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MHD THREE DIMENSIONAL FLOW PAST A POROUS PLATE

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This paper analysed the effects of magnetic field on the three dimensional flow of an incompressible viscous fluid past a porous plate under the following conditions : (*i*) The fluid is electrically conducting (*ii*) The free stream velocity is uniform (*iii*) The plate is subjected to a periodic suction velocity distribution (*iv*) The plate temperature is constant (*v*) A magnetic field of uniform strength is applied in the direction normal to the plate. Approximate solutions for the velocity field, temperature and skin friction are derived and the effects of Hartmann number (M) & Suction parameter (α) on the velocity field and skin friction are discussed with the help of graph and tables.

Key Words : **Hartmann Number; Reynolds Number; Prandtl Number; Permeability Parameter; Suction Prameter; Eckert Number**

Nomenclature

Bo - Magnetic field component along y^*-axis.

Cp - Specific heat of the fluid at constant pressure.

E - Eckert number

g - Acceleration due to gravity

k - Thermal conductivity of the fluid.

K^* - Permeability of the porous medium.

K - Permeability parameter

L - Half-wave length of the periodic suction velocity.

M - Hartmann number

p^* - Pressure

P - Prandtl number

R - Reynolds number

T^* - Temperature of the fluid

T^*_w - Temperature of the plate

T^*_∞ - Temperature of the fluid far away from the plate

u^*, v^*, w^* - Velocity components in x^*, y^* and z^* directions

u, v, w - Dimensionless velocity components

U - Free stream velocity

v_0 - Constant veloity

V - Basic steady distribution

x^*, y^*, z^* - Coordinate system

y, z - Dimensionless coordinates

ρ - Density of the fluid

ν - Kinematic viscosity

μ - viscosity

α - Suction parameter.

1. Introduction

The problem MHD laminar flow through a porous medium has become very important in recent years particularly in the fields of agricultural engineering to study the undergound water resources, seepage of water in river beds; in chemical engineering for filtration and purification process; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs.

Yamamto Iwamura[1] expressed the equations of flow through a highly porous medium under the influence of temperature differences. Raptis[2] studied the free convective flow through a porous medium bounded by an infinite vertical plate with oscillating plate temperature and constant saction. Raptis and Perdikis[3] further analysed the free convective flow through a highly porous medium bounded by an infinite vertical porous plate with constant suction when the free stream velocity oscillates about a mean constant value.

In above studies the investigators have restricted themselves to two dimensional flows. But there may arise situations where the flow fields may be essentially three dimensional. Singh *et al.*[4] have investigated the three dimensional convective flow and heat transfer in a porous medium. Singh[5] analysed the three dimensional flow and heat transfer along a porous plate in the presence of viscous dissipative heat. Further Singh[6] studied the effects of a uniform magnetic field applied normal to the free convection flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate with a slightly sinusoidal transverse suction velocity distribution. Kumari *et al.*[7] studied flow and heat transfer of a visco-elastic fluid over a flat plate with a magnetic field and a pressure gradient. Ahmed and Sarma[*] discussed the problem of three-dimensional free convective flow of an incompressible viscous fluid through a porous medium with uniform free stream velocity.

The purpose of the present paper is to study the hydromagnetic effrects of electrically conducting three dimensional flow of viscous incompressible fluid through a porous medium which is bounded by an infinite vertical porous plate with periodic suction at constant temperature.

2. Mathematical Analysis

Consider three-dimensional flow of viscous incompressible fluid through a highly porous medium which is bounded by a vertical infinite porous plate. We choose a coordinate system with plate lying vertically on x^* - z^* plane such that x^* axis is taken along the plate in the direction of the flow and is taken along the plate in the direction of the flow and y^* axis is perpendicular to the plane of the plate and directed into the fluid which is flowing with free stream velocity U. All physical quantities will be independent of x^*, however, the flow remains three dimensional due to the variation of the suction velocity distribution of the form.

$$v^*(z^*) = -V\left(1 + \varepsilon \, Cos \frac{\pi z^*}{L}\right).$$

The negative sign in the above equation indicates that the suction is towards the plate. The equations which govern the problem are :

Continuity Equation :

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0. \qquad \dots (1)$$

Momentum Equation :

$$v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \nu\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) - \frac{\sigma B_0^2 u^*}{\rho}. \qquad \dots (2)$$

$$v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right). \qquad \dots (3)$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \nu\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \frac{\sigma Bo^2 w^*}{\rho}. \qquad \dots (4)$$

