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SOME GENERALIZATIONS OF α-HOMEOMORPHISMS IN TOPOLOGICAL SPACES

R. DEVI* AND K. BALACHANDRAN**

Department of Mathematics
**Kongu Nadu Arts and Science College, Coimbatore* 641 029, *India*
***Bharathiar University, Coimbatore* 641 046, *India*

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In this paper we introduce new forms of homeomorphisms in topological spaces and investigate its group structure of their subgroup. Further we study some of their properties of their mappings from the quotient space to other space.

Key Words : α* Homeomorphisms; Generalized α-Homeomorphisms; α-Generalized Homeomorphisms; Group Structure

1. INTRODUCTION

Let X, Y and Z be topological spaces on which no separation axioms are assumed unless explicitly stated. Let S be a subset of X. The closure (resp. interior) of S will be denoted by $\bar{S}$ or Cl S (resp. S^0 or Int S). A subset A of a space X is called α-set or α-open[11] if $A \subset A^{0-0}$. The complement of an α-set is called α-closed set. The family of all α-open sets of (X, τ) is denoted by τ^α. For a subset B of (X, τ), τ^α-Cl (B) and τ^α - Int(B) represents the closure of B and the interior of B with respect to τ^α respectively. Levine[4] generalized the concept of closed sets to generalized closed sets. Using the above sets Devi *et al.*[2,3,5&6] have defined generalized α-closed sets, α-generalized closed sets, generalized α-continuous maps, α-generalized continuous maps, $g\alpha$ closed and αg-closed maps and proved normality and α-regularity are preserved under the last two maps. Tadros[13] introduced α-homeomorphisms and generalized some results concerning these maps. In this paper, we introduce a new class of homeomorphisms called α^*-homeomorphisms and define generalized α-homeomorphisms and α-generalized homeomorphisms which are generalizations of α-homeomorphisms[13] and α^*-homeomorphisms. We also investigate some properties of the above homeomorphisms from the quotient space to other spaces.

2. PRELIMINARIES

In this section we state some basic definitions for our study and introduce the notion of α^*-homeomorphisms in a topological space.

Definition 2.1[4] — A subset A of (X, τ) is said to be generalized closed (written as g-closed) in (X, τ) if Cl $(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ).

Definition 2.2[5,6] — A subset A of (X, τ) is said to be generalized α-closed set (resp. α-generalized closed set) written as g α-closed (resp. α g-closed) set if τ^α-Cl $(A) \subset U$ (resp. τ^α Cl $(A) \subset U$) whenever $A \subset U$ (resp. $A \subset U$) and U is α-open (resp. open) in (X, τ). A subset A of

(X, τ) is said to be generalized α-open (resp. α-generalized open) written as $g\alpha$ - open (resp. $\alpha\ g$ open) if its complement X-A is $g\ \alpha$-closed (resp. $\alpha\ g$-closed).

Definition 2.3 — A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called g-closed[7] (resp. α-closed[10], α-open[11], pre-α-closed[3], $g\ \alpha$-closed[3], $g\alpha$-open[3], αg-closed[3], αg-open[3]) if for each closed (resp. closed, open, α-closed, closed, open, closed, open) set F of (X, τ), $f(F)$ is g-closed (resp. α-closed, α-open, α-closed, $g\alpha$-closed, $g\alpha$-open, $\alpha\ g$-closed, $\alpha\ g$-open) set in (Y, σ).

Definition 2.4 — A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called $g\ \alpha$-continuous[5] (resp. $\alpha\ g$-continuous[2], $g\ \alpha$-irresolute[2], $\alpha\ g$-irresolute[2]) if for each closed set (resp. closed, $g\ \alpha$-closed, $\alpha\ g$-closed) set F of $(X, \tau), f^{-1}(F)$ is $g\ \alpha$-closed (resp. $\alpha\ g$-closed, $g\ \alpha$-closed, $\alpha\ g$-closed) set in (Y, σ).

Definition 2.5 — A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called α-homeomorphism[13] if it is bijective, α^*-continuous[13] (or α-irresolute) and α^*-open[13] (or pre -α-open)[3].

Definition 2.6 — A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called α^*-homeomorphism if it is bijective, continuous and α-open.

Remark 2.7 : Every homeomorphism is α^*-homeomorphism. Converse is not true by the following example.

Example 2.8 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$, $\sigma = \{\phi, \{a, b\}, \{a, b, c\}, Y\}$. α-closed sets of (Y, σ) are ϕ, $\{c\}$, $\{d\}$, $\{c, d\}$, Y. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity mapping. Here f is not homeomorphism, since for $V = \{c\}$ closed in (X, τ), we have $f(\{c\}) = \{c\}$ is not closed in (Y, σ). However, f is α^*-homeomorphism.

Remark 2.9 : Every α^*-homeomorphism is an α-homeomorphism.

PROOF : Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an α^*-homeomorphism. Then f is continuous, α-open map. Therefore, f is α-continuous and semi-open map. By Theorem 4.16 of Noiri[12], f is α-irresolute. Similarly, we can prove f^{-1} is α-irresolute. Therefore, f is an α-homeomorphism. Converse need not be true by the following example.

Example 2.10 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{c\}, \{b, c\}, X\}$ $\sigma = \{\phi, \{a\}, \{a, b, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined as follows $f(a) = b$, $f(b) = d$, $f(c) = a$, $f(d) = c$. For $V = \{a, b, c\}$ which is open in Y, $f^{-1}(\{a, b, c\}) = \{a, c, d\}$ which is not open in (X, τ). Therefore, f is not α^*-homeomorphism. However, f is an α-homeomorphism.

Definition 2.11[3] — A topological space (X, τ) is said to be αT_b (resp. αT_d) if every $\alpha\, g$-closed set is closed (resp. g-closed).

Definition 2.12[2] — A topological space (X, τ) is $\alpha T_{1/2}$ if every $g\ \alpha$-closed set is α-closed.

Definition 2.13[1] — A bijective $f: (X, \tau) \rightarrow (Y, \sigma)$ is called generalized semi-homeomorphism (resp. semi-generalized homeomorphism) if f is both gs-continuous and gs-open (resp. sg-continuous and sg-open).

