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D - SUPERCONTINUOUS FUNCTIONS

J. K. KOHLI

Department of Mathematics, Hindu College, University of Delhi, Delhi 110 007, *India*

AND

DAVINDER SINGH

Department of Mathematics, University of Delhi, Delhi 110 007, *India*

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The notion of a *D*-superfontinuous function is introduced. Basic properties of *D* supercontinuous functions are studied. The class of *D* supercontinuous functions properly includes the class of clopen maps of Reilly and Vamanamurthy (*Indian J. pure appl. Math.* **14** (1983), 767-772). The notion of *D* supercontinuous function is independent of the notion of a supercontinuous function introduced by Munshi and Bassan (*Indian J. pure appl. Math.* **13**(2) (1982), 229-236. Behaviour of (completely) G_δ-regular spaces under *D* supercontinuous function is investigated.

Key Words and Phrases : Supercontinuous Function; *D*-supercontinuous Function; Strongly Continuous Function; Perfectly Continuous Function; Clopen Map; Completely Continuous Function; Strongly θ-continuous Function; D-Open (closed) Set; (Completely) $G\delta$-Regular Space; *D*-Regular Space

1. INTRODUCTION

Several strong forms of continuity occur in the literature[1, 5-9]. Certain of these strong forms of continuity coincide with continuity if the domain space is suitably augmented. In this paper, we introduce a new strong form of continuity called '*D*-supercontinuous function' which coincides with continuity if domain is a *D*-regular space[2]. It turns out that the notion of a *D*-supercontinuous function is independent of the notion of a supercontinuous function[6] and that of a strongly θ-continuous function[7]. The class of *D*-supercontinuous functions neither contains nor is contained in the class of completely continuous functions[1] and properly includes the class of clopen maps[9] and hence contains all perfectly continuous functions of Noiri[8] which in its turn include all strongly continuous functions of Levine[5]. Basic properties of D-supercontinuous functions are discussed in Section 3. In section 4, we consider the notions of D-quotient topology and a D-quotient space. Section 5 is devoted to the study of G_δ-regular and completely G_δ-regular spaces, wherein their behaviour under *D*-supercontinuous functions is investigated. In Section 6 we consider *D*-regularization of a space and conclude with alternative proofs of some results of preceding sections.

2. Preliminaries and Basic Definitions

Definition 2.1 — A function $f: X \rightarrow Y$ from a topological space X into a topological space Y is said to be ***D*-supercontinuous at a point** $x \in X$ if for every open set U containing $f(x)$ there exist an open F_σ - set V containing x such that $f(V) \subset U$. The function f is said to be D - supercontinuous if it is D - supercontinuous at each $x \in X$.

Every D-supercontinuous function is continuous. However, the converse is not true. For example, Let $X = \{a, b, c\}$ and $\tau_1 = \{\phi, X, \{a\}, \{a, c\}\}$. Let $Y = \{x, y, z\}$ and $\tau_2 = \{\phi, Y, \{x\}, \{x, y\}\}$, and let $f: X \rightarrow Y$ be defined by $f(a) = x$, $f(b) = y = f(c)$. Then f is continuous, but f is not D - supercontinuous at the point a.

Definition[5] 2.2 — A function $f: X \rightarrow Y$ is said to be ***strongly continuous*** if $f(\overline{A}) \subset f(A)$ for all $A \subset X$.

Definition[1] 2.3 — A function $f: X \rightarrow Y$ is said to be ***completely continuous*** if for each open set $V \subset Y, f^{-1}(V)$ is regularly open.

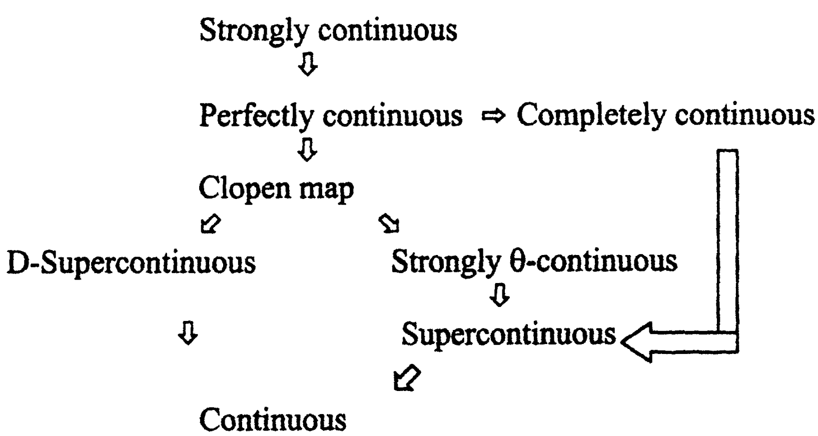
Definition[7] 2.4 — A function $f: X \rightarrow Y$ is said to be ***strongly θ - continuous*** if for each $x \in X$ and each open set V containing $f(x)$ there is an open set $U \subset X$ with $x \in U$ and $f(\overline{U}) \subset V$.