Energy Equation :

$$\rho C_p\left(v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial Z^*}\right) = k\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial Z^{*2}}\right) + \mu \phi, \qquad \dots (5)$$

where

$$\phi^* = 2\left\{\left(\frac{\partial v^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial Z^*}\right)^2\right\} + \left\{\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*} + \frac{\partial w^*}{\partial Z^*}\right)^2 + \left(\frac{\partial u^*}{\partial Z^*}\right)^2\right\} \qquad \dots (6)$$

The boundary conditions of the petroleum are :

$$y^* = 0;\ u^* = 0,\ v^* = -V\left(1 + \varepsilon \cos \frac{\pi z^*}{L}\right) w^* = 0,\ T^* = T_w^*$$

$$y^* \to \infty;\ u^* = U,\ p^* = p_\infty^*. \qquad \dots (7)$$

We now introduce the following non-dimensional variables :

$$y = \frac{y^*}{L},\ z = \frac{z^*}{L},\ u = \frac{u^*}{U},\ v = \frac{v^*}{U},\ w = \frac{w^*}{U}$$

$$p = \frac{p^*}{pU^2},\ \theta = \frac{T^* - T_\infty^*}{T^* w - T_\infty^*}$$

Reylonds Number (R) $R = \frac{UL}{v}$

Hartmann Number (M) : $M = \frac{\sigma Bo^2 \, v L}{U \mu}$

Prandtl Number (P) : $P = \frac{\mu C_p}{k}$

Eckert Number (E) : $E = \frac{U^2}{Cp\,(T^* w - T^*_\infty)}$

Permeability Parameter (K) : $K = \frac{K^* U^2}{v^2}$

Suction Parameter (α) : $\alpha = \frac{V}{U}$.

With the help of above non-dimensional variables eqs. (1) to (5) become :

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \qquad \text{... (8)}$$

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - Mu, \qquad \text{... (9)}$$

$$v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{R}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right), \qquad \text{... (10)}$$

$$v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{R}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - Mw \qquad \text{... (11)}$$

and

$$v \frac{\partial \theta}{\partial y} \; w \frac{\partial \theta}{\partial z} = \frac{1}{PR}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + \frac{E}{R}\phi, \qquad \text{... (12)}$$

where

$$\phi = 2\left\{ \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right\} + \left\{ \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 \right\}.$$

The corresponding boundary conditions become :

$$y = 0;\; u = 0,\; v = -\alpha(1 + \varepsilon \cos \pi z),\; w = 0,\; \theta = 1$$

$$y \to \infty;\; u = 1,\; p = p_\infty,\; w = 0 \;\; \theta = 0. \qquad \text{... (13)}$$

In order to solve these differential equations, we assume that :

$$u(y, z) = u_0(y) + \varepsilon u_1(y, z),$$

$$v(y, z) = v_0(y) + \varepsilon v_1(y, z),$$

$$w(y, z) = w_0(y) + \varepsilon w_1(y, z),$$

$$p(y, z) = p_0(y) + \varepsilon p_1(y, z)$$

and $$\theta(y, z) = \theta_0(y) + \varepsilon \theta_1(y, z). \quad \ldots (14)$$

On substituting eq. (14) in eqs. (8), (9) and (12) and equating the terms free from ε, we get the following system of differential equations :

$$v_0' = 0, \quad \ldots (15)$$

$$u_0'' + \alpha R u_0 - MRu_0 = 0 \quad \ldots (16)$$

and $$\theta_0'' + \alpha RP\, \theta_0 = -EPu_0'^2, \quad \ldots (17)$$

where primes denote the differentiation with respect to y.

The corresponding boundary conditions become :

$$y = 0;\ u = 0;\ v_0 = -\alpha,\ w_0 = 0,\ \theta_0 = 1$$

$$y \to \infty,\ u_0 = 1,\ p_0 = p_\infty\ w_0 = 0,\ \theta_0 = 0. \quad \ldots (18)$$

The solutions of eqs. (15) to (17) under the boundary conditions (18) are

$$v_0 = -\alpha, \quad \ldots (19)$$

$$u_0(y) = 1 - e^{-my} \quad \ldots (20)$$

and $$\theta_0(y) = e^{\alpha PRy} + E_1\left(e^{-\alpha PRY} - e^{-2my}\right), \quad \ldots (21)$$

where $$m = \frac{1}{2}\left\{\alpha R + \sqrt{\alpha^2 R^2 + 4MR}\right\}$$

and $$E_1 = \frac{EPm}{2(2m - \alpha PR)}.$$

On substituting eq. (14) in eqs. (8) to (12) and equating the coefficient of ε, we get the following system of differential equations :