3. GENERALIZED α-HOMEOMORPHISMS

In this section we introduce the concepts of generalized α-homeomorphisms and $g\ \alpha c$-homeomorphisms and study the group structure of the latter.

Definition 3.1 — A map $f: (X, \tau) \to (Y, \sigma)$ is called generalized α-homeomorphism if and only if it is bijective, g α-continuous and g α-open.

Remark 3.2 : (i) Every α^*-homeomorphism is a g α-homeomorphism.

(*ii*) Every g α-homeomorphism is a gs-homeomorphism.

Converses of (i) and (ii) are not true by the following examples.

Example 3.3 — Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$, $\sigma = \{\phi, \{a, b\}, Y\}$. Then g α-closed sets of (X, σ) and (Y, σ) are ϕ, $\{b\}$, $\{c\}$, $\{b, c\}$, X and ϕ, $\{c\}$, $\{b, c\}$, $\{a, c\}$, Y respectively.

Let $f: (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = b$, $f(b) = a$, $f(c) = c$. Here f is not α^*-homeomorphism, since the image of closed set $\{b, c\}$ in (X, τ) is $\{a, c\}$ which is not α-closed in (Y, σ). However, f is g α-homeomorphism.

Example 3.4 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, Y\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be an identity map. Then f is not g α-homeomorphism, since for $V = \{b, c\}$ closed in (Y, σ), $f^{-1}(\{b, c\}) = \{b, c\}$ is not g α-closed in (X, τ). However, f is a gs-homeomorphism.

Proposition 3.5 — *For any bijection $f: (X, \tau) \to (Y, \sigma)$ the following statements are equivalent:*

(i) $f^{-1} : (Y, \sigma) \to (X, \tau)$ is g α-continuous.

(ii) f is g α-open.

(iii) f is g α-closed.

Proposition 3.6 — *Let $f: (X, \tau) \to (Y, \sigma)$ be a bijective, g α continuous map. Then the following statements are equivalent* :

(i) f is $g\alpha$-open map.

(ii) f is a g α-homeomorphism.

(iii) f is a g α-closed map.

Remark 3.7 : The class of g α-homeomorphisms is not closed under the composition of maps by the following example.

Example 3.8 — Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$, $\sigma = \{\phi, \{a, b\}, Y\}$, $\eta = \{\phi, \{a, \{b\}, \{a, b\}, Z\}$. The set of all g α-closed sets of (X, τ); (Y, σ) and (Z, η) are ϕ, $\{b\}$, $\{c\}$, $\{b, c\}$, X; ϕ, $\{c\}$, $\{b, c\}$, $\{a, c\}$, Y and ϕ, $\{c\}$, $\{b, c\}$, $\{a, c\}$, Z respectively. Let $f: (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = b$, $f(b) = a$, $f(c) = c$ and $g : (Y, \sigma) \to (Z, \eta)$ be an identity map. Then $g \bullet f$ is not a g α-homeomorphism. However, f and g are g α-homeomorphisms.

Next we define another class of maps called $g\alpha$ c-homeomorphisms which is stronger than the class of g α-homeomorphisms and we prove that this class form a group with binary operation as the composition of maps.

Definition 3.9 — A map $f: (X, \tau) \to (Y, \sigma)$ is called $g\alpha c$-homeomorphism if and if only it is bijective, g α-irresolute and its inverse f^{-1} is also g α-irresolute.

Definition 3.10 — Two spaces are said to be $g\alpha c$-homemorphic if there exists a $g\alpha c$-homeomor- phism from (X, τ) onto (Y, σ)

Remark 3.11 : Every $g\alpha$-homeomorphism is a $g\alpha$ c-homeomorphism. Converse is not true by the following example.

Example 3.12 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$, $\sigma = \{\phi, \{a, b\}, \{a, b, c\}, Y\}$, $g\alpha$-closed sets of (X, τ) and (Y, σ) are ϕ, $\{a\}$, $\{b\}$, $\{d\}$, $\{a, b\}$, $\{b, d\}$, $\{a, d\}$, $\{a, b, d\}$, X and ϕ, $\{c\}$, $\{d\}$, $\{c, d\}$, $\{a, c, d\}$, $\{b, c, d\}$, Y respectively. Let $f: (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = d$, $f(b) = a$, $f(c) = b$, $f(d) = c$. Then for $\{b\}$ which is $g\alpha$-closed in (X, τ), $f(\{b\}) = \{a\}$ which is not $g\alpha$-closed in (Y, σ). Therefore, it is not $g\alpha c$-homeomorphism. However, f is $g\alpha$-homeomorphism.

Remark 3.13 : (*i*) If $f: (X, \tau) \to (Y, \sigma)$ is an α-homeomorphism, then it is a $g\alpha c$-homeomorphism.

(ii) If f is α^*-homeomorphism then it is a $g\alpha c$-homeomorphism.

Proof of (i) follows by Theorem 3.2[3] and proof of (ii) follows by (i). Converse of the above Remark is not always true by the following example.

Example 3.14 — Let $X = Y = \{a, b, c\}$, $\tau = \sigma = \phi, \{a\}, \{b, c\}, X\}$ and let $f: (X, \tau) \to (Y, \sigma)$. We define f as $f(a) = b$, $f(b) = a$, $f(c) = c$. Here f is not α-homeomorphism. However, f is a $g\alpha c$-homeomorphism.

Example 3.15 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, $\sigma = \{\phi, \{a, b\}, \{a, b, c\}, Y\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be an identity mapping. Then for $V = \{a, c, d\}$ closed in (X, τ), $f(\{a, c, d\}) = \{a, c, d\}$ is not α - closed in (Y, σ). Therefore, f is not $g\ \alpha$ c-homeomorphsim. However, f is an α^*-homeomorphism.

Remark 3.16 : Converse of the Remark 3.13 is true if (X, τ) and (Y, σ) are $\alpha T_{1/2}$.