Definition 2.5[9] — A function $f: X \rightarrow Y$ is said to be ***cl-open*** if for each $x \in X$ each open set $V \subset Y$ with $f(x) \in V$ there is a clopen set $U \subset X$ with $x \in U$ and $f(U) \subset V$.

Definition[6] 2.6 — A function $f: X \rightarrow Y$ is said to be ***supercontinuous at a point*** $x \in X$ if for every open set U containing $f(x)$ there exists an open set N containing x such that $f((\overline{N})^0) \subseteq U$. The function f is supercontinuous if it is supercontinuous at each $x \in X$.

Definition[8] 2.7 — A function $f: X \rightarrow Y$ is said to be ***perfectly continuous*** if for every open set $V \subset Y, f^{-1}(V)$ is clopen in X.

The following diagram well illustrates the relationships that exist among D - supercontinuous functions and various strong forms of continuous functions defined above.



However, none of the above implications is reversible as will be shown in the sequel.

Noiri[8] gave examples to show that a clopen map need not be perfectly continuous and that a perfectly continuous map need not be strongly continuous. Moreover, Noiri showed that a completely continuous map need not be perfectly continuous. The notion of D-supercontinuous function is independent of the notions of supercontinuous function, strongly θ-continuous function and completely continuous function as is exhibited by the following examples.

Example 2.1 — Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X \{a, b\}, \{d\}, \{a, b, d\}\}$ and let f denote the identity function on X. Then f is supercontinuous but fails to be D-supercontinuous.

Example 2.2 — Let $X = \{a, b, c, d, \tau_1 = \{\phi, X \{a, b\}, \{d\}, \{a, b, d\}\}$ and $\tau_2 = \{\phi, X \{a, b\}\}$. Let f denote the identity function from (X, τ_1) onto (X, τ_2). Then f is completely continuous but not D-supercontinuous.

Example 2.3 — Let X denote the mountain chain space due to Heldermann[2]. Then X is a regular space which is not D-regular. The identity function defined on X is strongly θ-continuous but not D-supercontinuous.

Example 2.4 — Let X denote the set of positive integers endowed with the cofinite topology. The identity function defined on X is D-supercontinuous but neither supercontinuous nor completely continuous nor strongly θ-continuous.

3. Basic Properties of D - Supercontinuous Functions

A set G in a topological space X is said to be ***d* - *open*** if for each $x \in G$, there exists an open F_σ - set H such that $x \in H \subseteq G$. The complement of a d-open set will be referred to as a ***d* - *closed*** set.

Theorem 3.1 — *For a function $f: X \to Y$, the following statements are equivalent :*

(a) f is D-supercontinuous.

(b) Inverse image of every open subset of Y is d - open in X.

(c) Inverse image of every closed subset of Y is d - closed in X.

(d) For each point x of X and for each open set U containing $f(x)$, there is a d-open set V containing x such that $f(V) \subset U$.

We omit the proof.

Definition[2] 3.1 — A space X is called ***D-regular*** if X has a base consisting of open F_σ - sets.

Since in a D-regular space every open set is d-open, in view of Theorem 3.1 it follows that every continuous function defined on a D-regular space is D-supercontinuous.

Definition 3.2 — Let X be a topological space and let $A \subset X$. A point $x \in X$ is said to be a ***d* - *adherent point*** of A if every open F_σ-set containing x intersects A. Let $[A]_d$ denote the set of all d-adherent points of A. Clearly the set A is d-closed if and only if $[A]_d = A$.

