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ON FREE CONVECTION OF A DUSTY CONDUCTING FLUID

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This study deals with unsteady laminar free convection flow of an incompressible, viscous, electrically conducting dusty fluid through a porous medium, which is bounded by an infinite vertical plane surface of constant temperature, in the presence of a uniform magnetic field, acting perpendicular to the surface. Laplace transform techniques are used to derive the solution of the governing equations in the transformed domain. The inversion process is carried out using a numerical method based on Fourier series expansion.

The effects of material parameters such as Grash of number, Prandtl number, permeability parameter and mass concentration of particle phase on the velocity of the fluid, the velocity of the particle phase, the temperature of the fluid, the temperature of the particle phase as well as for the induced magnetic and the electric fields are all discussed. Numerical computations are presented graphically and discussed.

Key Words : Free Convection; Conducting Fluid; Unsteady Motion; Dusty Fluid

1. INTRODUCTION

The study of the flow of dusty fluids is of practical importance, particularly the flow through packed beds, sedimentation, environmental pollution, and centrifugal separation of particles. Considering blood as a two-phase fluid, this study also gives some insight into the flow of the blood through veins or arteries.

The study of the boundary layer of fluid particle suspension flow is important in determining the investigations on boundary layer behaviour in particulate suspensions. Among these are studies[1&2] on the steady flow of a two-phase suspension past a semi-infinite flat plate. Over the years, many analytical or approximate solutions and the solution of the magnetohydrodynamic (MHD) dusty fluid boundary layer solutions have been obtained in [3-7]. On the other hand there have been numerous theoretical and experimental studies of heat and mass transfer induced by natural convection in fluids. These studies have many applications in physical systems where heat transport by buoyancy-induced convective motion takes place, such as chemical reactor, nuclear reactor, combustion systems, pneumatic transport, etc. In some of these applications the fluid may contain suspended dust particles.

In recent years, the requirement of modern technology has stimulated interest in fluid flow studies, which involve the interaction of several phenomena. One such study is related to the effects of free convection flow through a porous medium, which plays an important role in agriculture, engineering, and petroleum industries and heat transfer. The convection problem in a porous medium also has important applications in geothermal reservoirs and geothermal energy extractions. It is clear that in order to utilize the geothermal energy to the maximum, one should have a complete and precise knowledge of the amount of perturbations needed to generate convection current in geothermal fluids. Also, the knowledge of quantity of perturbations essential to initiate convection currents in mineral fluids found in the earth's crust helps one to utilize mineral energy to extract the minerals[8]. The free convection effects on the Stokes problem for an infinite verticalplate in a dusty fluid have been studied analytically in [9]. Investigation of the natural convection of clear fluid in a rectangular cavity for transient heating of the vertical walls has been conducted by several authors[10-13]. They found analytical and numerical solutions, and they have shown that a number of initial flow types are possible, which ultimately lead to two types of steady flow determined by relative values of dimensionless parameters, e.g., Prandtl number, Rayleigh number describing the flow. In [14], natural convection of a dusty fluid in an infinite rectangular channel with differentially heated vertical walls and adiabatic horizontal walls has been studied.

The study is an extension of the publications cited in reference [6]-[14] taking into account the effect of the electro-magnetic field, the volume fraction of particle phase and the permeability of the porous medium. The objective is to determine the nature of the subsequent transient motion driven by buoyancy forces.

2. Mathematical Formulation

We will consider unsteady free convection flow of electrically conducting, incompressible, viscous dusty fluid (e.g. seawater, rainwater and sewage) past an infinite vertical plate. The x-axis is taken in the vertical direction along the plate and the y-axis normal to it. All the influence of the density variation with temperature is considered only in the body force term. Constant magnetic field of strength H_0 acts in the direction of the y-axis. These produce an induced magnetic field $\boldsymbol{h}$ and an induced electric field $\boldsymbol{E}$ (assumed to be small). Due to infinite plane surface assumption, the flow variables are functions of y and t only, except for pressure P.

The electromagnetic quantities satisfy Maxwell's' equations[15]

$$curl\,\boldsymbol{h} = \boldsymbol{J} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}, \qquad \text{... (2.1)}$$

$$curl\,\boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{h}}{\partial t}, \qquad \text{... (2.2)}$$

$$div\,\boldsymbol{h} = 0,\ div\,\boldsymbol{E} = 0 \qquad \text{... (2.3)}$$

and

$$\boldsymbol{B} = \mu_0 (\boldsymbol{H}_0 + \boldsymbol{h}),\ \boldsymbol{D} = \varepsilon_0 \boldsymbol{E}, \qquad \text{... (2.4)}$$

where $\boldsymbol{J}$ is the electric current density, μ_0 and ε_0 is the magnetic and electric permeability, respectively and $\boldsymbol{B}, \boldsymbol{D}$ are the magnetic and electric induction vectors, respectively.

Ohm's law supplements these equations

$$\boldsymbol{J} = \sigma_0 (\boldsymbol{E} + \mu_0 \boldsymbol{u} \wedge [\boldsymbol{H_0} + \boldsymbol{h}]),$$

where σ_0 is the electric conductivity and $\boldsymbol{u}$ is the velocity vector. This equation can be linearized by neglecting small quantities of the second order giving

$$\boldsymbol{J} = \sigma_0 (\boldsymbol{E} + \mu_0 \boldsymbol{u} \wedge \boldsymbol{H_0}). \qquad \text{... (2.5)}$$

As mentioned above the applied magnetic field $\boldsymbol{H_0}$ has components $\boldsymbol{H_0} = (0, H_0, 0)$. The velocity components are $\boldsymbol{u} = (u\,(y, t), 0, 0)$. We assume that the induced magnetic field has the components $\boldsymbol{h} = (h(y, t), 0, 0)$. It can be seen from the above equations that the vectors $\boldsymbol{E}$ and $\boldsymbol{J}$ will have non-vanishing components only in the z-direction, i.e., $\boldsymbol{E} = (0, 0, E(y, t))$ and $\boldsymbol{J} = (0, 0, J(y, t))$ where J is given from eq. (2.5) in the form

$$J = \sigma_0 (E + \mu_0 H_0 u). \qquad \dots (2.6)$$

The vector eqs. (2.1) and (2.2) reduce to the following scalar equations

$$\frac{\partial h}{\partial y} = -\left[J + \varepsilon_0 \frac{\partial E}{\partial t} \right], \qquad \dots (2.7)$$

$$\frac{\partial E}{\partial y} = -\mu_0 \frac{\partial h}{\partial t}. \qquad \dots (2.8)$$

Eliminating J between eqs. (2.6) and (2.7), we obtain

$$\frac{\partial h}{\partial y} = -\sigma_0 \mu_0 H_0 u - \left[\sigma_0 E + \varepsilon_0 \frac{\partial E}{\partial t} \right]. \qquad \dots (2.9)$$

Eliminating E between eqs. (2.8) and (2.9), we get

$$\left(D^2 - \sigma_0 \mu_0 \frac{\partial}{\partial t} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \right) h = -\sigma_0 \mu_0 H_0 \frac{\partial u}{\partial y}, \qquad \dots (2.10)$$

where D is operator defined as

$$D \equiv \frac{\partial}{\partial y}. \qquad \dots (2.11)$$

Finally, the body forces in the direction of x-axis (Lorentz force), is given by

$$F_x = (\boldsymbol{J} \wedge \boldsymbol{B})_x = -\mu_0 H_0 J = \mu_0 H_0 \left[\frac{\partial h}{\partial y} + \varepsilon_0 \frac{\partial E}{\partial t} \right]. \qquad \dots (2.12)$$

If N is assumed to be the number density of particles i.e., $(N = \rho_p / m)$, and $(\tau_m = m/S)$ where $(S = 3\,\pi_p\,\mu)$, is the Stokes resistance coefficient, m is the mass of each particle, r_p is the particle diameter and τ_m, is called relaxation time during which the velocity of the particle phase relative to the fluid is reduced to $(1/e)$ times its initial value. Also $(\tau_T = mC_s / 2\pi k r_p)$ is thermal relaxation time of particle phase, i.e., time τ_T, the temperature of the particle phase relative to the fluid is $(1/e)$ times the initial value[14].

In most studies of dusty fluid flows, certain simplifying assumptions are usually made for dilute suspension[14]. The present study of laminar free convection is under the assumptions :

(*a*) Boussinesq's approximation is valid.

(*b*) The number density N of particles is constant.

(*c*) The solid particles are sparsely distributed and they are non-interacting, so that the pressure locally has the same velocity vector and temperature. Due to this assumption of defnciency of randomness in local particle motion, the pressure associated with the particle cloud is negligible. Hence the fluid pressure P will be the same as the total pressure of the mixture.

(*d*) In Stokes resistance coefficient neglecting the term, which represents the effect of electromagnetic field means that the electromagnetic field effects on the fluid and doesn't effect on the particle.

Following Saffman's model[16] of a dusty fluid, the governing equations under the stated conditions, neglecting the rate of work done by the particles due to the force of interaction with the fluid and the viscous dissipation of fluid is given by Marble[1] in the form of

Continuity equation

$$\frac{\partial v}{\partial y} = 0, \qquad \text{... (2.13)}$$

Momentum equation

$$\rho(1-\phi)\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial y^2} + \frac{\rho_p}{\tau_m}(u_p - u) + \rho g(T - T_\infty) - \frac{\mu}{K}u + F_x, \qquad \text{... (2.14)}$$

$$\rho_p\frac{\partial u_p}{\partial t} = -\frac{\rho_p}{\tau_m}(u_p - u), \qquad \text{... (2.15)}$$

$$C_p\rho\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + \frac{C_s\rho_p}{\tau_T}(T_p - T), \qquad \text{... (2.16)}$$

and

$$C_s\rho_p\frac{\partial T_p}{\partial t} = -\frac{C_s\rho_p}{\tau_T}(T_p - T), \qquad \text{... (2.17)}$$

where u, v are the velocity components, T is the temperature, ρ is the density, F_x is the body force, ϕ is the volume fraction of particle phase, K is the permeability of the porous medium and a subscript p in them denotes corresponding entities of particle phase. μ and C_p are the viscosity and specific heat of fluid, C_s is the specific heat particles.