PROOF : Let V be α - closed in (Y, σ), then V is $g\alpha$-closed in (Y, σ). Since f is $g\ \alpha$ -irresolute, $f^{-1}(V)$ is $g\ \alpha$-closed in (X, τ). But (X, τ) is $\alpha T_{1/2}$ implies $f^{-1}(V)$ is α-closed in (X, τ). Hence f is α-irresolute. Similarly, we can prove f is pre-α-closed. Hence, f is an α-homeomorphism.

Remark 3.17 : The class of $g\alpha c$-homeomorphisms is closed under the composition of maps.

For a topological space we introduce the following notations:

$$g\alpha ch\ (X, \tau) = \{f/f: (X, \tau) \to (X, \tau) \text{ is a } g\alpha c\text{-homeomorphism}\},$$

$$g\alpha h\ (X, \tau) = \{f/f: (X, \tau) \to (X, \tau) \text{ is a } g\alpha\text{-homeomorphism}\},$$

$$\alpha h\ (X, \tau) = \{f/f: (X, \tau) \to (X, \tau) \text{ is an } \alpha\text{-homeomorphism}\},$$

$$\alpha^*\ h\ (X, \tau) = \{f/f: (X, \tau) \to (X, \tau) \text{ is an } \alpha\text{ -homeomorphism}\}$$

and

$$h\,(X, \tau) = \{f/f: (X, \tau) \to (X, \tau) \text{ is a homeomorphism}\}.$$

Theorem 3.18 — *For a topological space the following implications hold :*

(*a*) $h(X, \tau) \subset \alpha^* h(X, \tau) \subset g\ \alpha h\ (X, \tau)$ *and*

(*b*) $h(X, \tau) \subset \alpha h(X, \tau) \subset g\ \alpha ch\ (X, \tau) \subset g\ \alpha h(X, \tau)$

PROOF : (*a*) It follows from Remark 2.7, 3.2[12]

(*b*) Proof for $h(X, \tau) \subset \alpha h(X, \tau)$ follows from Corollary 5.1.[12] The other implications are obtained by Remarks 2.11 and 2.13.[12]

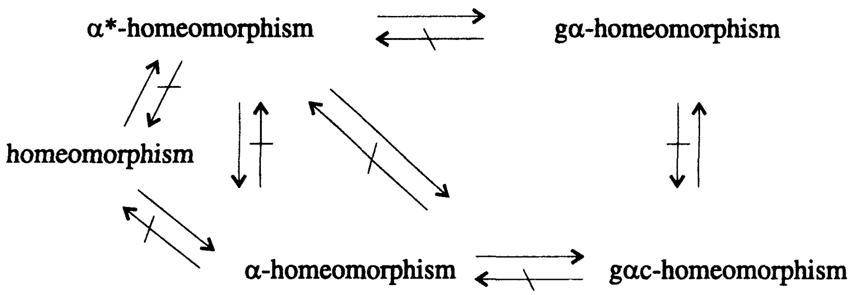
Theorem 3.19 — *The set $g\ \alpha\ ch\ (X, \tau)$ is a group which contains $\alpha h\ (X, \tau)$ and $h\ (X, \tau)$ as subgroups.*

PROOF : A binary operation $\mu : g\alpha ch\,(X, \tau)\, X\, g\alpha ch\,(X, \tau) \to g\alpha ch\,(X, \tau)$ is defined by $\mu\,(f, g) = g \bullet f$ (The composition of f and g) for f and g belongs to $g\,\alpha\, ch\,(X, \tau)$. By Remark 3.16 and definitions $g\alpha ch\,(X, \tau)$ is a group with μ as binary operation. But $h(X, \tau)$ and $\alpha h(X, \tau)$ are subsets of $g\alpha ch\,(X, \tau)$, by Theorem 3.18(b) and they are groups with binary operation μ of $g\alpha ch\,(X, \tau)$, $h(X, \tau)$ and $\alpha h\,(X, \tau)$ are subgroups of $g\alpha ch\,(X, \tau)$.

Remark 3.20 : The following example shows that $\alpha h(X, \tau)$ is a proper subgroup of $g\alpha ch(X, \tau)$ and $\alpha^* h(X, \tau)$ and $\alpha h(X, \tau)$ are proper subsets of $g\alpha ch(X, \tau)$.

Example 3.21 — Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. Let $f : (X, \tau) \to (X, \tau)$ be defined as $f(a) = b$, $f(b) = a$, $f(c) = c$. Here f is not α-homeomorphism. However, f is $g\alpha$-homeomorphism. Hence $\alpha h(X, \tau)$ is a proper subgroup of $g\alpha h(X, \tau)$. Here $g\alpha ch(X, \tau)$ is a group of order 6, $\alpha h(X, \tau)$ is a group of order 2. Also $\alpha^* h(X, \tau)$ is a set consisting of 2, α^*-homeomorphisms and $g\alpha h(X, \tau)$ is a set consisting of 6 $g\alpha$-homeomorphisms and $h(X, \tau)$ is a group of order 2.

Remark 3.22 : From Example 5.1 of Tadros13, Remark 2.7, Examples 2.10, 3.3, 3.14, 3.15, 3.12 we have the following implications :



Theorem 3.23 — *If $f : (X, \tau) \to (Y, \sigma)$ is a $g\alpha c$-homeomorphism then it induces an isomorphism from the group $g\alpha ch(X, \tau)$ onto $g\,\alpha\, ch(Y, \sigma)$.*

PROOF : The homeomorphism $f_* \, L\, g\,\alpha\, ch(X, \tau) \to g\alpha ch\,(Y, \sigma)$ is induced from f by $f_*\,(h) = f \bullet h \bullet f^{-1}$ for every $h \in g\,\alpha\, ch(X, \tau)$. Then f_* is an isomorphism by usual argument.

Remark 3.24 : Let $X = Y = \{a, b, c\}$ $\tau = \{\phi, \{a\}, \{b, c\}, X\}$, $\sigma = \{\phi, \{a\}, Y\}$ Here by Theorem 3.33 there exists no $g\,\alpha\, c$-homeomorphism from $(X, \tau) \to (Y, \sigma)$. That is (X, τ) and (Y, σ) are not $g\alpha\, c$-homeomorphic. Further, $g\,\alpha\, ch(X, \tau)$ is a group of order 6 and $g\,\alpha\, ch(Y, \sigma)$ is a group of order 2.