Theorem 3.2 — *A function f from a space X into a space Y is D - supercontinuous if and only if $f([A]_d) \subset \overline{f(A)}$ for every $A \subset X$.*

Proof : Suppose f is D-supercontinuous. Since $\overline{f(A)}$ is closed in Y, by Theorem 3.1 $f^{-1}(\overline{f(A)})$ is d-closed in X. Again, since $A \subset f^{-1}(\overline{f(A)})$, $[A]_d \subset [f^{-1}(\overline{f(A)})]_d = f^{-1}(\overline{f(A)})$ and so $f([A]_d) \subset f(f^{-1}(\overline{f(A)})) \subset \overline{f(A)}$. Conversely, suppose $f([A]_d) \subset \overline{f(A)}$ for every $A \subset X$. Let F be any closed set in Y. Then $f([f^{-1}(F)]_d) \subset \overline{f(f^{-1}(F))} \subset \overline{F} = F$ and hence $[f^{-1}(F)]_d \subset f^{-1}(F)$. Thus $[f^{-1}(F)]_d = f^{-1}(F)$ which shows that f is D- supercontinuous.

Theorem 3.3 — *A function f from a space X into a space Y is D-supercontinuous if and only if $[f^{-1}(B)]_d \subset f^{-1}(\overline{B})$ for every $B \subset Y$.*

PROOF : Suppose f is D-supercontinuous. Then $f^{-1}(\overline{B})$ is d-closed in X for every $B \subset Y$ and thus $f^{-1}(\overline{B}) = [f^{-1}(\overline{B})]_d$. Hence, $[f^{-1}(B)]_d \subset f^{-1}(\overline{B})$. Conversely, let F be any closed set in Y. Then $[f^{-1}(F)]_d \subset f^{-1}(\overline{F}) = f^{-1}(F)$. Since $f^{-1}(F) \subset \overline{f^{-1}(F)} \subset [f^{-1}(F)]_d$, $f^{-1}(F) = [f^{-1}(F)]_d$ which in turn implies that f is D-supercontinuous.

Definition 3.3 — A filter base $\mathfrak{F}$ is said to ***d - converge*** to a point x (written as $f \to x$) if every open F_σ - set containing x contains a member of $\mathfrak{F}$.

Theorem 3.4 — *A function $f: X \to Y$ is D - supercontinuous if and only if for each $x \in X$ and each filter base $\mathfrak{F}$ that d-converges to x, $f(\mathfrak{F}) \to f(x)$*

PROOF : Assume that f is D-supercontinuous and let $\mathfrak{F} \xrightarrow{d} x$. Let W be an open set containing $f(x)$. Then $x \in f^{-1}(W)$ and $f^{-1}(W)$ is d-open. Let H be an open F_σ - set such that $x \in H \subset f^{-1}(W)$ and so $f(H) \subset W$. Since $\mathfrak{F} \xrightarrow{d} x$, there exists $U \in \mathfrak{F}$ such that $U \subset H$ and hence $f(U) \subset f(H) \subset W$. Thus $f(\mathfrak{F}) \to f(x)$.

Conversely, let W be an open subset of Y containing $f(x)$. Now the filter base $\mathcal{N}_x$ consisting of all open F_σ - sets containing x d-converges to x and so by hypothesis $f(\mathcal{N}_x) \to f(x)$.

Hence there exists a member $f(N)$ of $f(\mathcal{N}_x)$ such that $f(N) \subset W$. Since $N \in \mathcal{N}_x$, N is an open F_σ - set containing x. Thus f is D-supercontinuous.

Theorem 3.5 — *If $f: X \to Y$ is D-supercontinuous and $f(X)$ is endowed with the subspace topology, then $f: X \to f(X)$ is D-supercontinuous.*

PROOF : Since $f: X \to Y$ is D-supercontinuous, for every open subset U of Y, $f^{-1}(U \cap f(X)) = f^{-1}(U) \cap f^{-1}(f(X)) = f^{-1}(U) \cap X = f^{-1}(U)$ is d-open. Hence $f: X \to f(X)$ is D-supercontinuous.

A function $f: X \to Y$ is said to be ***d - open (d - closed)*** if $f(A)$ is open (closed) in Y for every open F_σ - set (closed G_δ - set) A in X.

Every open function is d-open but the converse is not true.

Example 3.1 — Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$. Let $Y = \{x, y, z\}$, $\mu = \{\phi, Y, \{x\}\}$. Let $f: (X, \tau) \to (Y, \mu)$ be defined by $f(a) = x$, $f(b) = y$, $f(c) = z$. Then f is a d-open function, which is not an open function.