The relevant boundary conditions are

$$\left.\begin{array}{l} y = 0: u = u_p = 0, T = T_w, h = 0, \\ y \to \infty: u = u_p = 0, T = T_\infty, h = 0 \end{array}\right] \qquad \text{.. (2.18)}$$

Integration of eq. (2.13) gives v as a function of t or constant in this work we will take it as being equal to zero.

Combining eqs. (2.12) and (2.14), we get

$$\rho(1-\phi)\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial y^2} + \frac{\rho_p}{\tau_m}(u_p - u) + g\rho\beta(T - T_\infty) - \frac{\mu}{K}u + \mu_0 H_0\left[\frac{\partial h}{\partial y} + \varepsilon_0\frac{\partial E}{\partial t}\right]. \qquad \text{... (2.19)}$$

Introduce the non-dimensional quantities

$$y' = \frac{Uy}{\upsilon},\ u' = \frac{u}{U},\ u'_p = \frac{u_p}{U},\ t' = \frac{U^2 t}{\upsilon},\ h' = \frac{h}{H_0},\ V'_H = \frac{V_H}{U}$$

$$\theta=\frac{T-T_\infty}{T_0},\ E'=\frac{E}{\mu_0 H_0 U},\ P=\frac{C_p\mu}{k},\ G=\frac{\upsilon g\beta T_0}{U^3},\ K'=\frac{U^2K}{\upsilon^2},\ \gamma=\frac{C_s}{C_p}, \qquad \text{... (2.20)}$$

$$R=\frac{\upsilon}{\upsilon_m},\ f=\frac{\rho_p}{\rho},\ \tau_T=\frac{3P\gamma}{2}\tau_m,\ \varepsilon_0'=\mu_0\,\varepsilon_0\,U^2,\ \beta_0=\frac{\upsilon}{\tau_m U^2},$$

where $V_H=H_0\sqrt{\mu_0/\rho}$, is Alfven velocity, T_0 is characteristic temperature, U is characteristic of velocity, P and G are Prandtl and Grashof number, $(\upsilon=\mu/\rho)$ is the kinematic viscosity of fluid, $[\upsilon_m=1/\mu_0\,\sigma_0]$ is the magnetic diffusivity, f is the mass concentration of the particle phase and β_0 is the dust parameter.

With the help of the non-dimensional quantities above, eqs. (2,.9), (2.10) and (2.14)-(2.17) are reduced to the non-dimensional equations (dropping primes for convenience)

$$(1-\phi)\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+f\beta_0\,(u_p-u)-\frac{u}{K}+G\,\theta+V_H^2\left(\frac{\partial h}{\partial y}+\varepsilon_0\frac{\partial E}{\partial t}\right), \qquad \text{... (2.21)}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P}\frac{\partial^2\theta}{\partial y^2}+\frac{2f\beta_0}{3P}(\theta_p-\theta), \qquad \text{... (2.22)}$$

$$\frac{\partial u_p}{\partial t}=\beta_0\,(u-u_p), \qquad \text{... (2.23)}$$

$$\frac{\partial^2 h}{\partial y^2}-R\frac{\partial h}{\partial t}-\varepsilon_0\frac{\partial^2 h}{\partial t^2}=-R\frac{\partial u}{\partial y}, \qquad \text{... (2.24)}$$

$$\frac{\partial\theta_p}{\partial t}=\frac{2\beta_0}{3P\gamma}(\theta-\theta_p) \qquad \text{... (2.25)}$$

and

$$\frac{\partial h}{\partial y}=-Ru-\left[RE+\varepsilon_0\frac{\partial E}{\partial t}\right]. \qquad \text{.. (2.26)}$$

The reduced boundary conditions are

$$\left.\begin{array}{l} y=0:\ u=u_p=0,\ \theta=\theta_0,\ h=0,\\ y\to\infty:\ u=u_p=0,\ \theta=0,\ h=0 \end{array}\right] \qquad \text{... (2.27)}$$

where $\theta_0=(T_w-T_\infty)/T_0$ is a constant.

3. Solution in the Transformed Domain

Taking Laplace transform with parameter s (denoted by bar) of both sides of equations (2.21)-(2.26), and using the homogeneous initial conditions, we get

$$(1-\phi)\, s\, \bar{u} = \frac{\partial^2 \bar{u}}{\partial y^2} + f \beta_0 (\bar{u}_p - \bar{u}) - \frac{\bar{u}}{K} + G\, \bar{\theta} + V_H^2 \left(\frac{\partial \bar{h}}{\partial y} + \varepsilon_0\, s\, \bar{E} \right), \quad \text{... (3.1)}$$

$$s\, \bar{\theta} = \frac{1}{P} \frac{\partial^2 \bar{\theta}}{\partial y^2} + f \gamma_0 (\bar{\theta}_p - \bar{\theta}), \quad \text{... (3.2)}$$

$$\left[\frac{\partial^2}{\partial y^2} - sR - \varepsilon_0 s^2 \right] \bar{h} = -R \frac{\partial \bar{u}}{\partial y}, \quad \text{... (3.3)}$$

$$s\, \bar{u}_p = \beta_0 (\bar{u} - \bar{u}_p), \quad \text{... (3.4)}$$

$$s\, \bar{\theta}_p = \gamma_0 (\bar{\theta} - \bar{\theta}_p) \quad \text{... (3.5)}$$

and

$$\frac{\partial \bar{h}}{\partial y} = -R\, \bar{u} - (R + \varepsilon_0 s)\, \bar{E}. \quad \text{... (3.6)}$$

From (3.4) and (3.5) we get

$$\bar{u}_p = \frac{\beta_0}{\beta_0 + s} \bar{u} \quad \text{... (3.7)}$$

and

$$\bar{\theta}_p = \frac{\gamma_0}{\gamma_0 + s} \bar{\theta}, \quad \text{... (3.8)}$$

where

$$\gamma_0 = \frac{2\beta_0}{3P\gamma}.$$

Eliminating $\bar{u}_p$ and $\bar{E}$ between (3.6), (3.7) and (3.1) we obtain

$$\left[\frac{\partial^2}{\partial y^2} - \left\{ \left(\frac{\beta_0 f}{\beta_0 + s} + (1-\phi) + \alpha^2 \varepsilon_0 \right) s + \frac{1}{K} \right\} \right] \bar{u} = -G\, \bar{\theta} - \alpha^2 \frac{\partial \bar{h}}{\partial y}, \quad \text{... (3.9)}$$

where

$$\alpha^2 = \frac{R V_H^2}{R + \varepsilon_0 s}.$$

Eliminating $\bar{\theta}_p$ between (3.8) and (3.2), we get

$$\left(\frac{\partial^2}{\partial y^2} - \left[1 + \frac{\gamma_0 f}{\gamma_0 + s} \right] Ps \right) \bar{\theta} = 0. \quad \text{... (3.10)}$$

Now, we can rewrite eqs. (3.9), (3.10) and (3.3) in the form

$$(D^2 - A_0)\, \bar{u} = -G\, \bar{\theta} - \alpha^2 D\, \bar{h}, \quad \text{... (3.11)}$$

$$(D^2 - B_0)\, \bar{\theta} = 0 \quad \text{... (3.12)}$$

and $$(D^2 - C_0)\,\bar{h} = -R\,D\,\bar{u}. \qquad \text{... (3.13)}$$

The corresponding reduced boundary conditions are

$$\left.\begin{array}{l} y=0:\ \bar{u}=\bar{u}_p=0,\ \bar{\theta}=\dfrac{\theta_0}{s},\ \bar{h}=0 \\ y\to\infty:\ \bar{u}=\bar{u}_p=0,\ \bar{\theta}=0,\ \bar{h}=0 \end{array}\right] \qquad \text{... (3.14)}$$

Eliminating $\bar{\theta}$ and $\bar{h}$ between eqs. (3.12), (3.13) and (3.11), we arrive at the following sixth order differential equation satisfied by $\bar{u}$

$$(D^6 - AD^4 + BD^2 - C)\,\bar{u} = 0, \qquad \text{... (3.15)}$$

where

$$\left.\begin{array}{l} A = A_0 + B_0 + C_0 + \alpha^2 R, \\ B = A_0 B_0 + B_0 C_0 + C_0 A_0 + \alpha^2 R_0, \\ C = A_0 B_0 C_0, \end{array}\right] \qquad \text{... (3.16)}$$

and

$$\left.\begin{array}{l} A_0 = \left(\alpha^2 \varepsilon_0 + 1 - \phi + \dfrac{f\beta_0}{\beta_0 + s}\right) s + \dfrac{1}{K}, \\ B_0 = \left(1 + \dfrac{\gamma_0 f}{\gamma_0 + s}\right) Ps, \\ C_0 = (R + \varepsilon_0\, s)\, s. \end{array}\right] \qquad \text{... (3.17)}$$