Remark 3.25 : The following example show that the converse of Theorem 3.23 is not always true. That is there exists a map which induces an isomorphism $f_* : g\alpha ch(X, \tau) \to g\alpha ch(Y, \sigma)$ but $g\,\alpha\,c$-homeomorphism.

Example 3.26 — Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ $\sigma = \{\phi, \{a\}, Y\}$, $g\,\alpha$-closed sets of (X, τ) are ϕ, $\{c\}$, $\{b, c\}$, $\{a, c\}$, X and $g\,\alpha$-closed sets of (Y, σ) are ϕ, $\{b\}$, $\{c\}$. $\{b, c\}$, Y. Define a map $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is not $g\alpha c$-homeomorphism, since the image of $g\,\alpha$-closed set $\{c\}$ is $\{a\}$, which is not $g\,\alpha$-closed in (Y, σ). Let $h_c : (X, \tau) \to (X, \tau)$ and $h_a : (Y, \sigma) \to (Y, \sigma)$ be defined as $h_c(a) = b$, $h_c(b) = a$, $h_c(c) = c$ and $h_a(a) = a$, $h_a(b) = c$, $h_a(c) = b$ respectively. Then we have $g\alpha ch(X, \tau) = \{h_c, I_x\}$ and $g\alpha ch\,(Y, \sigma) = \{h_a, I_y\}$ hold where $I_x : (X, \tau) \to (X, \tau)$ and $I_y : (Y, \sigma) \to (Y, \sigma)$ are identity maps. The induced homeomorphism $f_* : g\alpha ch(X, \tau) \to g\alpha ch(Y, \sigma)$ is an isomorphism, since $f_*(h_c) = f \bullet h_c \bullet f^{-1} = h_a$ and $f_*(I_x) = I_y$, the induced homeomorphism $f_* : g\alpha ch(X, \tau) \to g\alpha ch(Y, \sigma)$ is an isomorphism.

4. α-GENERALIZED HOMEOMORPHISMS

Here we introduce a new class of maps called $\alpha\,g$-homeomorphisms and this class of maps induces αgc-homeomorphisms[2] and $g\,\alpha$-homeomorphisms.

Definition 4.1 — A map $f: (X, \tau) \to (Y, \sigma)$ is called α-generalized homeomorphism if and only if it is bijective, αg-continuous and $\alpha\,g$-open.

Remark 4.2 : Every g-homeomorphism, $g\alpha$-homeomorphism is an αg-homeomorphism.

PROOF : Since every g-closed set, $g\alpha$-closed set is αg-closed.

Converse of the above Remark is not true by the following examples.

Example 4.3 — Let $X = Y = \{a, b, c, d\}$ $\tau = \{\phi, \{c\}, \{b, c\}, X\}$, $\sigma = \{\phi, \{c\}, \{c, d\}, \{a, c\}, \{a, c, d\}, Y\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be an identity mapping, αg-closed sets of (X, τ) are ϕ, $\{a\}$, $\{b\}$, $\{d\}$, $\{a, b\}$, $\{b, d\}$, $\{a, d\}$, $\{a, c\}$, $\{c, d\}$, $\{a, b, d\}$, $\{a, b, c\}$, $\{a, c, d\}$, $\{b, c, d\}$, X; $\alpha\,g$-closed of (Y, σ) are ϕ, $\{a\}$, $\{b\}$, $\{d\}$, $\{a, b\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{a, b, c\}$, $\{b, c, d\}$, $\{a, b, d\}$, Y. For $V = \{a, d\}$, closed in (X, τ), $f(\{a, d\}) = \{a, d\}$ is not g-closed in (Y, σ); therefore f is not g-homeomorphism. However, f is an αg-homeomorphism.

Example 4.4 — Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$, $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = b$, $f(b) = a$, $f(c) = c$ then f is not $g\alpha$-homeomorphism, since the inverse image of a closed set $\{b, c\}$ in (Y, σ) is $\{a, c\}$ which is not $g\alpha$-closed in (X, τ). However, f is an αg-homeomorphism.

Remark 4.5 : The following examples show that $\alpha\,g$-homeomorphisms and sg-homeomorphisms are independent concepts.

Example 4.6 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{c\}, \{b, c\}, X\}$, $\sigma = \{\phi, \{a\}, \{a, b, c\}, Y\}$. sg-closed sets of (X, τ) are ϕ, $\{a\}$, $\{b\}$, $\{d\}$, $\{a, b\}$, $\{a, d\}$, $\{b, d\}$, $\{a, b, d\}$, X. $\alpha\,g$-closed sets of (X, τ) are ϕ, $\{a\}$, $\{b\}$, $\{d\}$, $\{a, b\}$, $\{b, d\}$, $\{a, d\}$, $\{a, c\}$, $\{c, d\}$, $\{a, b, d\}$, $\{a, b, c\}$,

$\{a, c, d\}$, $\{b, c, d\}$, X. sg-closed sets of (Y, σ) = $\{\phi\ \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, Y\}$ = αg-closed sets of (Y, σ). Let $f: (X, \tau) \to (Y, \sigma)$ be an identity mapping. Then f is not sg-homeomorphism, since for $V = \{b, c, d\}$ closed in $(Y, \sigma), f^{-1}(\{b, c, d\})$ = $\{b, c, d\}$ is not sg-closed in (X, τ). However, f is an αg-homeomorphism.

Example 4.7 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, X\}$, $\sigma = \{\phi, \{a\}, \{b, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, Y\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = b$, $f(b) = a$, $f(c) = d$, $f(d) = c$. Then for $V = \{b, c\}$ closed in (X, τ). $f(\{b, c\}) = \{a, d\}$ is not αg-closed in (Y, σ). Hence f is not αg-homeomorphism. However, f is a sg-homeomorphism.

Remark 4.8 : Every αg-homeomorphism is a gs-homeomorphism, since every αg-closed set is gs-closed set. Converse is not true by the following example.