Theorem 3.6 — *If $f: X \to Y$ is D - supercontinuous and $g: Y \to Z$ is continuous, then gof is D - supercontinuous. In particular, the composition of D - supercontinuous function is D - supercontinuous.*

Corollary 3.1 — Let $f: X \to Y$ be D - supercontinuous. If Z is a space containing Y as a subspace, then the function $h: X \to Z$ defined by $h(x) = f(x)$ for each $x \in X$ is D - supercontinuous.

PROOF : Since h is the composition of D - supercontinuous function $f: X \to Y$ and the inclusion mapping $i: Y \to Z$, by Theorem 3.6 it is D - supercontinuous.

Remark 3.1 : Theorem 3.6 shows that the study of D - supercontinuous functions from categorial view point is useful. It seems an interesting and a rewarding exercise to study the category of topological spaces and D-supercontinuous functions.

Theorem 3.7 — *Let $f: X \rightarrow Y$ be a d - open, D - supercontinuous surjection and let $g: Y \rightarrow Z$ be any function. Then gof is D - supercontinuous if and only if g is continuous.*

PROOF : Sufficiency is obvious. To prove necessity, let gof be D - supercontinuous and let G be an open subset of Z. Then $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is d - open in X. Since f is a d - open subrjection, $f[f^{-1}(g^{-1}(G))] = g^{-1}(G)$ is open. Hence g is continuous.

Theorem 3.8 — *Let $\{f_\alpha : X \rightarrow X_\alpha \mid \alpha \in \Lambda\}$ be a family of functions and let $f: X \rightarrow \prod_{\alpha \in \Lambda} X_\alpha$ be defined by $f(x) = (f_\alpha(x))$. Then f is D - supercontinuous if and only if each $f_\alpha : X \rightarrow X_\alpha$ is D - supercontinuous.*

PROOF : Let $f: X \rightarrow \prod_{\alpha \in \Lambda} X_\alpha$ be D - supercontinuous. Then $f_\alpha = p_\alpha \circ f$, where p_α denotes the projection of X onto α - coordinate space X_α. Hence by Theorem 3.6 each f_α is D - supercontinuous.

Conversely, suppose that each $f_\alpha : X \rightarrow X_\alpha$ is D - supercontinuous. To show that the function f is D - supercontinuous, in view of Theorem 3.1 it is sufficient to show that $f^{-1}(U)$ is d - open for each open set U in the product space $\prod_{\alpha \in \Lambda} X_\alpha$.

Since the finite intersections and arbitrary unions of d - open sets are d - open, it suffices to prove that $f^{-1}(S)$ is d - open for every subbasic open set S in the product space $\prod_{\alpha \in \Lambda} X_\alpha$. Let $U_\beta \times \prod_{\alpha \neq \beta} X_\alpha$ be a subbasic open set in $\prod X_\alpha$. Then $f^{-1}\left(U_\beta \times \prod_{\alpha \neq \beta} X_\alpha\right) = f^{-1}(p_\beta^{-1}(U_\beta)) = f_\beta^{-1}(U_\beta)$ is d - open. Hence f is D - supercontinuous.

Theorem 3.9 — *Let $f: X \rightarrow Y$ be a function and $g: X \rightarrow X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, be the graph function. Then g is D - supercontinuous if and only if f is D - supercontinuous and X is D - regular.*

PROOF : To prove necessity, suppose that g is D - supercontinuous. By Theorem 3.6 $f = p_y \circ g$ is D-supercontinuous, where p_y is the projection from $X \times Y$ onto Y. Let U be any open set in X and let $x \in U$. Then $U \times Y$ is an open set containing $g(x)$. Since g is D - supercontinuous, there exists an open F_σ-set W containing x such that $g(W) \subset U \times Y$. Thus $x \in W \subset U$, which shows that U is d-open and so X is D - regular.

To prove sufficiency, let $x \in X$ and let W be an open set containing $g(x)$. There exist open sets $U \subset X$ and $V \subset Y$ such that $(x, f(x)) \in U \times V \subset W$. Since X is D-regular, there exists an open F_σ - set G_1 in X containing x such that $x \in G_1 \subset V$. Since f is D-supercontinuous, these exists an open F_σ-set G_2 in X containing x such that $f(G_2) \subset V$. Let $G = G_1 \cap G_2$. Then G is an open F_σ - set containing x and $g(G) \subset U \times V \subset W$, which implies that g is D - supercontinuous.