Eliminating $\bar{u}$ and $\bar{\theta}$ among eqs. (3.11)-(3.13), we find that $\bar{h}$ satisfies the same equation as $\bar{u}$ i.e.,

$$(D^6 - ADsp4 + D^2 - C)\,\bar{h} = 0. \qquad \text{... (3.18)}$$

Eq. (3.15) and (3.18) can be factorized as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)\,\bar{u} = 0, \qquad \text{... (3.19)}$$

where k_1^2, k_2^2 and k_3^2, are the roots of the auxiliary equation

$$k^6 - Ak^4 + Bk^2 - C = 0. \qquad \text{... (3.20)}$$

These roots are given by

$$\left.\begin{aligned} k_1^2 &= \frac{1}{2}\,[(A_0 + C_0 + \alpha^2 R) + \sqrt{(A_0 - C_0 + \alpha^2 R)^2 + 4\alpha^2 C_0 R}\,], \\ k_2^2 &= \frac{1}{2}\,[(A_0 + C_0 + \alpha^2 R) - \sqrt{(A_0 - C_0 + \alpha^2 R)^2 + 4\alpha^2 C_0 R}\,], \\ k_3^2 &= B_0. \end{aligned}\right] \quad \ldots (3.21)$$

The solution of eq. (3.12), which is bounded at infinity, is given by

$$\bar{\theta}(y, s) = \frac{\theta_0}{s}\, e^{-k_3 y}, \quad \ldots (3.22)$$

satisfying the boundary conditions in (3.14). The solutions of equation (3.15) and (3.18) have the form:

$$\bar{u} = \sum_{i=1}^{3} \bar{u}_i \;, \; \bar{h} = \sum_{i=1}^{3} \bar{h}_i \quad \ldots (3.23)$$

where $\bar{u}_i, \bar{h}_i$ are the solution of equations

$$(D^2 - k_i^2)\,\bar{u}_i = 0,\; (D^2 - k_i^2)\,\bar{h}_i = 0,\; (i = 1, 2, 3). \quad \ldots (3.24)$$

For $\bar{u}$ and $\bar{h}$ we are seeking solutions, which are bounded at infinity, we obtain

$$\bar{u}\,(y, s) = \sum_{i=1}^{3} A_i\,(s)\, e^{-k_i y} \quad \ldots (3.25)$$

and

$$\bar{h}\,(y, s) = \sum_{i=1}^{3} A_i'\,(s)\, e^{-k_i y}, \quad \ldots (3.26)$$

where $A_i\,(s)$ and $A_i'\,(s)$ are some parameters depending on s only.

The compatibility between these eqs. and (3.11) and (3.13) gives

$$\left.\begin{aligned} A_i'\,(s) &= \left(\frac{R k_i}{k_i^2 - C_0}\right) A_i\,(s),\; A_3\,(s) = M\,(k_3^2 - C_0), \\ M(s) &= \frac{G\,\theta_0}{s(k_2^2 - k_3^2)\,(k_3^2 - k_1^2)}, \end{aligned}\right] \quad \ldots (3.27)$$

taking into account the relations between the roots k_1^2, k_2^2 and k_3^2.

Changing (3.27) into (3.26), we get

$$\bar{h}(y, s) = R \sum_{i=1}^{3} \frac{k_i A_i(s)}{(k_i^2 - C_0)} e^{-k_i y}. \quad \text{... (3.28)}$$

Under the boundary condition (3.14) for $\bar{u}$ and $\bar{h}$, we get

$$\sum_{i=1}^{3} A_i(s) = 0, \quad \sum_{i=1}^{3} \left(\frac{k_i A_i(s)}{k_i^2 - C_0} \right) = 0. \quad \text{... (3.29)}$$

Solving eq. (3.29) for $A_1(s)$ and $A_2(s)$ taking into consideration $A_3(s)$ from (3.27) we arrive at

$$\left.\begin{aligned} A_1(s) &= M(s)\,(k_1^2 - C_0) \left(\frac{k_2 - k_3}{k_1 - k_2} \right) \left(\frac{k_2 k_3 + C_0}{k_1 k_2 + C_0} \right), \\ A_2(s) &= M(s)\,(k_2^2 - C_0) \left(\frac{k_1 - k_3}{k_2 - k_1} \right) \left(\frac{k_1 k_3 + C_0}{k_1 k_2 + C_0} \right). \end{aligned}\right] \quad \text{... (3.30)}$$

Substituting from eqs. (3.23) and (3.28) into equation (3.6), we get

$$\bar{E}(y, s) = \left(\frac{RC_0}{R + \varepsilon_0 s} \right) \sum_{i=1}^{3} \left(\frac{A_i(s)}{k_i^2 - C_0} \right) e^{-k_i y}. \quad \text{... (3.31)}$$

Substituting from eqs. (3.22) and (3.25) into eqs. (3.8) and (3.7) respectively, we obtain in the transformed domain the velocity and the temperature of particle phase

$$\left.\begin{aligned} \bar{u}_p(y, s) &= \left(\frac{\beta_0}{\beta_0 + s} \right) \sum_{i=1}^{3} a_i(s)\, e^{-k_i y}, \\ \bar{\theta}_p(y, s) &= \frac{2\beta_0 \theta_0}{(3Ps + 2\beta_0)\, s} e^{-k_3 y}. \end{aligned}\right] \quad \text{... (3.32)}$$

By taking Laplace transform of eq. (2.6) and using the non-dimensional quantities (2.21) we get

$$\bar{J} = \sigma_0 (\bar{E} + \bar{u}). \quad \text{... (3.33)}$$

Substituting from eqs. (3.25) and (3.31) into eq. (3.33), we obtain in the transformed domain the electric current density in the form

$$\bar{J}(y, s) = \sigma_0 \sum_{i=1}^{3} \left(\frac{k_i^2 (R + \varepsilon_0 s) - \varepsilon_0 s C_0}{(R + \varepsilon_0 s)(k_i^2 - C_0} \right) A_i(s)\, e^{-k_i y}. \quad \text{... (3.34)}$$

4. Inversion of the Laplace Transform

We shall outline the numerical inversion method used to find the solution in physical domain. Let $\bar{f}(s)$ be the Laplace transform of a function $f(t)$. The inversion formula for Laplace transform can be written as

$$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st}\, \bar{f}(s)\, ds,$$

where c is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(s)$. Takings $s = c + iy$, the above integral takes the form

$$f(t) = \frac{e^{ct}}{2\pi} \int_{-\infty}^{\infty} e^{ity}\, \bar{f}(c+iy)\, dy.$$

Expanding the function $h(t) = \exp(-ct)\, f(t)$ in Fourier series in the interval $[0, 2T]$ we obtain the approximate formula[17]

$$f(t) = f_\infty(t) + E_D,$$

where $$f_\infty(t) = \frac{c_0}{2} + \sum_{k=1}^{\infty} c_k \;, \text{ for } 0 \le t \le 2T, \qquad \text{... (4.1)}$$

and $$c_k = \frac{e^{ct}}{T} \operatorname{Re}\,[e^{ikt\,\pi/T}\bar{f}(c + ik\pi/T)]. \qquad \text{... (4.2)}$$

E_D, the discretization error can be made arbitrarily small be choosing c large enough[17].

As the infinite series in (4.1) can only be summed up to a finite number N of terms, the approximate value of $f(t)$ becomes

$$f_N(t) = \frac{c_0}{2} + \sum_{k=1}^{N} c_k, \text{ for } 0 \le t \le 2T. \qquad \text{... (4.3)}$$

Using the above formula to evaluate $f(t)$, we introduce a truncation error E_T that must be added to the discretization error to produce the total approximation error.

Two methods are used to reduce the total error. First, the "korrecktur" method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and hence to accelerate convergence.

The Korrecktur-method uses the following formula to evaluate the function $f(t)$

$$f(t) = f_\infty(t) - e^{-2cT} f_\infty(2T + t) + E'_D,$$

where the new discretization error $|E'_D| << |E_D|$[17]. Thus, the approximate value of $f(t)$ becomes

$$f_{NK} = f_N(t) - e^{-2cT} f_{N'}(2T + t). \quad \text{... (4.4)}$$

N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm, which is used to accelerate the convergence of the series in (4.3). Let $N = 2n + 1$ where n is a natural number and let

$$s_m = \sum_{k=1}^{m} c_k$$

be the sequece of partial sums of (4.3). We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \ \varepsilon_{1,m} = s_m$$

and

$$\varepsilon_{p+1,m} = \varepsilon_{p-1,m+1} + 1 / \{\varepsilon_{p,m+1} - \varepsilon_{p,m}\}, \ p = 1, 2, 3, \ .....$$

It can be shown that[17] the sequence

$$\varepsilon_{1,1}, \varepsilon_{3,1}, \ldots, \varepsilon_{N,1}$$

converges to $f(t) + E_D - c_0/2$ faster than the sequence of partial sums

$$s_m \ (m = 1, 2, 3, 4, 5, ...).$$

The actual procedure used to invert the Laplace transforms consists of using eq. (4.4) together with the ε-algorithm. The values of c and T are chosen according to the criteria outlined in [17].

5. RESULTS AND DISCUSSIONS

In order to invert Laplace-transform in the above equations, we used the numerical technique outlined above to obtain the velocity distribution of fluid and particle phase, the temperature distribution of fluid and particle phase, induced magnetic field distribution and induced electric field distribution. The computations were carried out for the physical parameters (R = 1.2, θ_0 = 1 and β_0 = 1). All distributions are shown in Figs. (1a-8b).

As result of the numerical computations, we conclude the following points:

(1*a*) Figs. (1*a*, 2*a*) show the velocity and temperature profiles of fluid and Figs. (1*b*, 2*b*) the velocity temperature profiles of particle phase are plotted Vs y for various values of volume fraction ϕ and mass concentration f at certain values of physical parameters [P = 7, G = 3 and K = 1.5][18]. It is observed that by increasing mass concentration f, the velocity and temperature of fluid and the particle phase decreases. From these figures and by comparing, it is evident that at any time, the velocity and temperature of the fluid is always greater than the velocity and temperature of particle phase, and the difference between the curves of velocities and temperature of the fluid, and the difference between the curves of velocities and temperature of the particle phase is insignificant at neighborhood of the plate and far from the plate. Also, the difference between the velocities of fluid and particle phase increases as the relaxation time of the particle phase τ_m increases.