Example 4.9 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}. \{a, b\}, \{a, b, c\}, X\}$, $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, Y\}$, gs-closed sets of (X, τ) are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, d\}$, $\{a, c, d\}$, $\{b, c, d\}$, X. αg-closed sets of (x, τ) are ϕ, $\{c\}$, $\{d\}$, $\{a, d\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, d\}$, $\{a, c, d\}$, $\{b, c, d\}$, X. gs-closed of sets (Y, σ) are ϕ, $\{c\}$, $\{d\}$, $\{a, d\}$, $\{b, c\}$, $\{a, c\}$, $\{b, d\}$, $\{c, d\}$, $\{b, c, d\}$, $\{a, c, d\}$, Y. αg-closed sets of (Y, σ) are ϕ, $\{c\}$, $\{d\}$, $\{c, d\}$, $\{b, c\}$, $\{b, c, d\}$, $\{a, c, d\}$, Y. Let $f: (X, \tau) \to (Y, \sigma)$ be an identity map. Then for $\{b, c\}$ closed in (Y, σ), $f^{-1}(\{b, c\}) = \{b, c\}$ is not αg-closed in (X, τ). Therefore, f is not αg-homeomorphism. However, f is a gs-homeomorphism.

Proposition 4.10 — *For any bijection* $f: (X, \tau) -. (Y, \sigma)$ *the following statements are equivalent:*

(a) $f^{-1}: (Y, \sigma) \to (X, \tau)$ is αg-continuous.

(b) f is αg-open.

(c) f is αg-closed.

Proposition 4.11 — *Let* $f: (X, \tau) \to (Y, \sigma)$ *be a bijective and* αg*-continuous map then the following statements are equivalent* :

(a) f is an αg-open map.

(b) f is an αg-closed map.

(c) f is an αg-homeomorphism.

The composition of αg-homeomorphisms is not always an αg-homeomorphism by the following example.

Example 4.12 — Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\phi, \{b\}, \{a, b\}, X\}$ $\sigma = \{\phi, \{a\}, Y\}$, $\eta = \{\phi, \{b\}, \{a, b\}, Z\}$ Let $f: (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = b$, $f(b) = a$, $f(c) = c$ and $g: (Y, \sigma) \to (Z, \eta)$ be defined as $g(a) = a$, $g(b) - c$, $g(c) = b$. Then $f \bullet g$ is not αg-homeomorphism. However, f and g are αg-homeomorphism.

Remark 4.13 : Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be αg-homeomorphisms. Then $f \bullet g$ is an αg-homeomorphism if (Y, σ) is αT_b.

Definition 4.14 — A bijection $f: (X, \tau) \to (Y, \sigma)$ is called αgc-homeomorphism if f is αg-irresolute and its inverse f^{-1} is αg-irresolute. We say that the topological spaces (X, τ) and (Y, σ) are αgc-homeomorphic, if there exists an αgc-homeomorphism from $(X, \tau) \to (Y, \sigma)$.

Remark 4.15 : (*i*) Every homeomorphism is an αgc-homeomorphism and every αgc-homeomorphism is an αg-homeomorphism. But the converses are not true by Examples 4.16, 4.17.

(*ii*) gc-homeomorphism and αgc-homeomorphism are independent concepts by Examples 4.18, 4.19.

(*iii*) α-homeomorphism and αgc-homeomorphism are independent concepts by Examples 4.20, 4.21.

(*iv*) $g\alpha c$-homeomorphism and. αgc-homeomorphism are independent concepts by Examples 4.22, 4.23.

Example 4.16 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}. X\}$, $\sigma = \{\phi, \{a, b\}, \{a, b, c\}, Y\}$. αg-closed sets of (X, τ) are ϕ, $\{c\}$, $\{d\}$, $\{c, d\}$, $\{a, c, d\}$. $\{b, c, d\}$, $\{a, d\}$, $\{b, d\}$, $\{a, b, d\}$; X αg-closed sets of (Y, σ) are ϕ, $\{c\}$, $\{d\}$, $\{c, d\}$, $\{a, d\}$, $\{b, d\}$, $\{a, c, d\}$, $\{b, c, d\}$, $\{a, b, d\}$, Y Let $f: (X, \tau) \to (Y, \sigma)$ be an identity map. Therefore, for $V = \{a, c, d\}$ closed in (X, τ), $f(\{a, c, d\})$ is not closed in (Y, σ). Therefore, f is not homeomorphism. However, f is an αgc-homeomorphism.

Example 4.17 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{c\}, \{b, c\}, X\}$, $\sigma = \{\phi, \{a\}, \{a, b, c\}, Y\}$, αg-closed sets of (X, τ) are ϕ, $\{a\}$, $\{b\}$, $\{d\}$, $\{a, b\}$, $\{b, d\}$, $\{a, d\}$, $\{a, c\}$, $\{c, d\}$, $\{a, b, c\}$, $\{a, c, d\}$, $\{a, b, d\}$, $\{b, c, d\}$, X. αg-closed sets of (Y, σ) are ϕ, $\{b\}$, $\{c\}$, $\{d\}$, $\{b, d\}$, $\{b, c\}$, $\{c, d\}$, $\{a, d\}$, $\{a, c, d\}$, $\{a, b, d\}$, $\{b, c, d\}$, Y. Let $f: (X, \tau) \to (Y, \sigma)$ be an identity map. Then f is not αgc-homeomorphisms. Since for $V = \{c\}$, which is αg-closed in (Y, σ), $f^{-1}(\{c\}) = \{c\}$ is not αg-closed in (X, τ) However, f is an αg-homeomorphism.

Example 4.18 — Let $X = Y = \{a, b, c, d\}$ $\tau = \{\phi, \{c\}, \{b, c\}, X\}$, $\sigma = \{\phi, \{a\}, \{a, b, c\}, Y\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = a, f(b) = b, f(c) = d, f(d) = c$. Then f is not αgc-homeomorphism.

