$$

Where,

$$T_{\mu\gamma}=\rho\, u_\mu u_\gamma-\lambda x_\mu x_\gamma \qquad \ldots (2)$$

Here, ρ = rest energy density of cloud of strings with particles attached to them,

λ = tension density of strings,

u^{μ} = cloud 4 - velocity,

x^{μ} = direction of anisotropy

and ϕ = displacement field (Lyra field) vector defined by :

$$\phi_{\mu} = [\beta(t), 0, 0, 0].$$

We have,

$$u^{\mu} u_{\mu} = -x^{\mu} x_{\mu} = 1 \text{ and } u^{\mu} x_{\mu} = 0. \quad \text{... (3)}$$

We consider,

$$\rho = \rho_p + \lambda, \quad \text{... (4)}$$

where ρ_p is the rest energy density of particles.

3. Field Equations and Their Solutions

The LRS metric for the spatially homogeneous Bianchi type-1 cosmological model is :

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2, \quad \text{... (5)}$$

Where A, B, C are functions of t only.

We choose

$$u^{\mu} = u_{\mu} = (1, 0, 0, 0) \quad \text{... (6)}$$

and x^{μ} to be along x-axis, so that,

$$x^{\mu} = (0, A^{-1}, 0, 0). \quad \text{... (7)}$$

The Eienstein equations (1) are

$$\ddot{B} + \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} = \lambda - \frac{3}{4}\beta^2. \quad \text{... (8)}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} = -\frac{3}{4}\beta^2. \quad \text{... (9)}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} = -\frac{3}{4}\beta^2. \quad \text{... (10)}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=-\rho-\frac{3}{4}\beta^2. \qquad \text{... (11)}$$

The energy conservation equation is

$$\dot{\rho}+\frac{3}{2}\beta\dot{\beta}+\left(\rho+\frac{3}{2}\beta^2\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\lambda\frac{\dot{A}}{A}=0. \qquad \text{... (12)}$$

Assuming,

$$A=A_0\,t^{\alpha},\, B=B_0\,t^{k},\, C=C_0\,t^{\gamma}$$

$$\lambda=\lambda_0\,t^{\theta},\, \rho=\rho_0\,t^{\delta},\, \beta=\beta_0\,t^{\chi}$$

a solution for (8)-(11) is given by,

$$A=A_0\,t^{\frac{-1+\sqrt{1-2D}}{4}},\quad D=5\,\rho_0+6\,\beta_0^2-\lambda_0<0$$

$$B=B_0\,t^{\frac{3+\sqrt{1-2D}}{8}}$$

$$C=C_0\,t^{\frac{3+\sqrt{1-2d}}{8}}$$

$$\lambda=\lambda_0\,t^{-2},\, \rho=\rho_0\,t^{-2},\, \beta=\beta_0\,t^{-1}$$

The LRS metric for Bianchi type-II cosmological model is:

$$ds^2=dt^2-(S\,dx+Sz\,dy)^2-(Rdy)^2-(Rdz)^2, \qquad \text{... (14)}$$

where R and S are functions of t only.

As in case (i) we take,

$$u^{\mu}=u_{\mu}=(1,0,0,0) \qquad \text{... (15)}$$

and

$$x^{\mu}=(0,\ S^{-1},\ 0,\ 0) \qquad \text{... (16)}$$

The Einstein equations (1) are

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}-\frac{3}{4}\frac{S^2}{R^4}=\frac{3}{4}\beta^2-\lambda. \qquad \text{... (17)}$$

$$\frac{\ddot{S}}{S}+\frac{\ddot{R}}{R}+\frac{\dot{S}\dot{R}}{SR}+\frac{1}{4}\frac{S^2}{R^4}=\frac{3}{4}\beta^2. \qquad \text{... (18)}$$

$$2\frac{\dot{S}\dot{R}}{SR}+\frac{\dot{R}^2}{R^2}-\frac{1}{4}\frac{S^2}{R^4}=-\rho-\frac{3}{4}\beta^2. \qquad \text{... (19)}$$

and the energy conservation equation is :

$$\dot{\rho}+\frac{3}{2}\beta\,\dot{\beta}+\left(\rho+\frac{3}{2}\beta^2\right)\left(\frac{\dot{S}}{S}+2\frac{\dot{R}}{R}\right)=\lambda\frac{\dot{S}}{S}, \qquad \dots (20)$$