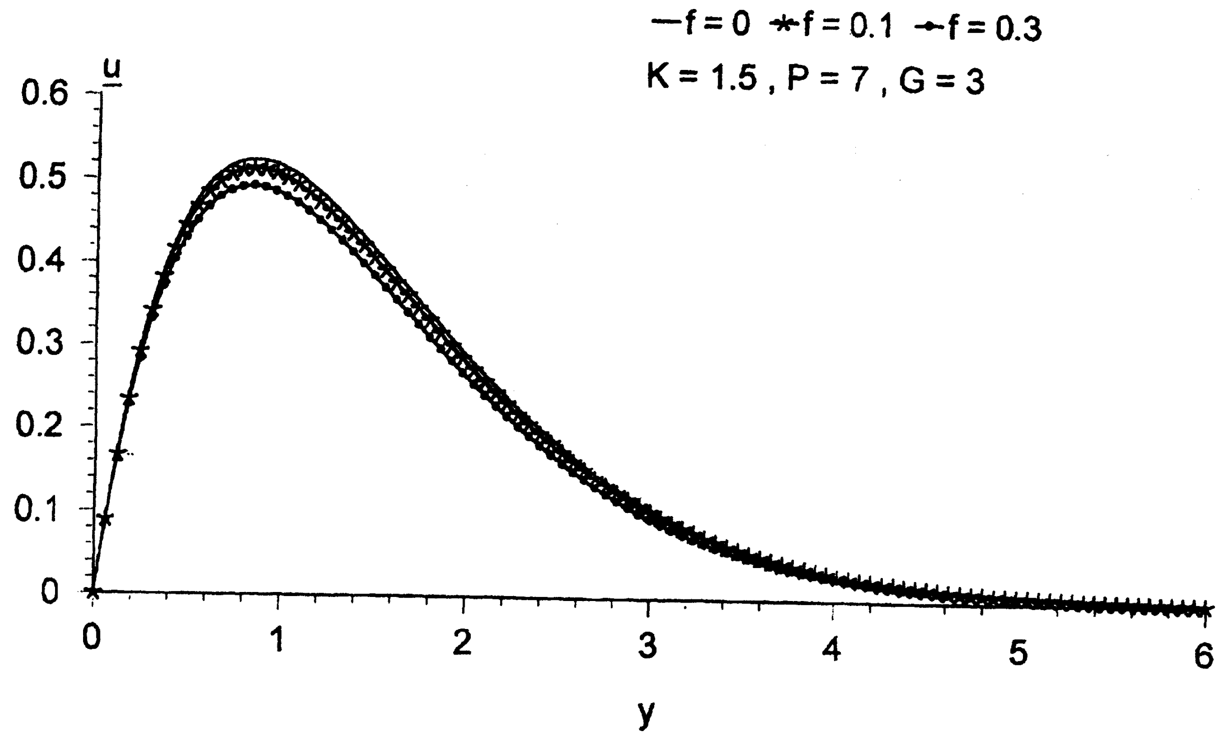

FIG. 1*a*. Velocity profiles of the fluid for different f, $t = 5$.

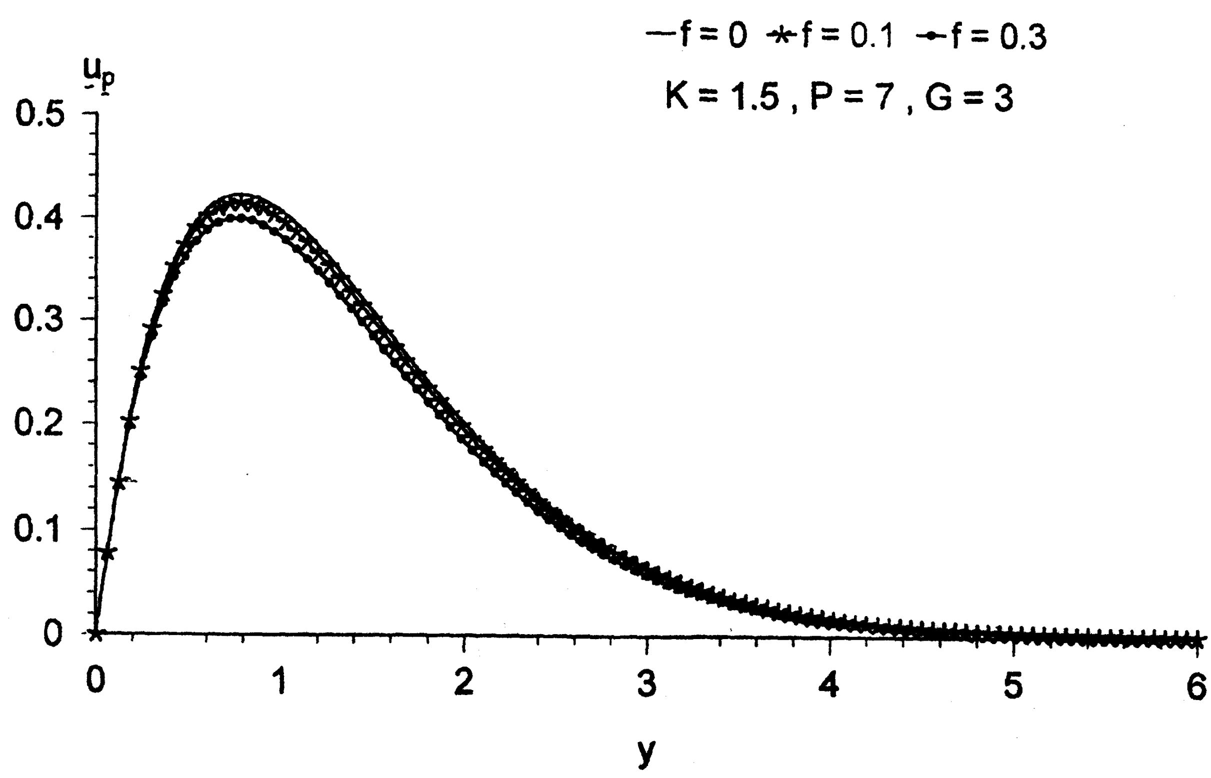

FIG. 1*b*. Velocity profiles of the particle phase for different f, $t = 5$.

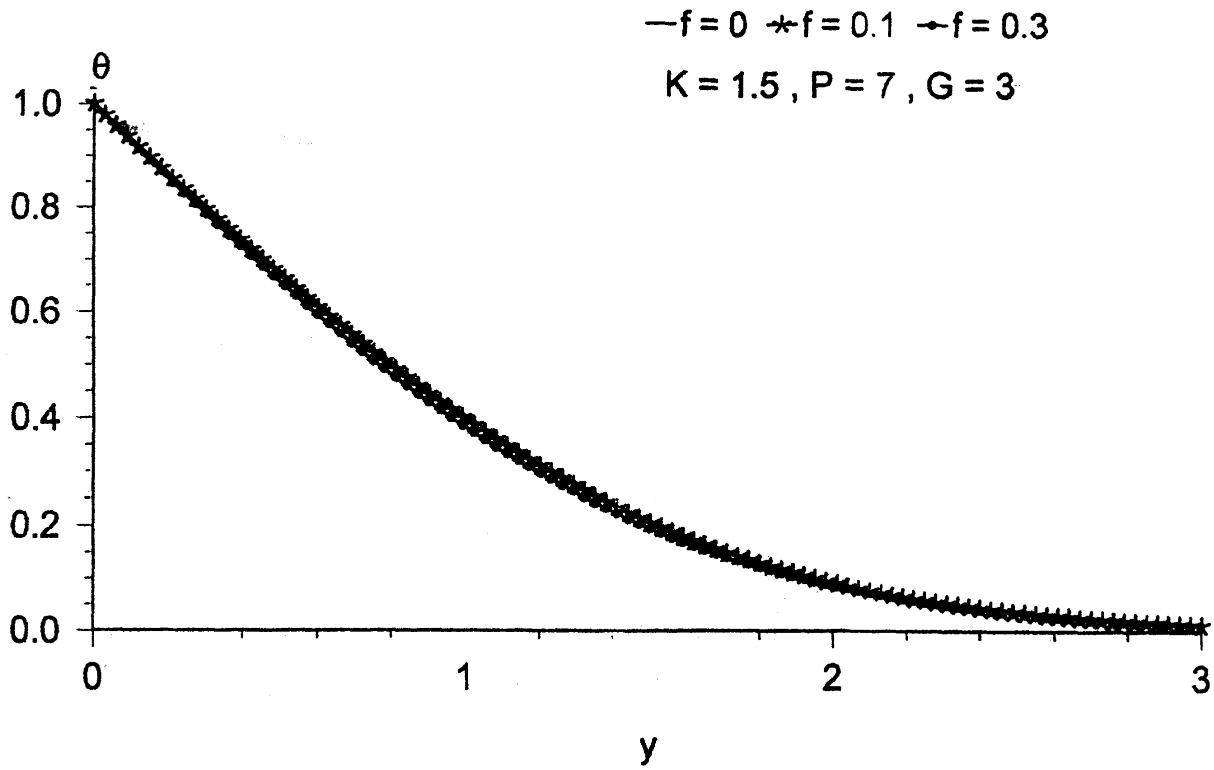

FIG. 2*a*. Temperature distribution of the fluid for different *f*, $t = 5$.