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \quad \ldots (22)$$

$$v\frac{\partial u_0}{\partial y} - \alpha\frac{\partial u_1}{\partial y} = \frac{1}{R}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - Mu_1, \quad \ldots (23)$$

$$-\alpha\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right), \quad \text{... (24)}$$

$$-\alpha\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right)-Mw_1 \quad \text{... (25)}$$

and

$$v_1\frac{\partial \theta_0}{\partial y}-\alpha\frac{\partial \theta_1}{\partial y}=\frac{1}{PR}\left(\frac{\partial^2 \theta_1}{\partial y^2}+\frac{\partial^2 \theta_1}{\partial z^2}\right)+\frac{2E}{R}\cdot\frac{\partial u_0}{\partial y}\cdot\frac{\partial u_1}{\partial y}. \quad \text{...(26)}$$

The corresponding boundary conditions become :

$$y = 0;\ u_1=0,\ v_1=-\alpha\cos\pi z,\ w_1=0,\ \theta_1 = 0$$

$$y\to\infty;\ u_1=0\ p_1 = 0\ w_1=0,\ \theta_1=0. \quad \text{... (27)}$$

In order to solve eqs. (24) and (25), we assume that :

$$v_1(y,z)=v_{11}(y)\cos\pi z, \quad \text{... (28)}$$

$$w_1(y,z)=-\frac{1}{\pi}v_{11}(y)\sin\pi z \quad \text{... (29)}$$

and

$$p_1(y,z)=p_{11}(y)\cos\pi z. \quad \text{... (30)}$$

On substituting these equations in eqs. (24) and (25), we get the following equations :

$$v_{11}''+R\alpha v_{11}'-\pi^2 v_{11}=Rp_{11}' \quad \text{... (31)}$$

and

$$v_{11}'''+R\alpha v_{11}''-(\pi^2+MR)v_{11}'=R\pi^2 p_{11}. \quad \text{... (32)}$$

The corresponding boundary conditions become :

$$y = 0;\ v_{11}=-\alpha,\ v_{11}'=0$$

$$y\to\infty;\ v_{11}=0,\ v_{11}'=0,\ p_{11}=0 \quad \text{... (33)}$$

On solving eqs. (31) and (32) under the boundary conditions (33), we get

$$v_1=\frac{\alpha}{r_2-r_3}[r_3 e^{-r_2 y}-r_2 e^{-r_3 y}]\cos\pi z \quad \text{... (34)}$$

and

$$w_1=\frac{\alpha r_2 r_3}{\pi(r_2-r_3)}\{e^{-r_2 y}-e^{-r_3 y}\}\ \sin\ \pi z, \quad \text{... (35)}$$

where

$$r=\frac{1}{2}\ \{m+\sqrt{m^2+4\pi^2}\},$$

$$r_3 = \frac{1}{2}\ \{n + \sqrt{n^2 + 4\,\pi^2}\}$$

and $$n = \frac{1}{2}\,\{R\,\alpha - \sqrt{R^2\,\alpha^2 + 4RM}\}.$$

In order to solve the eqs. (23) and (26), we assume that

$$u_1\,(y, z) = u_{11}\,(y)\cos \pi z \qquad \text{... (36)}$$

and $$\theta_1\,(y, z) = \theta_{11}\,(y)\cos \pi z. \qquad \text{... (37)}$$

On substituting eqs. (26) & (37) in eqs. (23) & (26), we get the following equations :

$$u_{11}'' + \alpha R u_{11}' - (\pi^2 + MR)\,u_{11} = Rv_{11}\,u_0' \qquad \text{... (38)}$$

and $$\theta_{11}'' + \alpha\,PR\,\theta_{11}' - \pi^2\,\theta_{11} = RP\theta_0'\,v_{11} - 2EPu_0'\,u_{11}'. \qquad \text{... (39)}$$

The corresponding boundary conditions become :

$$y = 0;\ u_{11} = 0,\ \theta_{11} = 0$$

$$y \to \infty;\ u_{11} = 0,\ \theta_{11} = 0 \qquad \text{... (40)}$$

On solving eqs. (38) & (39) under the boundary conditions (40), we get

$$u = e^{-my}\,\frac{\varepsilon R\,\alpha m}{r_2 - r_3}\left\{A_1\,e^{-(r_2 - m)\,y} - A_2\,e^{-(r_3 + m)\,y} - e^{-r_4 y}\right\}\cos \pi z \qquad \text{... (41)}$$

$$\theta = e^{\alpha\,PRy} + E_1\,(e^{-\alpha\,PRy} - e^{-2my}) + \frac{\varepsilon\,\alpha^2\,P^2R^2}{r_2 - r_3}$$