Assuming,

$$R=R_0\,t^{\alpha}, S=S_0\,t^{\delta}, \rho=\rho_0\,t^{\gamma},\ \lambda=\lambda_0\,t^{\theta}, \beta=\beta_0\,t^{\chi}$$

a solution for (17)-(19) is given by,

$$R=R_0\,t^{\frac{1+\sqrt{1+3k_1}}{6}},\ K_1=\lambda_0-3\,(\rho_0+\beta_0^2)>1$$

$$S=S_0\,t^{\frac{-2+\sqrt{1+3K_1}}{3}}$$

$$\rho=\rho_0\,t^{-2}, \lambda=\lambda_0\,t^{-2}, \beta=\beta_0\,t^{-1}$$

The LRS-metric for Bianchi type VI_0 cosmological model is :

$$ds^2=dt^2-Adx^2-Be^{-2m_1x}dy^2-Ce^{2m_1x}dz^2 \qquad \dots (21)$$

where A, B, C are function of t only and m_1 is a non-zero constant. Taking u^{μ} as in (15) and

$$x^{\mu} = (0, A^{-1/2}, 0, 0), \qquad \dots (22)$$

Einstein equations (1) are

$$\frac{1}{2}\frac{\dot{B}\,\dot{C}}{BC}-\frac{1}{4}\left(\frac{\dot{B}^2}{B^2}=\frac{\dot{C}^2}{C^2}-\frac{\dot{B}\,\dot{C}}{BC}\right)+\frac{m_1^2}{A}=\lambda-\frac{3}{4}\beta^2. \qquad \dots (23)$$

$$\frac{1}{2}\left(\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}\right)-\frac{1}{4}\left(\frac{\dot{A}^2}{A^2}+\frac{\dot{C}^2}{C^2}-\frac{\dot{C}\,\dot{A}}{CA}\right)-\frac{m_1^2}{A}=+\rho\frac{3}{4}\beta^2, \qquad \dots (24)$$

$$\frac{1}{4}\left(\frac{\dot{A}\,\dot{B}}{AB}+\frac{\dot{B}\,\dot{C}}{BC}+\frac{\dot{C}\,\dot{A}}{CA}\right)-\frac{m_1^2}{A}=\rho-\frac{3}{4}\beta^2. \qquad \dots (25)$$

The energy conservation equation is

$$\dot{\rho}+\frac{3}{2}\beta\,\dot{\beta}+\frac{1}{2}\left(\rho+\frac{3}{2}\beta^2\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\frac{1}{2}\lambda\frac{\dot{A}}{A}=0 \qquad \dots (26)$$

Assuming,

$$A=A_0\,t^{\alpha},\ B=B_0\,t^{\delta},\ C=C_0\,t^{\gamma},\ \rho=\rho_0\,t^{\theta},\ \lambda=\lambda_0\,t^{\chi},\ \beta=\beta_0\,t^{\gamma}$$

a solution for (23)-(25) is given by,

$$A = A_0 t^2$$

$$B = B_0 t^{\frac{\left(\frac{4m_1^2}{A_0} - 3\beta_0^2\right) - \left(4\rho_0 + 2\sqrt{\frac{4m_1^2}{A_0} - 3\beta_0^2}\right)}{2 + \sqrt{\frac{4m_1^2}{A_0} - 3\beta_0^2}}}$$

$$C = C_0 t^{\left(\frac{4m_1^2}{A_0} - 3\beta_0^2\right)^{1/2}}, \text{ where } \left(\frac{4m_1^2}{A_0} - 3\beta_0^2\right)^{1/2} > 1 + \sqrt{1 + 4\rho_0}$$

$$\rho = \rho_0 t^{-2}, \lambda = \lambda_0 t^{-2}, \beta = \beta_0 t^{-1}$$

(iv) The LRS metric for Bianchi type VIII ($\delta = -1$) and Bianchi type IX ($\delta = +1$) is

$$ds^2 = dt^2 - (Sdx - shdz)^2 - (Rdy)^2 - (Rfdz)^2, \qquad \text{... (27)}$$

where S, R are functions of t only and

$$f(y) = \begin{bmatrix} \text{Sin } y \\ \text{Sinh } y \end{bmatrix}, h(y) = \begin{bmatrix} \text{Cos } y \\ -\text{Cosh } y \end{bmatrix} \text{ for } \delta = \begin{bmatrix} +1 \\ -1 \end{bmatrix}$$