Example 4.19 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{c\}, \{b, c\}, X\}$, $\sigma = \{\phi, \{a\}, \{a, b, c\}, Y\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = d$, $f(b) = b$, $f(c) = a$, $f(d) = c$. Then f is not gc-homeomorphism, since for $V = \{d\}$ which is g-closed in (X, τ), $f(\{d\}) \doteq \{c\}$ is not g-closed in (Y, σ). However f is an αgc-homeomorphism.

Example 4.20 — Take (X, τ) and (Y, σ) as in Example 4.19. Let $f: (X, \tau) \to (Y, \sigma)$ be $f(a) = b$, $f(b) = d$, $f(a) = a$, $f(d) = c$. Then f is not αgc-homeomorphism, since for $V = \{a, d\}$, αg-closed in (Y, σ), f^{-1} $(\{a, d\}) = \{b, c\}$, is not αg-closed in (X, τ). However, f is an α-homeomorphism.

Example 4.21 — Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, $\sigma = \{\phi, \{d\}, \{c, d\}, Y\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined as an identity map. Then for $V = \{a, c, d\}$, α-closed in (X, τ), $f(\{a, c, d\}) = \{a, c, d\}$ is not α-closed in (Y, σ). Therefore, f is not α-homeomorphism. However, f is an αgc-homeomorphism.

Example 4.22 — Take (X, τ), (Y, σ) as in Example 4.19. Let $f: (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = b$, $f(b) = d$, $f(c) = a$, $f(d) = c$. Then is not αgc-homeomorphism, since for $V_2\{q, d\}$, αg-closed in (Y, σ) $f(\{a, d\}) = \{b, c\}$ f is not αg-closed in (X, τ). However, f is $g\alpha c$-homemorphism..

Example 4.23 — Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}\}$, $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, Y\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = b$, $f(b) = a$, $f(c) = c$. Then, f is not $g\ \alpha\ c$-homeomorphism, since for $V = \{b, c\}$, $g\alpha$-closed in (Y, σ), $f^{-1}(\{b, c\}) = \{a, c\}$ is not $g\alpha$-closed in (X, τ). However, f is an αgc-homeomorphism.

Theorem 4.24 — *(a) If $f: (X, \tau) \to (Y, \sigma)$ is α-irresolute and pre-α-closed then for every $g\alpha$-closed set A of (Y, σ), $f^{-1}(A)$ is αg-closed in (X, τ).*

(b) If $f: (X, \tau) \to (Y, \sigma)$ is continuous and pre-α-closed, then for every $\alpha\ g$-closed set A of $(X, \tau), f(A)$ is $\alpha\ g$-closed in (Y, σ).

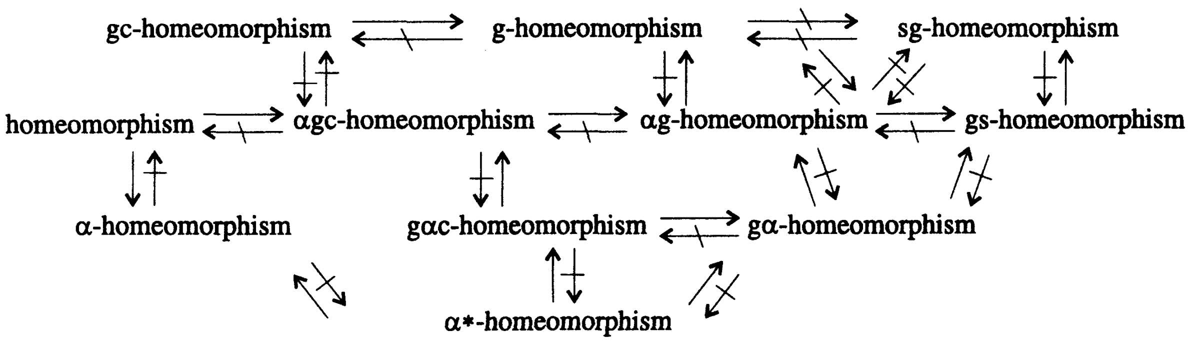
(c) If $f: (X, \tau) \to (Y, \sigma)$ is a homeomorphism then f and its inverse f^{-1} are pre-α-closed and α-irresolute.

PROOF : Parts (a) and (b) follow from Theorem 3.2[3] and Part (c) follows from Theorem 5.4 (i)[3].

Theorem 4.25 — *If $f: (X, \tau) \to (Y, \sigma)$ is an α-irresolute and pre-α-closed, then f is $g\alpha c$-irresolute.*

PROOF : $f: (X, \tau) \to (Y, \sigma)$ is an α- irresolute implies $f: (X, \tau) \to (Y, \sigma)$ is continuous and $f: (X, \tau) \to (Y, \sigma)$ is pre-α-closed implies $f: (X, \tau^{\alpha}) \to (Y, \sigma^{\alpha})$ is a closed map. By Theorems 6.1, 6.3[4], $f: (X, \tau^{\alpha}) \to (Y, \sigma^{\alpha})$ is gc-irresolute. Therefore, $f: (X, \tau) \to (Y, \sigma)$ is $g\alpha\, c$-irresolute.

Remark 4.26 : From Example 5.1 of Tadros[13], Remarks 2.7, 2.9, 3.2, 3.11, 3.13, 4.2, 4.5, 4.8, 4.5, Examples 2.8, 2.10, 3.3, 3.4, 3.12, 3.14, 3.15, 4.3, 4.4, 4.6, 4.7, 4.9, 4.16 to 4.23, Remark 4.21 we have the following diagram :



5. $g\alpha$ AND αg-HOMEOMORPHISMS FROM QUOTIENT SPACES INTO SPACES

Let R be an equivalence relation on a space (X, τ). Let $(X, R, \tau/R)$ be a quotient space of (X, τ) and $\pi: (X, \tau) \to (X/R, \tau/R)$ be the canonical projection where $\tau/R = \{U \subset X/R : \pi^{-1}(U) \in \tau\}$.

Proposition 5.1 — *Suppose the canonical projection $\pi: (X, \tau) \to (X/R, \tau/R)$ is an α-irresolute and pre-α-closed map.*

(i) A subset F is $g\alpha$-closed in $(X/R, \tau/R)$ if and only if the inverse image $\pi^{-1}(F)$ is $g\alpha$-closed in (X, τ).