$$\left\{B_1\,e^{-(r_2 - \alpha\,PR)y} - B_2\,e^{-(r_3 - \alpha\,PR)y} - B_3\,e^{-(r_2 + \alpha\,PR)\,y} + B_4\,e^{-(r_3 + \alpha\,PR)\,y}\right\}$$

$$+\ \frac{2m\,\alpha\,PR}{r_2 - r_3}\left\{B_5\,e^{-(r_2 + 2m)y} - B_6\,e^{-(r_3 - 2m)y} - B_7\,e^{-(m + r_4)\,y}\right\} - Ce^{-r_5}, \qquad \text{... (42)}$$

where $$r_4 = \frac{1}{2}\left\{\alpha R + \sqrt{\alpha^2\,R^2 + 4\,(\pi^2 + MR)}\right\},$$

$$A_1 = \frac{r_3}{(r_2 + m)^2 - \alpha R\,(r_2 + m) - (\pi^2 + MR)},$$

$$A_2 = \frac{r_2}{(r_3\,m)^2 - \alpha R\,(r_3 + m) - (\pi^2 + MR)},$$

$$B_1 = \frac{r_3}{(r_2 - \alpha PR)^2 - \alpha PR\,(r_2 - \alpha PR) - \pi^2},$$

$$B_2 = \frac{r_2}{(r_3 - \alpha PR)^2 - \alpha PR\,(r_3 - \alpha PR) - \pi^2},$$

$$B_3 = \frac{E_1\, r_3}{(r_2 + \alpha PR)^2 - \alpha PR'(r_2 + \alpha PR)\,\pi^2},$$

$$B_4 = \frac{E_1\, r_2}{(r_3 + \alpha PR)^2 - \alpha PR\,(r_3 + \alpha PR) - \pi^2},$$

$$B_5 = \frac{D_1}{(r_2 + 2m)^2 - \alpha PR\,(r_2 + 2m) - \pi^2},$$

$$B_6 = \frac{D_2}{(r_3 + 2m)^2 - \alpha PR\,(r_3 + 2m) - \pi^2},$$

$$B_7 = \frac{Em\, r_4}{(m + r_4)^2 - \alpha PR(m + r_4) - \pi^2},$$

$$D_1 = E_1\, r_3 + Em\, A_1\,(r_2 + m)$$

$$D_2 = E_1\, r_2 + Em\, A_2\,(r_3 + m)$$

$$r_5 = \frac{1}{2}\,[\alpha PR + \sqrt{\alpha^2 P^2 R^2 + 4\,\pi^2}]$$

and $$C = \frac{\alpha^2 P^2 R^2}{r_2 - r_3}\,(B_1 - B_2 - B_3 + B_4) - \frac{2m\,\alpha PR}{r_2 - r_3}\,(B_5 - B_6 - B_7).$$

Knowing the velocity field we can obtain the expressions for the shear stress components in the x^* and z^* directions in the non-dimensional form as :

$$\tau_x = \frac{\tau_x^*}{\rho\, UV} = \frac{\nu}{VL}\left(\frac{\partial u}{\partial y}\right)_0 = \frac{m}{\alpha R} = \varepsilon\,\{1 - F_1\,(\alpha, R, M)\}\ \cos\,\pi z \qquad \text{... (43)}$$

and $$\tau_z = \frac{\tau_z^*}{\mu\, V/L} = \frac{U}{V}\left(\frac{\partial w}{\partial y}\right)_0 = -\,\varepsilon\, F_2\,(\alpha, R, M)\,\sin\,\pi z, \qquad \text{... (44)}$$

where $$F_1\,(\alpha, R, M) = 1 - \frac{m}{r_2 - r_3}\ \{r_4 + A_2\,(r_3 + m) - A_1\,(r_2 + m)\} \qquad \text{... (45)}$$

and $$F_2(\alpha, R, M) = \frac{\alpha r_2 r_3}{\pi}. \qquad \ldots (46)$$

3. CONCLUSIONS

The effects of varying the Hartmann number (M) and Suction parameter (α) with $R = 1$, $z = 0$ & $\varepsilon = 0.2$ on velocity and skin friction are shown in Fig. 1 and tables.