Taking x^μ and u^μ as in (15) and (16),
Einstein equations (1) are

$$2\frac{\ddot{R}}{R} + \frac{1}{R^2}(\dot{R}^2 + \delta) - \frac{3}{4}\frac{S^2}{R^4} = \frac{3}{4}\beta^2 - \lambda. \qquad \text{... (28)}$$

$$\frac{\dot{S}\dot{R}}{SR} + \frac{S^2}{R^4} + \frac{\ddot{S}}{S} + \frac{\ddot{R}}{R} = \frac{3}{4}\beta^2. \qquad \text{... (29)}$$

$$2\frac{\dot{S}\dot{R}}{SR} + \frac{1}{R^2}(\dot{R}^2 + \delta) - \frac{1}{4}\frac{S^2}{R^4} = -\rho - \frac{3}{4}\beta^2, \qquad \text{... (30)}$$

The energy conservation equation is

$$\dot{\rho} + \frac{3}{2}\beta\,\dot{\beta} + \left(\rho + \frac{3}{2}\beta^2\right)\left(\frac{\dot{S}}{S} + 2\frac{\dot{R}}{R}\right) - \lambda\frac{\dot{S}}{S} = 0. \qquad \text{... (31)}$$

Assuming,

$$R = R_0 t^\alpha, S = S_0 t^\gamma, \rho = \rho_0 t^\xi, \lambda = \lambda_0 t^O, \beta = \beta_0 t^\chi$$

a solution for (28)-(31) is given by

$$R = R_0 t,$$

$$S = S_0 t$$

$$\rho = \rho_0 t^{-2}$$

$$\lambda = \lambda_0 t^{-2}$$

and $$\beta = \beta_0 t^{-1}.$$

4. Conclusion

We make two comments in conclusion :

(1) The solutions presented in some cosmologies here are relevant for early phases only when the strings and the Lyra field exist.

(2) During the early phases relevant in the different cosmologies, the strings and the energy density decay more rapidly than the Lyra field.

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References

1. A. D. Linde, *Rep. Prog. Phys.*, **42** (1979) 25
2. T. W. B. Kibble, *J. Phys.*, A, **9** (1976), 1387; *Phys. Rep.* **67** (1980) 183; Y. B. Zel'dovich, *Mon, Not. R. Astron Soc.* **192** (1980) 663.
3. A. Vilenkin, *Phys. Rev. D*, **23** (1981) 852.
4. A. Vilenkin, *Phys. Rev. Lett.*, **46** (1981) 1169; *Phys. Rev. D*, **24** 2082; T. W. B. Kibble and N. Turok, *Phys. Lett.*, **116 B** (1982) 141.
5. P. S. Letelier, *Phys. Rev. D*, (1983) **28** 2414.
6. K. D. Krori, T. Chaudhari, A. Mazumber and C. Mahanta **22** (1989) **123** (1990).
7. K. D. Krori and D. Goswami, *Gen. Rel. Grav.* **25** (1991) 117.
8. K. D. Krori and D. Goswami and K. Das, *Class-Quant. Grav* **10** (1992), (1993) 125.
9. K. D. Krori, T. Chaudharu and C. R. Mahanta, *Gen. Rel. Grav.*, **26** (1994) 265.
10. K. D. Krori, D. Goswami, M. Barua-Basu and K. Das. *Class-Quant. Grav.*, **11** (1994) L153.
11. K. D. Krori, P. Borgohain and D. (Kar) Das, *J. matj. Phys.*, **35**, 1032 (1994)
12. A. Pradhan and A. K. Vishwakarma, *Preprint IUCAA* - 27/99 (June, 1999).
13. D. K. Sen, *Z. Phys.* **149** (1957) 311.