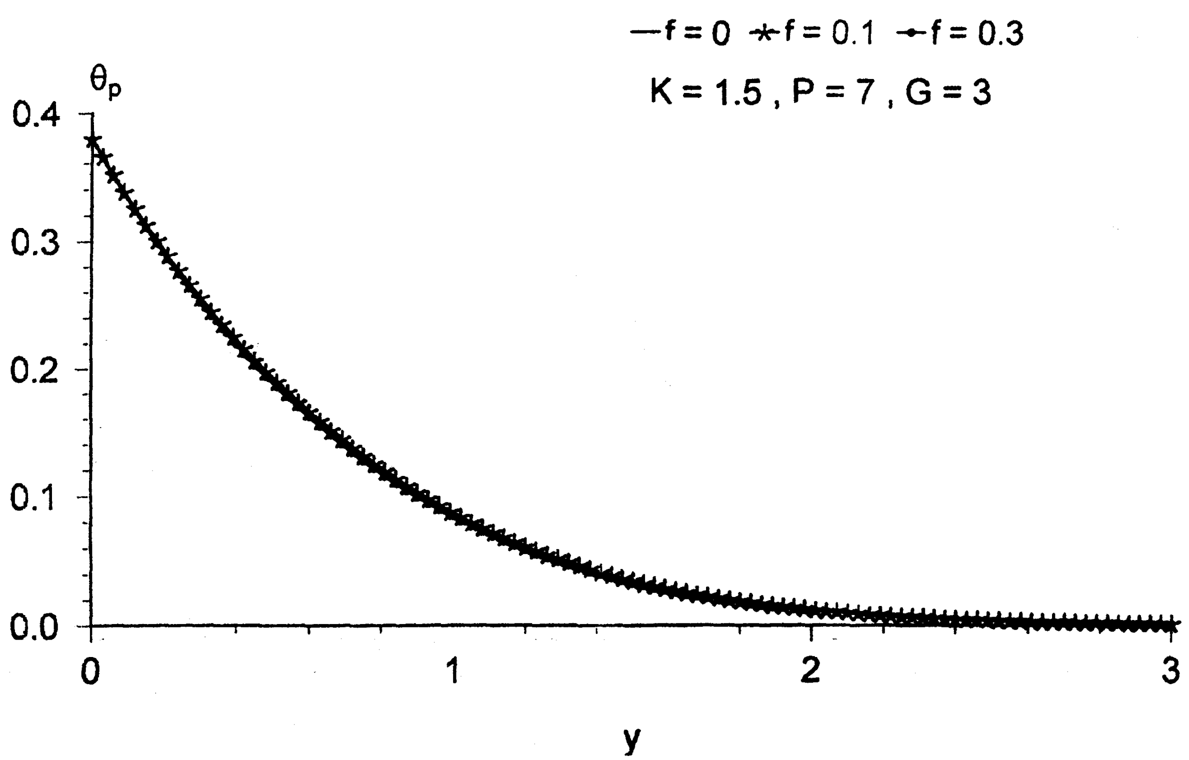

FIG. 2*b*. Temperature distribution of the particle phase for different *f*, $t = 5$.

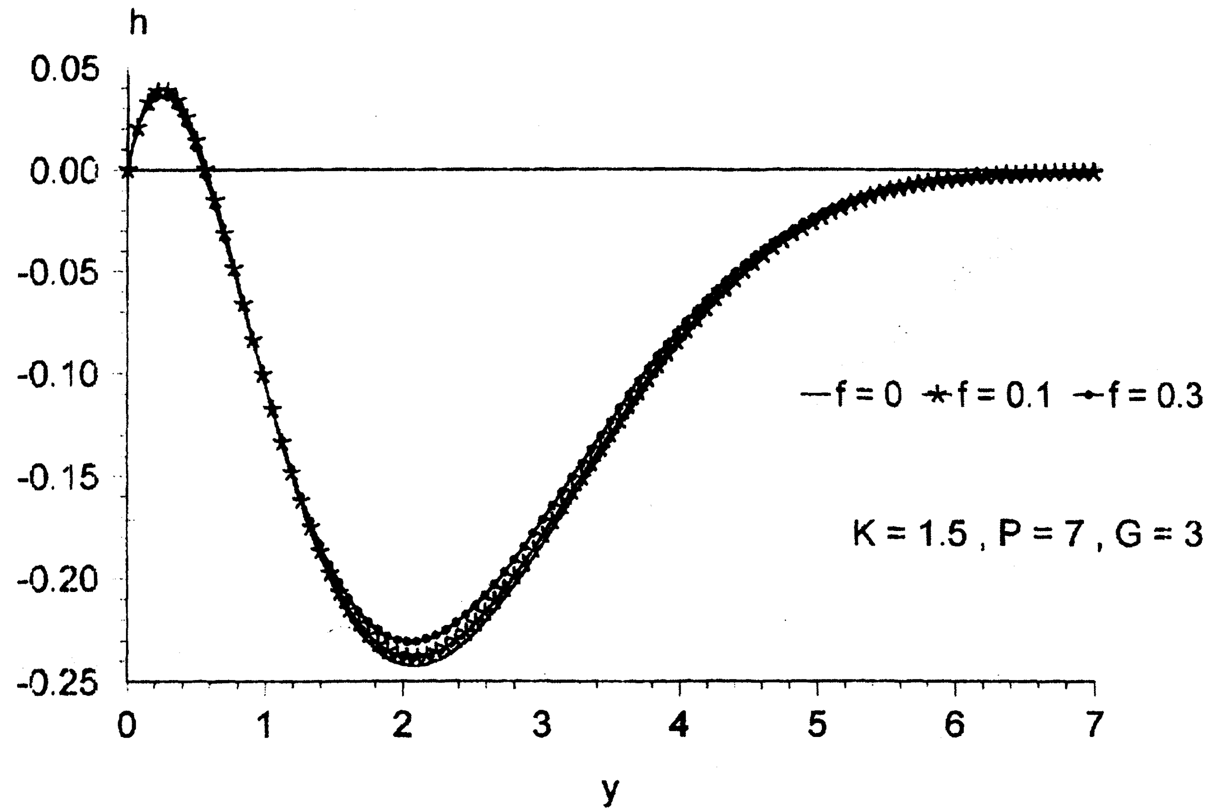

FIG. 3. Induced magnetic field distribution for different f, $t = 5$.

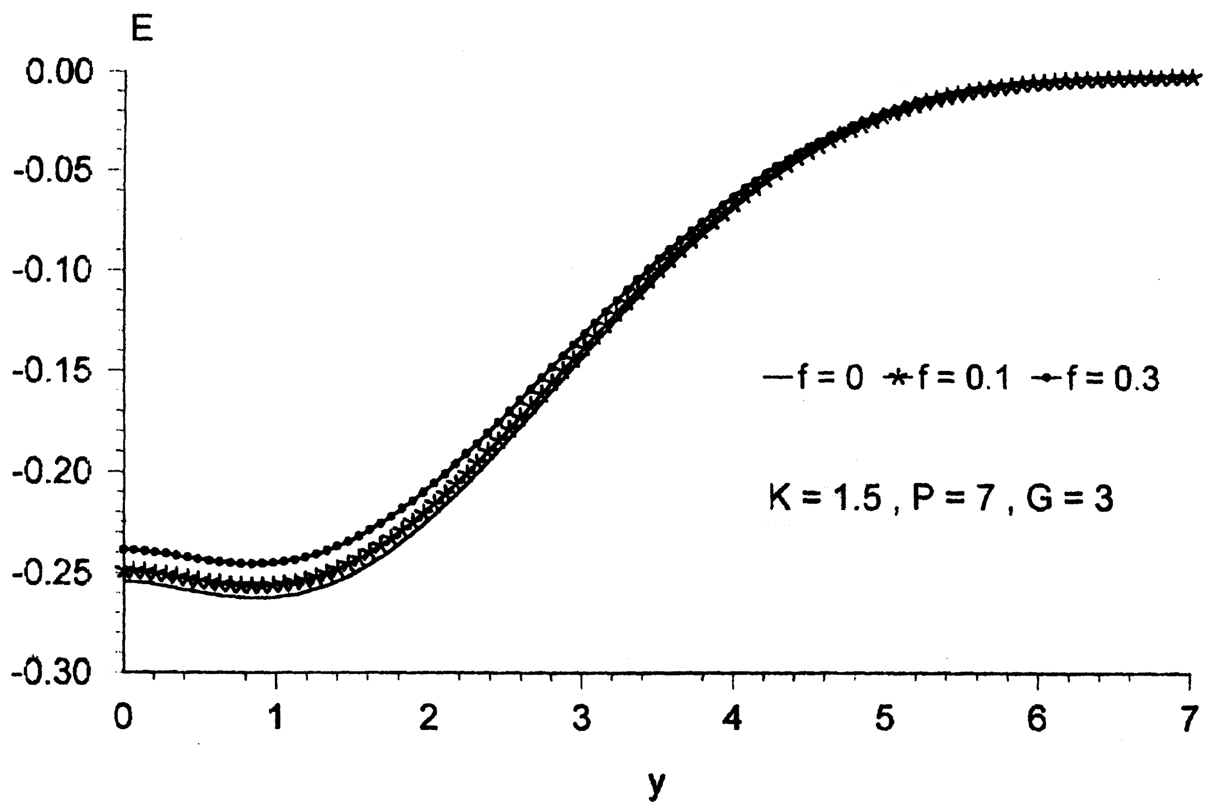

FIG. 4. Induced electric field distribution for different f, $t = 5$.