(ii) If $\pi^{-1}(F)$ is αg-closed in (X, τ) then F is αg-closed in $(X/R, \tau/R)$.

PROOF : (*i*) (Necessity) — Suppose that F is $g\alpha$-closed in $(X/R, \tau/R)$. Since π is α-irresolute and pre-α-closed, $\pi^{-1}(F)$ is $g\alpha$-closed in (X, τ) by Theorem 4.25.

(Sufficiency) Suppose that $\pi^{-1}(F)$ is $g\alpha$-closed in (X, τ) then $\pi(\pi^{-1}(F)) = F$ is $g\alpha$-closed in $(X/R, \tau/R)$ by using Theorem 4.25.

(*ii*) Suppose that $\pi^{-1}(F)$ is αg-closed in (X, τ) then by Theorem 4.24 (*b*), $\pi(\pi^{-1}(F)) = F$ is αg-closed in $(X/R, \tau/R)$, since π is continuous and pre- α-closed.

Remark 5.2 : Proposition 5.1 (*i*) and (*ii*) are true when $g\alpha$-closed sets and αg-closed sets are replaced by $g\alpha$-open sets and αg-open sets respectively.

Proposition 5.3 — *Suppose that* $\pi : (X, \tau) \to (X/R, \tau/R)$ *is an* α*-irresolute and pre-*α*-closed*

(*i*) *If* (X, τ) *is* $G\alpha O$*-compact, then* $(X/R, \tau/R)$ *is* $G\alpha O$*-compacts and*

(*ii*) *if* (X, τ) *is* αGO*-compact, then* $(X/R, \tau/R)$ *is compact.*

PROOF : (*i*) It follows from Remark 5.2.

(*ii*) It follows from Theorem 4.24 (*a*).

Now we investigate the topological properties of $g\alpha$-continuous maps (resp. $g\alpha$-irresolute maps) and $g\alpha$-homeomorphisms (resp. $g\alpha c$-homeomorphisms) from the quotient space into other space and also αg-continuous maps (resp. α-irresolute maps) and αg-homeomosphisms version.

Suppose that a map $f : (X, \tau) \to (Y, \sigma)$ satisfies the following conditions, that is (*) if xRy for x and $y \in X$ then $f(x) = f(y)$.

Then the induced map $\bar{f} : (X/R, \tau/R) \to (Y, \sigma)$ is well defined by $\bar{f}([x]) = f(x)$ for every $x \in X$, where $[x]$ denotes the equivalence class $\pi(x)$ containing x.

Theorem 5.4 — *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a map satisfying the condition (*) above. Suppose that the canonical projection* $\pi : (X, \tau) \to (X/R, \tau/R)$ *is an* α*-irresolute and pre-*α*-closed map, (i) the following statements are equivalent:*

(*a*) f *is* $g\alpha$*-continuous (resp.* $g\alpha$*-irresolute),*

(*b*) *the induced map* $f : (X/R, \tau/R) \to (Y, \sigma)$ *is* $g\alpha$*-continuous (resp.* $g\alpha$*-irresolute); and*

(*ii*) *If* f *is* α *g-continuous (resp.* α *g-irresolute) then its induced map* $\bar{f} : (X/R, \tau/R) \to (Y, \sigma)$ *is* α *g-continuous (resp.* α *g-irresolute).*

PROOF : (*i*) (*a*) $\to$ (*b*). Let V be a closed set (resp. $g\alpha$-closed set) of (Y, σ). Since $\pi^{-1}(\bar{f}^{-1}(V)) = f(V)$ and f is $g\alpha$-continuous (resp. $g\alpha$-irresolute), the set $\pi^{-1}(\bar{f}^{-1}(V))$ is $g\alpha$-closed in (X, τ). By using Proposition 5.1 (i) it is obtained that $\bar{f}^{-1}(V)$ is $g\alpha$-closed in $(X/R, \tau/R)$. Hence, f is $g\alpha$-continuous (resp. g α-irresolute).

(*b*) $\to$ (*a*) Let V be a closed set (resp. $g\alpha$-closed set) of (Y, σ). Since $\bar{f}^{-1}(V)$, say F, is $g\alpha$-closed in $(X/R, \tau/R)$, the inverse image $\pi^{-1}(F) = \pi^{-1}(\bar{f}^{-1}(V))$ is $g\alpha$-closed in (X, τ) by Proposition 5.1 (i). But $\pi^{-1}(\bar{f}^{-1}(V)) = f^{-1}(V)$. Hence, $f^{-1}(V)$ is $g\alpha$-closed in (X, τ). Hence, f is $g\alpha$-continuous (resp. g α-irresolute) map.

(*ii*) It follows from Proposition 5.1 (*ii*) in the place of Proposition 5.1 (*i*) as similar as proof of Necessity of (*i*).

Theorem 5.5 — *Suppose that* $\pi : (X, \tau) \to (X/R, \tau/R)$ *is an* α*-irresolute and pre-*α*-closed map and map* $f : (X, \tau) \to (Y, \sigma)$ *satisfies the following condition :*

(**) xRy for x and $y \in X$ *if and only if* $f(x) = f(y)$ *holds.*

(i) If f is $g\alpha$*-continuous, onto* $g\alpha$*-closed map, then* $\bar{f} : (X/R, \tau/R) \to (Y, \sigma)$ *is a* $g\alpha$*-homeomorphism.*

(ii) If f is an αg*-continuous, onto and* αg*-closed map, then* $\bar{f} : (X/R, \tau/R) \to (Y, \sigma)$ *is a* αg*-homeomorphism.*

PROOF : (i) follows from Theorem 5.4 (i) and Proposition 3.5 and (ii) follows from Theorem 5.4 (ii) and Proposition 4.6.