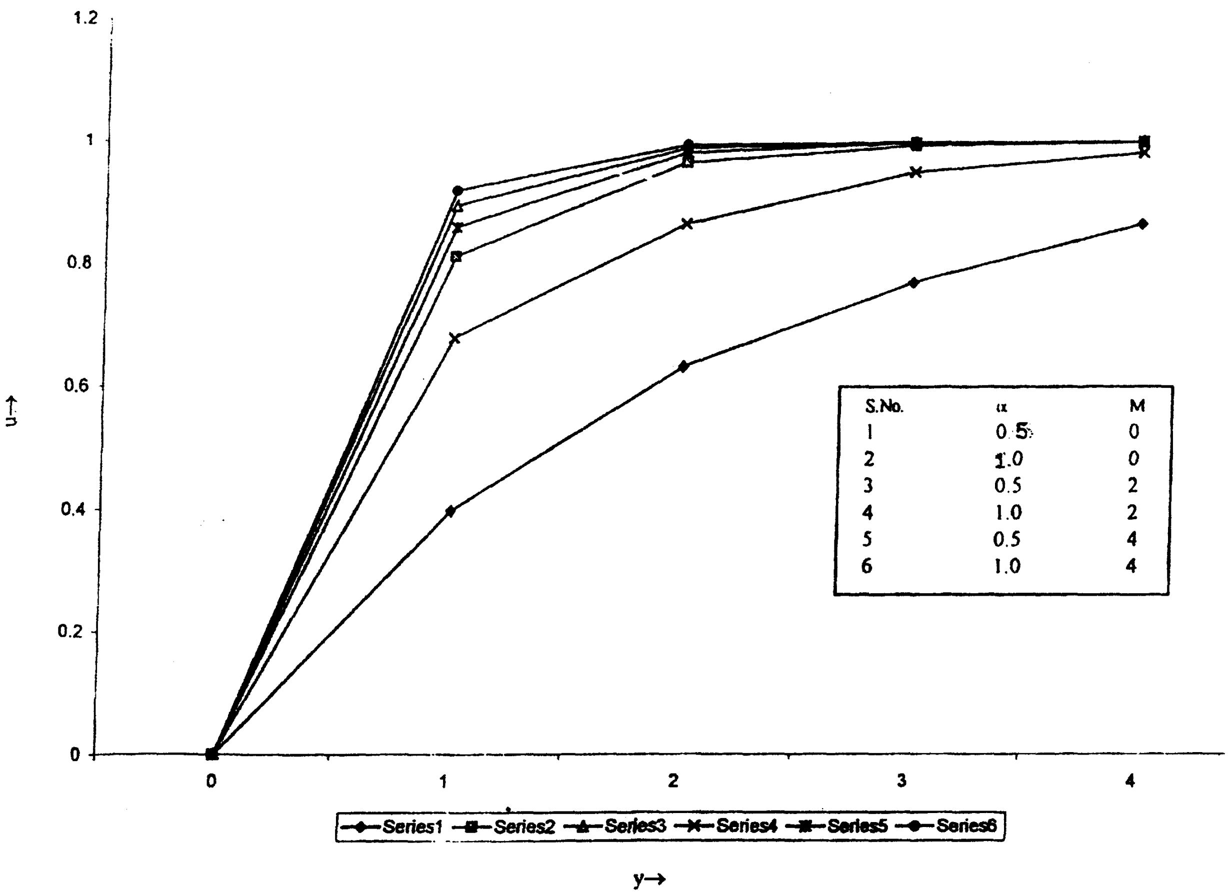

FIG. 1. Velocity profiles for $R = 1$, $Z = 0$, $\epsilon = 0.2$.

It is observed from the figure that the velocity increases with the increase of Hartmann number (M) and suction prameter (α) both. It is also observed from the figure that the velocity increases upto $y = 4$ and then after it becomes linear.

It is observed from the Table I that the function $F_1(\alpha, R, M)$ increases with the increase of suction parameter (α) in the absence of magnetic field but in the presence of magnetic field it decreases. We can also conclude from this figure that when we apply magnetic field, the function $F_1(\alpha, R, M)$ increases and when we increase magnetic field from $M = 2$ to $M = 4$ it decreases very slightly.

TABLE I

Function $F_1(\alpha, R, M)$ at $R = 1$

α	$M = 0$	$M = 2$	$M = 4$
0.5	1.6999	1.6967	1.6939
1.0	3.6796	3.6655	3.6524
1.5	5.9675	5.9341	5.9032

From Table II it is clear that the function $F_2(\alpha, R, M)$ decreases with the increase of magnetic field but it increases with the increase of suction parameter (α).

TABLE II

Function $F_2(\alpha, R, M)$ at $R = 1$

α	$M = 0$	$M = 2$	$M = 4$
0.5	−13.7762	−6.7958	−6.8673
1.0	−11.1099	−7.2075	−7.5672
1.5	−10.5515	−8.3515	−8.8642

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