Definition 3.4 [4] — A function $f: X \rightarrow Y$ is said to be ***D - continuous*** if for each $x \in X$ and each open F_σ - set V containing $f(x)$ there is an open set U containing x such that $f(U) \subset V$.

Lemma 3.1 — For a function $f: X \rightarrow Y$, the following statements are equivalent

(a) f is D - continuous.

(b) $f(\overline{A}) \subseteq [f(A)]_d$ for all $A \subseteq X$.

(c) $\overline{f^{-1}(B)} \subseteq f^{-1}([B]_d)$ for all $B \subseteq Y$.

(d) Inverse image of every d - closed set is closed.

(e) Inverse image of every d - open set is open.

PROOF : (a) $\Rightarrow$ (b) : Let $y \in f(\overline{A})$. Choose $x \in \overline{A}$ such that $f(x) = y$. Let V be an open F_σ - set containing y. Since f is D - continuous, $f^{-1}(V)$ is an open set containing x. This gives $f^{-1}(V) \cap A \neq \phi$ which in turn implies that $V \cap f(A) \neq \phi$ and consequently $y \in [f(A)]_d$. Hence $f(\overline{A}) \subset [f(A)]_d$.

(b) $\Rightarrow$ (c) — Let B be any subset of Y. Then $f(\overline{f^{-1}(B)}) \subseteq [f(f^{-1}(B))]_d$ and consequently $\overline{f^{-1}(B)} \subseteq f^{-1}([B]_d)$.

(c) $\Rightarrow$ (d) — Since a set A is d - closed if and only if $A = [A]_d$, therefore the implification (c) $\Rightarrow$ (d) is obvious.

(d) $\Rightarrow$ (e) — Obvious.

(e) $\Rightarrow$ (a) — This is immediate since every open F_σ - set is d - open and since a function is D - continuous if and only if the inverse image of every open F_σ - set is open.

Theorem 3.10 — *Let X, Y and Z be topological spaces and let the function $f: X \to Y$ be D continuous and $g: Y \to Z$ be D-supercontinuous. Then $g \circ f: X \to Z$ is continuous.*

PROOF : It is immediate in view of Lemma 3.1 and Theorem 3.1.

However, if $f: X \to Y$ is D - continuous and $g \circ f: X \to Z$ is continuous, $g: Y \to Z$ may not be D - supercontinuous.

Example 3.2 — Let R be the real line endowed with the cocountable topology.

Let $X = \{a, b\}$, $\tau = \{\phi, X, \{a\}\}/$

Let $f: R \to X$ be defined by :

$f(x) = a$, if x is irrational.

Let $Y = \{c, d\}$, $\mathcal{J} = \{\phi, Y, \{d\}\}$. Let $g: X \to Y$ be defined by $g(a) = d$, $g(b) = c$. Then $f: R \to X$ is D-continuous and $g \circ f: R \to Y$ is continuous but $g: X \to Y$ is not D - supercontinuous.

4. D - Quotient Topology and D - Quotient Spaces

Definition 4.1 — Let $f: X \to Y$ be a function from a topological space X onto a set Y. The topology on Y for which a subset $A \subset Y$ is open if and only if $f^{-1}(A)$ is d - open in X is called the ***D - quotient topology*** and the map is f is called the ***D - quotient map.***

It is clear that in general D - quotient topology on Y is coarser than the quotient topology on Y and the two coincide if X is D - regular.

Example 4.1 — Let $X = \{a, b, c\}$, $\tau = \{\phi, X\{a\}, \{a, b\}, \{a, c\}\}$ and $Y = \{x, y, z\}$. Let $f: X \to Y$ be defined by $f(a) = x$, $f(b) = y$, $f(c) = z$. Then the quotient topology on Y is given by $\{\phi, Y, \{x, \{x\}, y\}, \{x, z\}\}$ while the D - quotient topology on Y is indiscrete.

Theorem 4.1 — *Let f be a function from a topological space (X, τ_1) onto a topological space (Y, τ_2), where τ_2 is the D-quotient topology on Y. Then f is D - supercontinuous. Moreover, τ_2 is the finest topology on Y which makes $f: X \to Y$ D - supercontinuous.*

PROOF : The D - supercontinuity of f follows from the definition of D - quotient topology. Let τ_3 be a topology on Y such that $f: (X, \tau_1) \rightarrow (Y, \tau_3)$ is D - supercontinuous. Let G be a τ_3 open set in Y. By D - supercontinuity of f, $f^{-1}(G)$ is d - open in X. Now by the definition of the D - quotient topology, G is τ_2 - open and hence $\tau_3 \subset \tau_2$.

