*Indian J. pure appl. Math.*, **32**(3) : 343-344, March 2001


# SOME EQUIVALENT OF EXTREMALLY DISCONNECTED SPACES

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(*Received* 7 *July* 1999; *accepted* 26 *June* 2000)

This paper gives some equivalent characterizations of extremally disconnected spaces.

**Key Words : Open Sets; Closed Sets; Interior; Closure; Extremally Disconnected Space**

Extremally disconnected spaces play a prominent role in set-theoretical topology. Due to their peculiar properties extremally disconnected spaces provide curcial applications in the theory of Boolean algebra, in axiomatic set theory and in some branches of functional analysis (for example: in $C^*$-algebra) as well. There are many interesting equivalent of extremally disconnected spaces. See for example: problems 1H, 3N, 6M of [3] and, in case of Boolean algebra, Theorem 2.33 of [2].

In this short paper we prove some set-theoretical equivalent of extremally disconnected spaces. Proofs of these equivalent conditions are not trivial. But as a good exercises details are left to the readers.

Recall that a topological space $X$ is said to be extremally disconnected if the closure of every open set of $X$ is open in $X$. We denote by $A^o$ the interior of $A$ and by $A^-$ the closure of $A$.

*Lemma* 1 — Let $X$ be a topological space. If $A$ is a subset of $X$ and $B$ is an open subset of $X$ then

$$A^{-o} \cap B^{-o} = (A \cap B)^{-o}.$$

PROOF : Use the fact $A \cap B^{-o} \subset (A \cap B)^-$. □

As a Corollary to this we prove the Lemma 2.34 of [2].

*Corollary* 2 — If $A$ and $B$ are open subsets of a topological space $X$ then

$A^{-o} \cap B^{-o} = (A \cap B)^{-o}$.

***Theorem*** 3 — *The following are equivalent for a space X :*

*(a) X is extremally disconnected;*

*(b)* $A^- \cap B^- = (A \cap B)^-$ *for all open subsets A and B of X;*

*(c)* $A^- \cap B^- = 0$ *for all open subsets A and B of X with* $A \cap B = 0$;

*(d)* $K^{-o-} \cap A^- = 0$ *for all subsets K and all open subsets A of X with* $K \cap A = 0$;

*Research is supported by University Grants Commission, New Delhi, E-mail: kuru@doe.ernet.in.

*(e)* $K^{-o} \cap A^{-} = (K \cap A)^{-o}$ *for all subsets K and all open subsets A of X with* $K \cap A = 0$;

*(f)* $A^{-o} \cup B^{-o} = (A \cup B)^{-o}$ *for all open subsets A and B of X; and*

*(g)* $G^{o} \cup H^{o} = (G \cup H)^{o}$ *for all closed subsets G and H of X.*

PROOF : (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (a) $\Leftrightarrow$ (e) $\Rightarrow$ (d) $\Rightarrow$ (c) $\Rightarrow$ (a) $\Rightarrow$ (f) $\Rightarrow$ (g) $\Rightarrow$ (c) $\Rightarrow$ (a). □

*Remark* : Above Theorem seems to be an isolated result, but this will be useful to the readers those are interested in generalization of concepts having closed relation with extremally disconnected spaces. For example: in generalization of certain properties of spaces with filters and $\infty$-disconnected (extremally disconnected) spaces (cf. [1]).

## REFERENCES


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