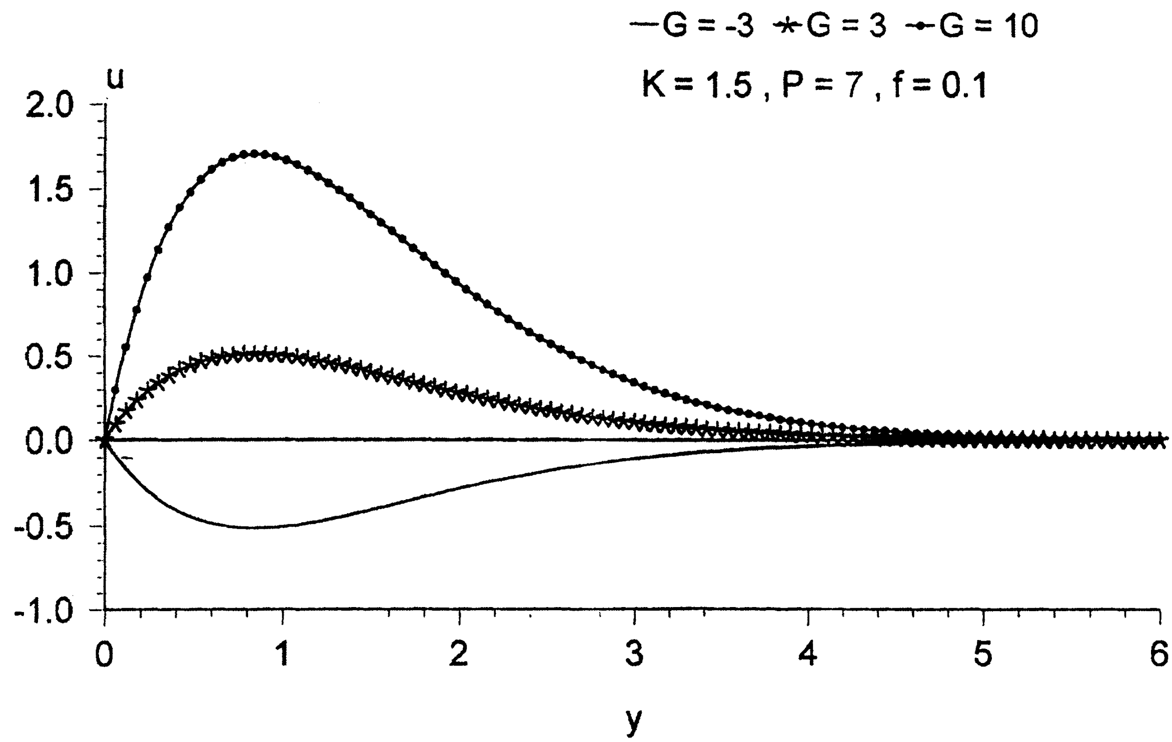

FIG. 5*a*. Velocity profiles of the fluid for distribution for different G, t = 5.

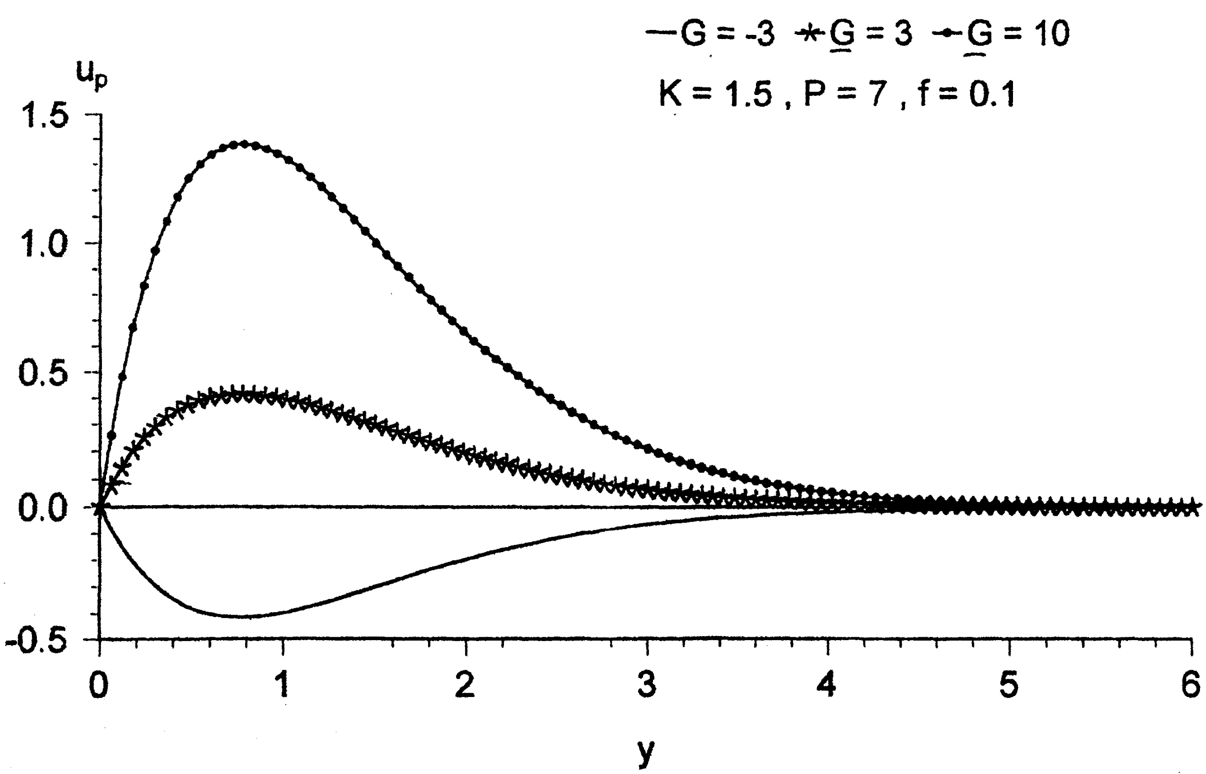

FIG. 5*b*. Velocity profiles of the particle phase for different G, t = 5.

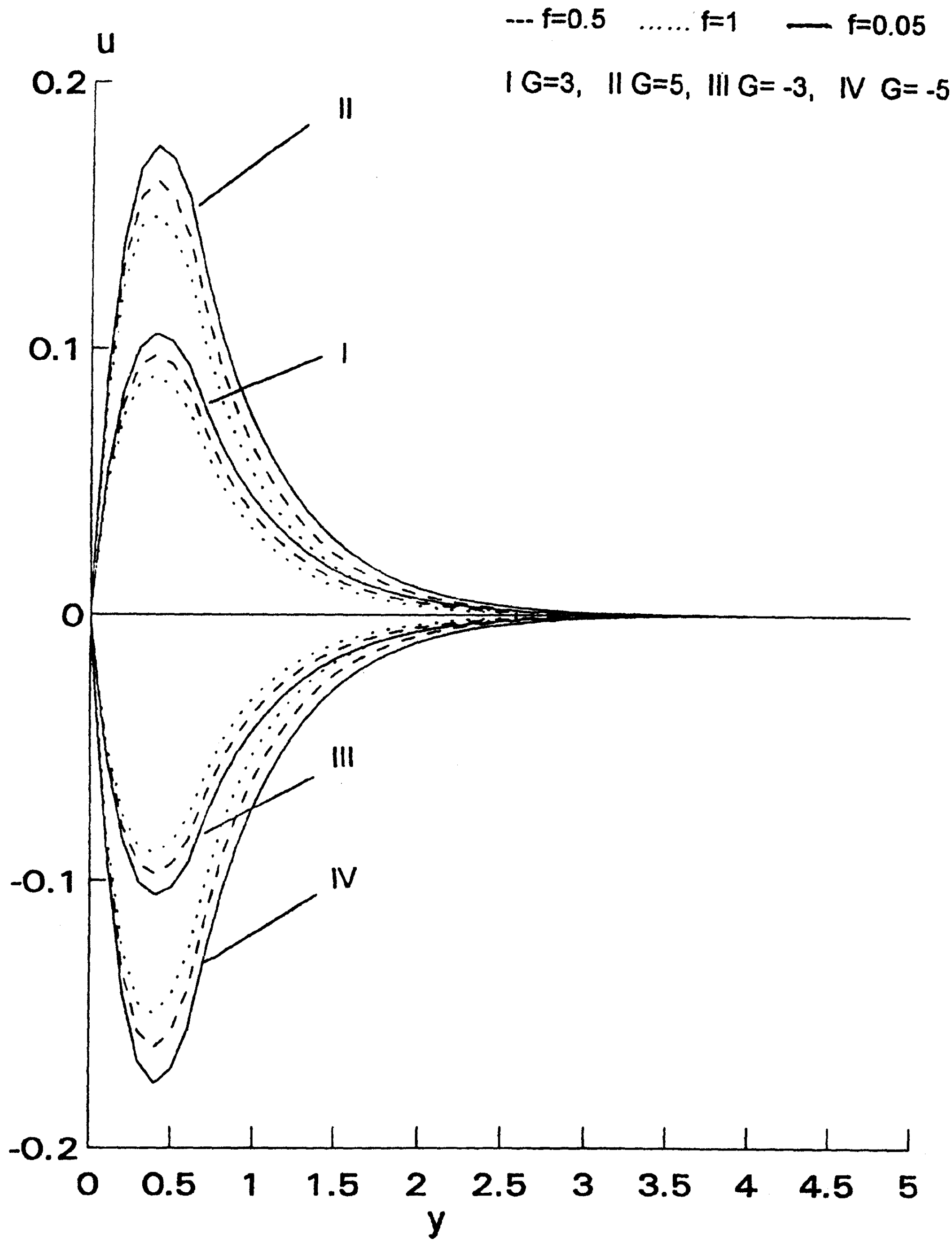

FIG. 6a. Velocity profiles of the fluid for different G.