Theorem 4.2 — *Let $f: X \rightarrow Y$ be a D - quotient map. Then a function $g: Y \rightarrow Z$ is continuous if and only if $g \circ f$ is D - supercontinuous.*

PROOF : If U is an open set in Z and $g \circ f$ is D - supercontinuous, then $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$, which is d - open in X. Since f is a D - quotient map, $g^{-1}(U)$ is open in Y. Again by the definition of D - quotient topology, the converse is immediate.

5. G_δ - REGULAR SPACES AND COMPLETELY G_δ - REGULAR SPACES

In this section we study the behaviour of G_δ-regular spaces and completely G_δ regular spaces under D - supercontinuous functions.

Definition[3] 5.1 — A space X is said to be a G_δ **- *regular*** if for every closed G_δ- set F and a point $x \notin F$, there exist disjoint open sets U and V such that $F \subset U$ and $x \in V$.

Definition[3] 5.2 — A space X is said to be ***completely*** G_δ**- *regular*** if for every closed G_δ - set A and a point $x \notin A$, there is a continuous function f from X into the closed unit interval [0, 1] such that $f(x) = 0$ and $f(A) = 1$.

Theorem 5.1 — *Let $f: X \rightarrow Y$ be a D - supercontinuous, open bijection from a G_δ- regular space X onto Y. Then Y is a regular space.*

PROOF : Let A be any closed subset of Y and let $y \notin A$. Then $f^{-1}(A) \cap f^{-1}(y) = \phi$. Since f is D - supercontinuous by Theorem 3.1 $f^{-1}(A)$ is d-closed and so $f^{-1}(A) = \bigcap_{\alpha \in \Lambda} F_\alpha$, where each F_α is a closed G_δ - set. Since f is one - one $f^{-1}(y)$ is a singleton and so there exists $\alpha_0 \in \Lambda$ such that $f^{-1}(y) \notin F_{\alpha 0}$. By G_δ - regularity of X, there exist disjoint open sets U and V containing $F_{\alpha 0}$ and $f^{-1}(y)$, respectively. Since f is an open one - one map, $f(U)$ and $f(V)$ are disjoint open sets containing A and y, respectively. Hence Y is regular.

Definition 5.3 — A function $f: X \rightarrow Y$ is said to be a ***d - homeomorphism*** if f is a bijection such that both f and f^{-1} are D - supercontinuous.

Lemma 5.1 — A space X is completely G_δ - regular if and only if for every d - closed set A and a point $x \notin A$, there is a continuous function f from X into the closed interval [0, 1] such that $f(x) = 0$ and $f(A) = 1$.

PROOF : Since every closed G_δ- set is d-closed sufficiency is obvious. To prove necessity let A be a d-closed set in X not containing the point x. Then there exists an open F_α - set G containing x such that $G \cap A = \phi$. Now X - G is a closed G_δ - set in X. So by complete G_δ - regularity of X there exists a continuous function f from X into [0, 1] such that $f(x) = 0$ and $f(X - G) = 1$. Since $A \subset X - G, f(A) = 1$.

Theorem 5.2 — *A d-homeomorphic image of a completely G_δ - regular space is completely regular.*

PROOF : Let $f: X \to Y$ be a d - homeomorphism of a completely G_δ - regular space X onto a space Y. Let A be a closed set in Y and let $y \notin A$. Then $f^{-1}(y) = \{x\}$ is a singleton and x is not in the d - closed set $f^{-1}(A)$. By Lemma 5.1 there exists a continuous function $g: X \to [0, 1]$ such that $g(x) = 0$ and $g(f^{-1}(A)) = 1$. Let $h = g \circ f^{-1}$. Since f is a d-homeomorphism, h is well defined and f^{-1} is D - supercontinuous and so h is continuous. Moreover, $h(y) = 0$ and $h(A) = 1$ and thus Y is completely regular.

Next we quote the following results pertaining to (completely) G_δ - regular spaces which are immediate consequences of the results in Section 4 and 5 of [3] and immediately follow on substituting $P = G_\delta$ - set in the respective results therein.