Theorem 5.6 — *Suppose that* $\pi : (X, \tau) \to (X/R, \tau/R)$ *is an* α*-irresolute and pre-*α*-closed map and a map* $f : (X, \tau) \to (Y, \sigma)$ *satisfies the following conditions.*

(**) xRy for x and $y \in X$ *if and only if* $f(x) = f(y)$ *holds.*

(i) If f is $g\alpha$*-continuous (resp.* $g\alpha$*-irresolute), onto and if* (X, τ) *is a* $G\alpha O$*-compact and* (Y, σ) *is Hausdorff, then its induced map* $\bar{f} : (X/R, \tau/R) \to (Y, \sigma)$ *is a* $g\alpha$*-homeomorphisms (resp.* $g\alpha c$*-homeomorphism).*

(ii) If f is αg*-continuous onto and if* (X, τ) *is an* αGO*-compact and* (Y, σ) *is Hausdorff, then its induced map* $\bar{f} : (X/R, \tau/R) \to (Y, \sigma)$ *is an* αg*-homeomorphism.*

PROOF : (*i*) By Theorem 5.4 (*i*), $\bar{f}$ is a bijective and $g\alpha$-continuous (resp. $g\alpha$-irresolute) map. Then it suffices to prove $\bar{f}^{-1}$ is $g\alpha$-continuous, (resp. $g\alpha$-irresolute). Let F be a closed (resp. $g\alpha$-closed) set of $(X/R, \tau/R)$ then $\pi^{-1}(F)$ is $g\alpha$-closed in (X, τ) and hence it is $G\alpha O$-compact relative to X, by Proposition 5.1 and Theorem (4.3 $(i)^2$. By Proposition 4.3 $(ii)^2$ $\bar{f}(F)$) is compact in (Y, σ) (resp. $G\alpha O$-compact relative to Y and hence it is compact in (Y, σ). By assumption that (y, σ) is Hansdorff, $\overline{fown20f}(F)$ is closed in (y, σ). Therefore, $\bar{f}^{-1}$ is a $g\alpha$-continuous (resp. $g\alpha$-irresolute).

(*ii*) The proof is similar to that of (*i*) using Theorem 4.4 (*ii*), 4.3 (*i*), $(ii)^2$.

In the following Theorem, let S be an equivalence relation on (Y, σ) and let $\pi^* : (Y, \sigma) \to (Y/S, \sigma/S)$ be the canonical projection. Let $f : (X, \tau) \to (Y, \sigma)$ be a map satisfying the following conditions, that is,

(***) if xRy for x and $y \in X$, then $f(x)$ S $f(y)$ holds. Then the induced map $f_* : (X/R, \tau/R) \to (Y/S, \sigma/S)$, is well defined by $f_*([x]) = \pi^*(f(x))$ for every $[x] \in X/R$.

Theorem 5.7 — *Suppose that* $\pi : (X, \tau) \to (X/R, \tau/R)$ *and* $\pi^* : (Y, \sigma) \to (Y/S, \sigma/S)$ *are* α*-irresolute and pre-*α*-closed maps.*

(i) Let $f:(X,\tau)\to(Y,\sigma)$ be $g\alpha$-continuous (resp. $g\alpha$-irresolute) map satisfying the conditions (***) above.

(a) The induced map $f_*:(X/R,\tau/R)\to(Y/S,\sigma/S)$ is $g\alpha$-continuous (resp. $g\alpha$-irresolute).

(b) If there exists a $g\alpha$-continuous (resp. $g\alpha$-irresolute) map $j:(Y,\sigma)\to(X,\tau)$ such that $f_*\bullet\pi\bullet j=\pi^*$ and the converse of (***) holds, then f_* is an $g\alpha$-homeomorphisms (resp. $g\alpha c$-homeomorphism,

(ii) Let $f:(X,\tau)\to(Y,\sigma)$ be a α g-continuous map satisfying the conditions (***) above

(a) The induced map $f_*:(X/R,\tau/R)\to(Y/S,\sigma/S)$ is αg-continuous.

(b) If there exists an αg-continuous map $j:(Y,\sigma)\to(X,\tau)$ and such that $f_*\bullet\pi\bullet j=\pi^*$and the converse of (***) holds, then f_* is an α g-homeomorphism.

PROOF : (*i*) (*a*) Let $h=\pi^*\bullet f:(X,R)\to(Y/S,\sigma/S)$. Since $h(x)=h(y)$ for x and $y\in X$ such that $x\ R\ y$, we have $\bar{h}=f_*$. Let V be a closed (resp. $g\alpha$-closed) set of $(Y/S,\sigma/S)$, then $h^{-1}(V)=(\pi^*\bullet f)^{-1}(V)=f^{-1}(\pi^{*-1}(V))$ is $g\alpha$-closed (resp. $g\alpha$-irresolute). By theorem 5.4 (*i*) f_* is $g\alpha$-continuous (resp. $g\alpha$-irresolute), since $\bar{h}=f_*$.

(*b*) It follows from (*i*) and assumption that f_* is a $g\alpha$-continuous (resp. $g\alpha$-irresolute) bijection. We shall prove that f_* is $g\alpha$-closed (resp. $\bar{f}_*^{1}$ is $g\alpha$-irresolute). Let F be a closed (resp. $g\alpha$-closed) set of $(X/R,\tau/R)$. It follows from the continuity of π (resp. Theorem 4.25) and $g\alpha$-continuity of j (resp. $g\alpha$-irresoluteness of j) that $j^{-1}(\pi^{-1}(F))$ is a $g\alpha$-closed set. By a fact that $f_*(F)=\pi^*((\pi\bullet j)^{-1}(F))$ holds and Theorem 4.25, $f_*(F)$ is a $g\alpha$-closed set and hence f_* is a $g\alpha$-closed map (resp. $\bar{f}_*^{1}$ is $g\alpha$-irresolute). Therefore, by Proposition 3.5 (resp. Definition 3.9) f_* is a $g\alpha$-homeomorphism (resp. $g\alpha c$-homeomorphism).

(*ii*) (*a*) By using Theorem 5.4 (*ii*) and assumptions it can be proved similarly as (*i*) (*a*).

(*b*) By using Theorem 4.24 (*b*), Propositions 4.11 and assumptions, it can be proved similarly as (*i*) (*b*).

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