(*b*) In figs. (3, 4) the induced magnetic field and induced electric field distributions are plotted *vs*. *y*, for several values of mass concentration and for certain values of the physical parameters. It is revealed that the induced magnetic field decreases where as the induced electric field increases by increasing *f*.

(*c*) In Figs. (5*a*, 5*b*) the velocity of fluid and particle phase is plotted against *y*. It is obvious that the effect of cooling and heating by free convection currents occurs when ($G > 0$) and $G < 0$ respectively, in agreement with physical observations and with the obtained results. Also, we notice that an increase of G leads to an increase in the velocity of fluid and particle phase.

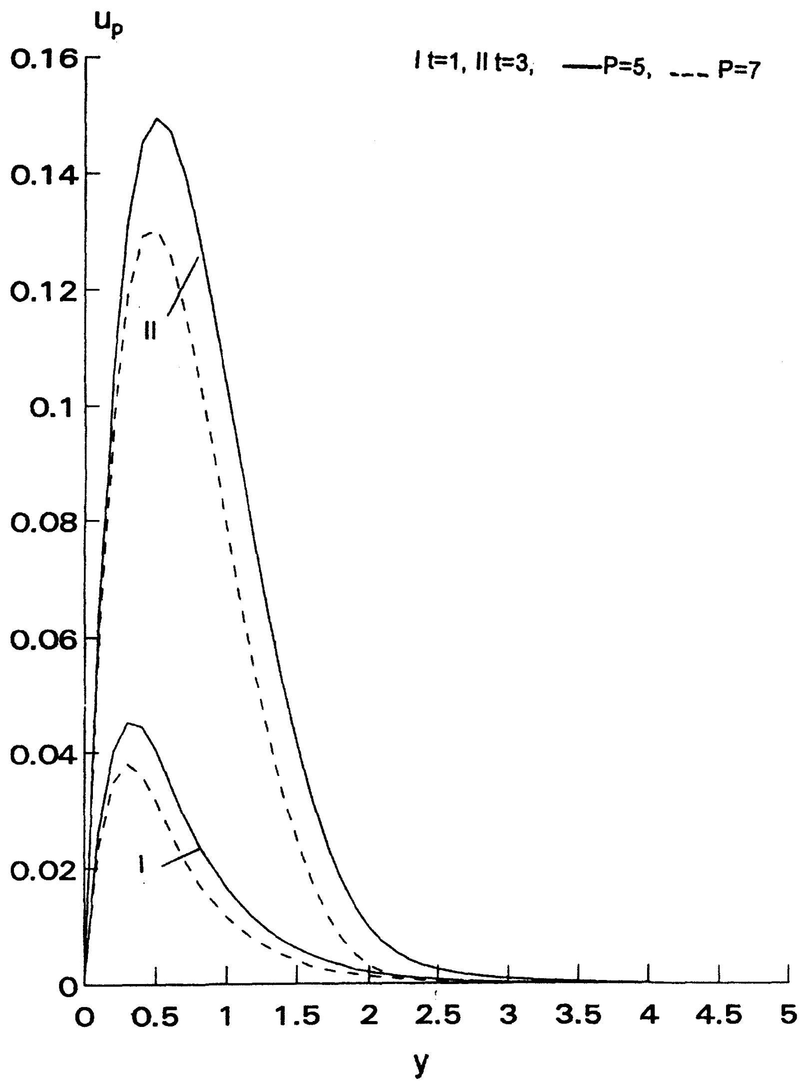

FIG. 6*b*. Velocity profiles of the particle phase for different *P*.

(*d*) In Fig. (6*a*), the velocity profile of the fluid is indicated for different values of Grashof number G and mass concentration f at a certain values of physical parameters [$P = 7$, $K = 1$ and $t = 1$]. It is evident that the increase of f decreases the velocity for cooling ($G > 0$) and increases the velocity for heating ($G < 0$). Furthermore, the graphs show that an increase of G leads to an increase in the velocity.

(*e*) In Figs. (6*b*, 7*a* and 7*b*), the velocity of the particle phase, temperature profiles of fluid and particle phase are plotted *vs.* y for different values of Prandtl number P at different times and certain values of the physical parameters [$G = 3$, $f = 0.05$ and $K = 1$]. The curves show that the increase of P not only leads to a decrease in the velocity at any time but also leads to a decrease in the temperature of fluid and particle phase surrounding the plate and makes the temperature more rapidly tend to zero. Also, due to acceleration, an increase in time leads to a rise in the values of velocities of the fluid and particle phase.

(*f*) For different values of the permeability parameter K, the velocity of fluid and particle phase is plotted in Figs. (8*a*, 8*b*). It is obvious that by increasing the value of K the velocity of fluid as well as the velocity of particle phase increases.

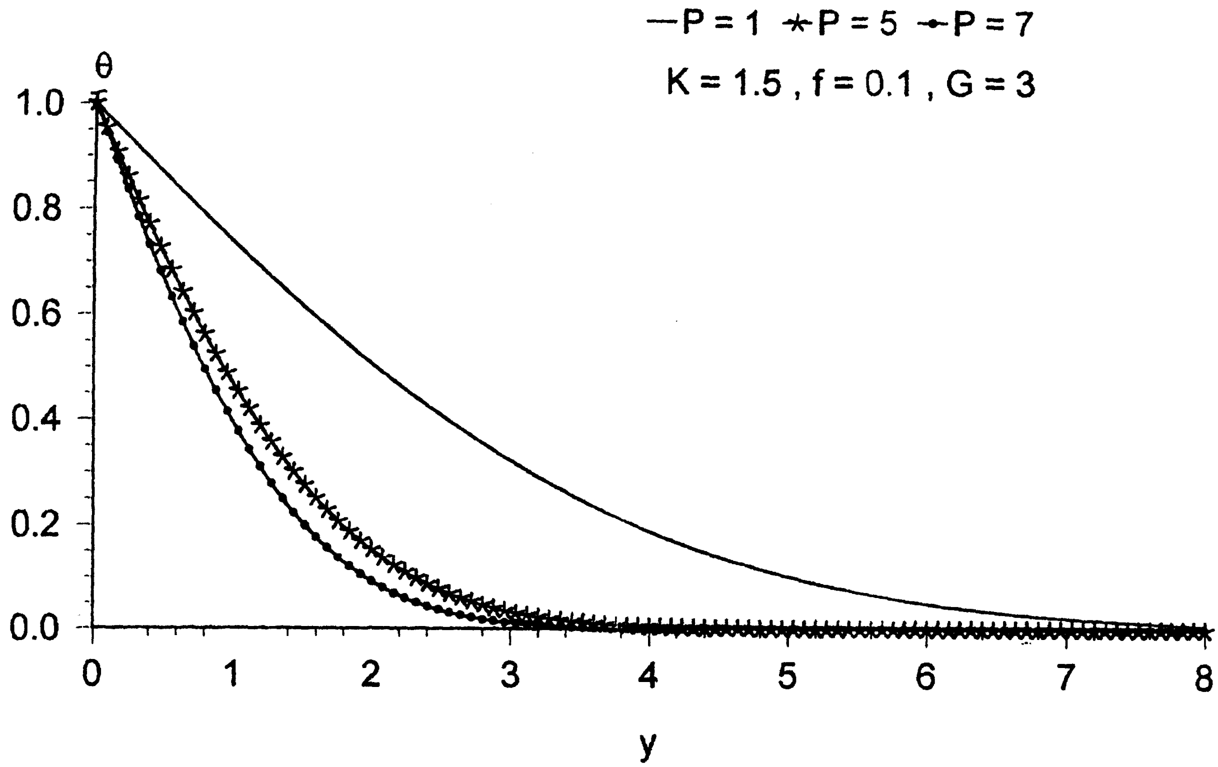

FIG. 7*a*. Temperature distribution of the fluid for different P, $t = 5$.

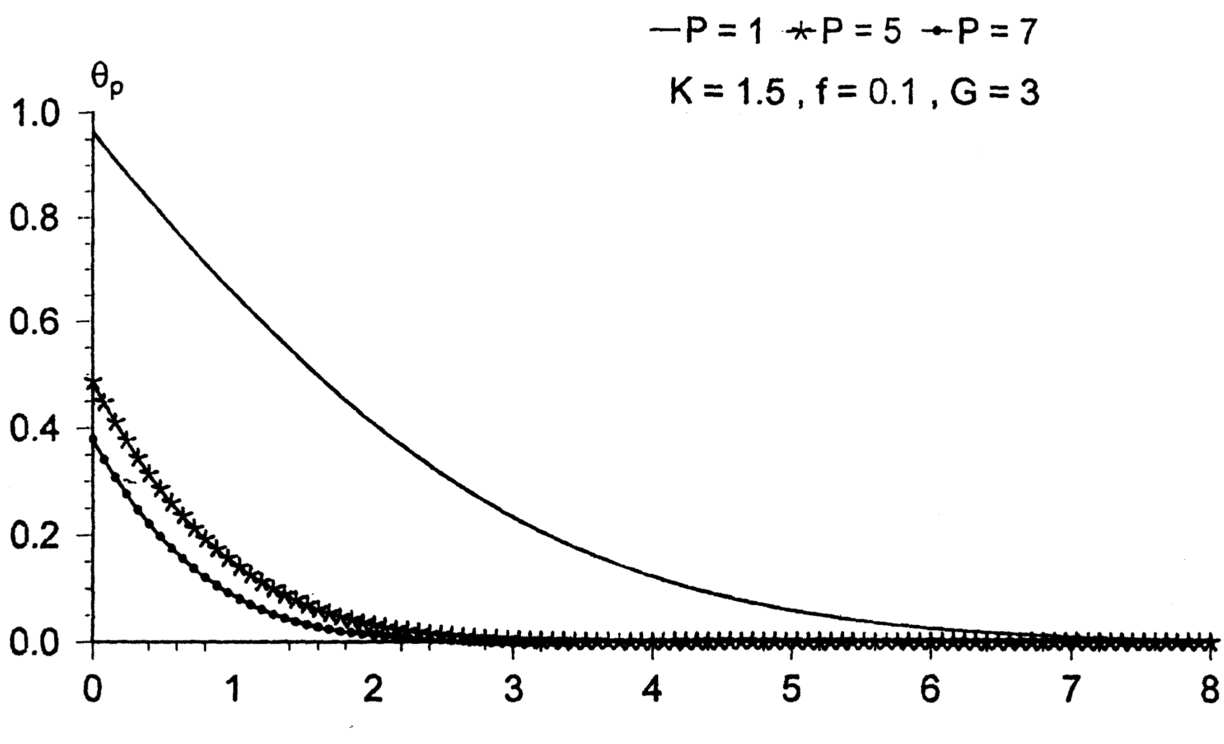

FIG. 7*b*. Temperature distribution of the particle phase for different P, $t = 5$.