Theorem 5.3 — *Let X be G_δ - regular space. Let $f: X \to Y$ be a continuous, open and closed surjection such that either X or Y is T_1. If $f^{-1}(y)$ is a G_δ - set for each $y \in Y$. Then Y is Hausdorff.*

Theorem 5.4 — *Let X be a Hausdorff G_δ - regular space and let A be a closed G_δ - set. Then the quotient space obtained from X by identifying A to a point is Hausdorff.*

Theorem 5.5 — *Let X be a completely G_δ - regular space. If K and F are disjoint subsets of X such that K is compact and F is a closed G_δ - set, then there exists a continuous function $f: X \to [0, 1]$ such that $f(K) = 0$ and $f(F) = 1$.*

Theorem 5.6 — *Let X be a completely G_δ - regular space. If K is a compact G_δ which is expressible as a countable intersection of open F_σ - sets, then K is a zero set in X.*

Theorem 5.7 — *Let $f: X \to Y$ be a D - continuous closed surjection defined on a regular space X. If either f is open or if $f^{-1}(y)$ is compact for each $y \in Y$, then Y is a G_δ - regular space.*

Theorem 5.8 — *Let $f: X \to Y$ be an open closed D - continuous surjection defined on a completely regular space X. Then Y is a completely G_δ- regular space.*

Theorem 5.9 — *A (completely) D - regular space is (completely) regular if and only if it is (completely) G_δ - regular.*

6. *D* - REGULARIZATION

In this section we show that if the domain of a D - supercontinuous function is retopologized in an appropriate way, then f is simply a continuous function.

D - REGULARIZATION

Let (X, τ) be a topological space, and let β denote the collection of all open F_σ - subsets of (X, τ). Since the intersection of two open F_σ -sets is an open F_σ - set, the collection β is a base for a topology τ^* on X called the ***D-regularization*** of τ. Clearly $\tau^* \subset \tau$. The space (X, τ) is D - regular if and only if $\tau^* = \tau$.

Throughout the section, the symbol τ^* will have the same meaning as in the above paragraph.

Theorem 6.1 — *The function* $f : (X, \tau) \to (Y, \mathcal{J})$ *is* D - *supercontinuous if and only if* $f : (X, \tau^*) \to (Y, \mathcal{J})$ *is continuous.*

Theorem 6.2 — *Let* (X, τ) *be a topological space. Then the following are equivalent.*

(a) (X, τ) is D - regular.

(b) Every continuous function from (X, τ) into a space $(Y, \mathcal{J})$ is D - supercontinuous.

PROOF : (a) $\Rightarrow$ (b) is obvious.

(b) $\Rightarrow$ (a) : Take $(Y, \mathcal{J}) = (X, \tau)$. Then the identity function 1_x on X is continuous, and hence D - supercontinuous. Thus by Theorem 6.1, $1_x : (X, \tau^*) \to (X, \tau)$ is continuous. Since $U \in \tau$ implies $1_x^{-1}(U) = U \in \tau^*$, $\tau \subset \tau^*$. Therefore $\tau^* = \tau$, and so (X, τ) is D - regular.

Many of the results studied in Section 3 follow now from Theorem 6.1 and the corresponding standard properties of continuous functions.

Theorem 6.3 — *Let* $f : (X, \tau) \to (Y, \mathcal{J})$ *be a function. Then*

(a) f is D - continuous if and only $f : (X, \tau) \to (Y, \mathcal{J}^*)$ is continuous.

(b) f is d - open if and only if $f : (X, \tau^*) \to (Y, \mathcal{J})$ is open.

PROOF : Obvious.

In the light of Theorems 6.1 and 6.3 Theorem 3.7 can be restated as follows. If $f : (X, \tau^*) \to (Y, \mathcal{J})$ is a continuous open subrjection and $g : (Y, \mathcal{J}) \to (Z, \nu)$ is a function, then g is continuous if and only if $g \circ f$ is continuous and Theorem 3.10 is simply the result that the composition $g \circ f$ of the continuous functions $f : (X, \tau) \to (Y, \mathcal{J}^*)$ and $g : (Y, \mathcal{J}^*) \to (Z, \nu)$ is continuous.

Moreover, D - quotient topology on Y determined by $f : (X, \tau) \to Y$ in Section 4 coincides with the usual quotient topology on Y determined by $f : (X, \tau^*) \to Y$.

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