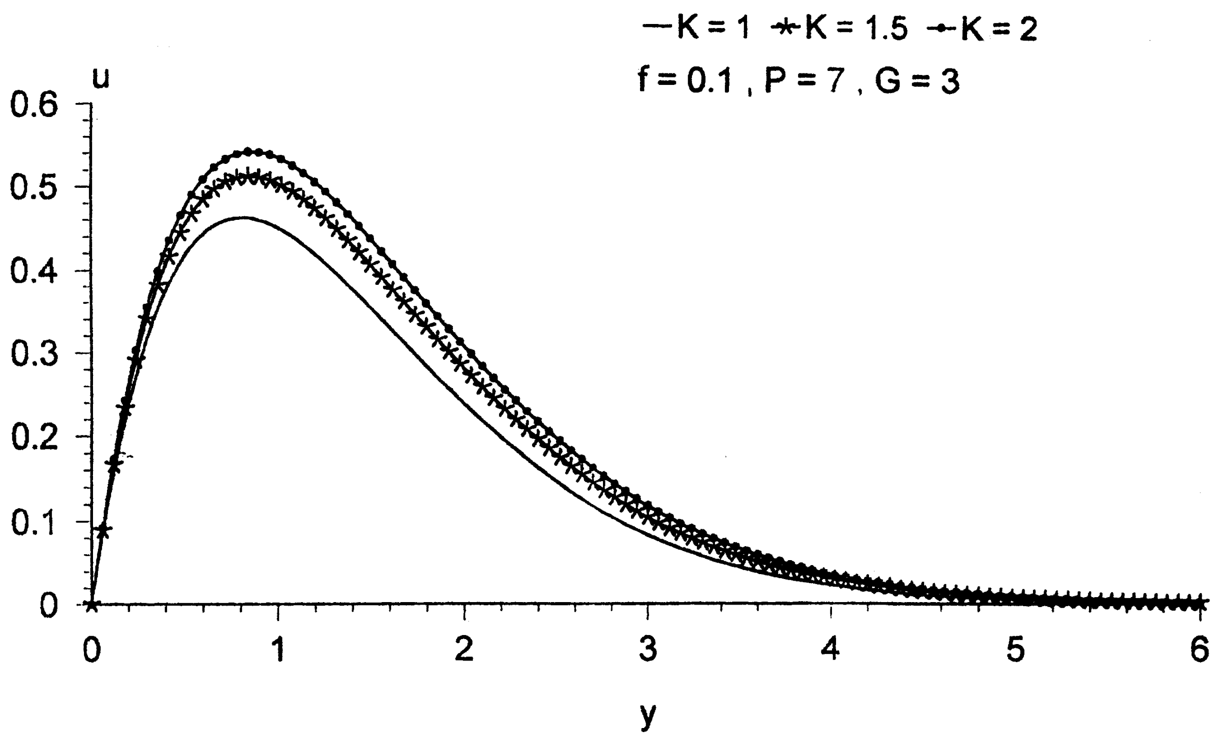

FIG. 8*a*. Velocity profiles of the fluid for different K, t = 5.

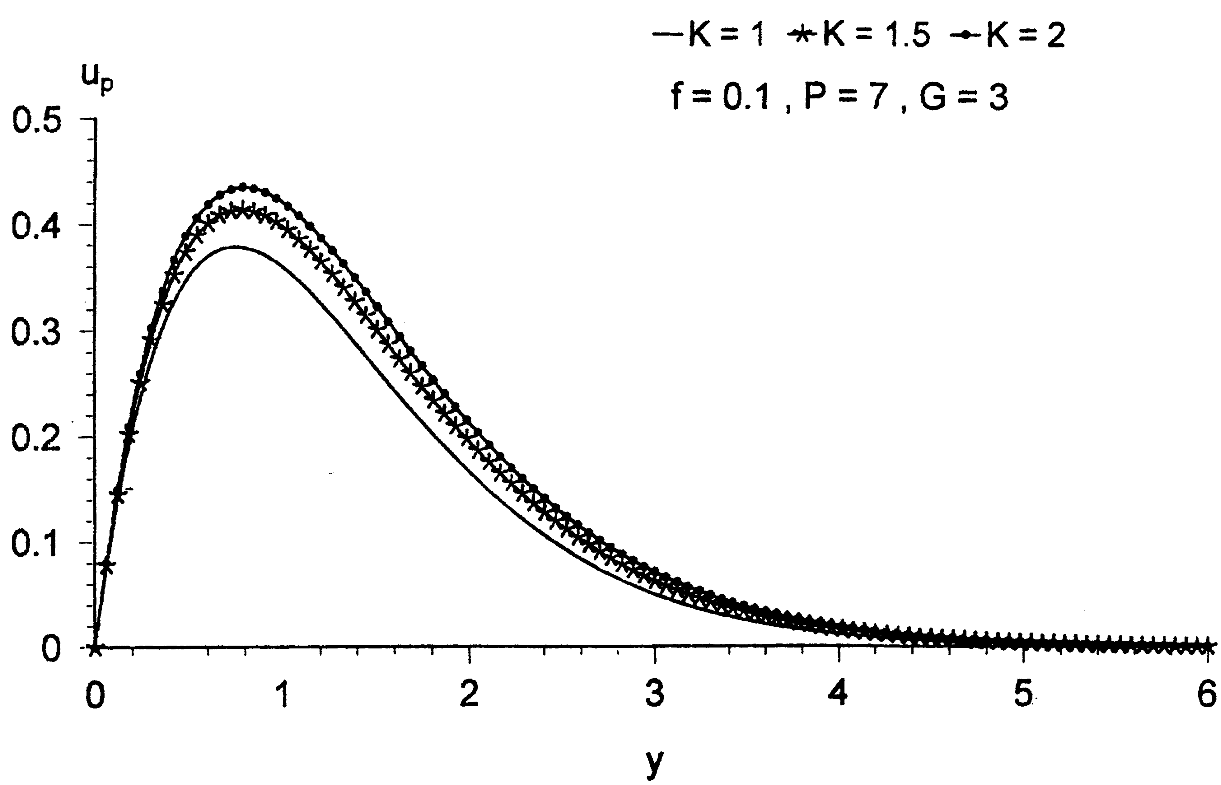

FIG. 8*b*. Velocity profiles of the particle phase for different K, t = 5.

Finally, the solutions and curves indicated that the velocity of the fluid and particle phase decreases exponentially with k_i, and from eq. (3.6) we deduce that the effect of the electromagnetic field is to decrease the velocity of the fluid as well as the velocity of the particle phase.

6. CONCLUSIONS

A number of numerical inversion methods have been developed over the last few years. To invert Laplace transform in the obtained equations in the present work we use a numerical technique based on Fourier expansion of function. As a result, it is clear that the effect of the mass concentration f leads the velocity and temperature of the fluid as well as the velocity and temperature of the particle phase to decrease. The obtained results qualitatively agree with the expectation, since, from the physical point of view, as long as the volume fraction ϕ takes finite value, the inertia of the fluid is reduced, and this produces a slight increase of the velocity of the fluid, which agrees with our results. A physically acceptable result is that the effect of the electromagnetic field is to decrease the velocity of the fluid and the velocity of the particle phase.

From the physical point of view, ($G > 0$) corresponds to the cooling of the surface by free convection currents and ($G < 0$) corresponds to the heating of the surface by free convection, which agrees with our results. The increasing of G leads to a rise in the value of the velocity, while the increase of P decreases the velocity of the fluid, as well as the velocity of particle phase that is near the plate, and makes it more rapidly tend to zero far from the plate. Moreover, the increase of the permeability parameter K make the velocity of the fluid and particle phase increase. In all discussed cases, due to acceleration, the increase in time leads to a rise in the values of the velocity of fluid and particle phase.

The electromagnetic flow has many applications such as electric heating, mathematical biology, bio fluid mechanics, biomedical engineering and the blood. To study the effect of the electric field on the particles, we must take another term in the governing equation; it will lead to discussion of the attraction force between the particles suspended in the fluid. (In forthcoming paper). For liquid metals it is usually negligible the term $\varepsilon_0 \dfrac{\partial E}{\partial t}$. If we put $\varepsilon_0 = 0$, the change will occur in A_0, B_0 and C_0 (eq. 3.17) which implies a change in the characteristic roots, which decreases the values of these roots. Hence we can deduce that this term has significance.

Finally, the problem of free convection of dusty fluid is more general than the problem of free convection of Newtonian fluid. Hence, if the dusty parameter is equal to zero ($\beta_0 = 0$) the problem is reduced to a laminar free convection flow of an incompressible, viscous, electrically conducting Newtonian fluid in the presence of constant magnetic field, perpendicular to the vertical plate. The corresponding results give its solution.

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