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FUZZY CONNECTEDNESS : A UNIFIED APPROACH

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The intent of this article is to initiate a unified theory in the context of fuzzy connectedness and many of its allied forms, done already in a fuzzy topological space. With the help of a generalized type of operator, it has ultimately been possible to arrive at certain unified results which when interpreted in different particular settings in fuzzy topological spaces, provide results, already known in the respective corresponding situations. It is thus revealed that different types of fuzzy connectedness, studied so far as different entities, can now be brought under a single proof.

Key Words : Fuzzy Operator System; Γ-Separated Fuzzy Sets; Γ-Separated Fuzzy Sets; Γ-Connectedness; Quasi-coincidence; Fuzzy Topological Space

1. INTRODUCTION AND PRELIMINARIES

It is well known that after the introduction of the fuzzy topological space (henceforth fts, for short) by Chang[4] in 1968, a large number of mathematicians have taken great interests in generalizing and extending different concepts of set topology into fuzzy setting. The concept of connectedness along with some of its allied forms is one of the directions that have hitherto been ventured with meticulous attention. However, the results obtained in connection with different contexts like fuzzy connectedness[10], semi-connectedness[6], δ-connectedness[5] etc. in an fts are seen to be quite parallel and analogous. This is chiefly due to the fact that the study of these variations of the concept of fuzzy connectedness has been effected only by replacing the fuzzy closure operator by fuzzy semiclosure operator or fuzzy δ-closure operator or the like. It can thus be conjectured that the use of a suitable generalized type of operator should unify all these different but similar results. This article is aimed at showing that the conjecture is indeed true.

We shall start with a set and an arbitrary operator on the class of all fuzzy sets on X, satisfying certain conditions; and thereby develop a generalized notion of fuzzy connectedness. Certain

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theorems are then derived, which serve as the unified results binding together the corresponding ones, known already in connection with the investigations of fuzzy connectedness and certain variant forms of it, these latter results being displaced in a few tables at the end with proper references. Our results incidentally show that many other similar results in some other particular settings could also be obtained with different interpretations of the unified operator, employed here.

In what follows, by an fts (X, τ) we shall consider a fuzzy topological space according to the definition of Chang[4]. Since a fuzzy set A in a nonempty set X is a function from X to I (= [0, 1]), I^X stands for the class of all fuzzy sets in X. The two particular constant fuzzy sets which take the respective constant values 0 and 1 at each point of X will be denoted by 0_X and 1_X respectively. A fuzzy point[9] with the singleton support x and the value $\alpha (0 < \alpha \leq 1)$ at x will be writen, as usual, by x_α. The usual notation $(1 - A)$ stands for the fuzzy complement of fuzzy set A in X, defined by $(1 - A)(x) = 1 - A(x)$, for each $x \in X$. If A, B are two fuzzy sets in X, then we write $A \leq B$ to mean that $A(x) \leq B(x)$ for each $x \in X$, while A is said to be quasi-coincident (q-coincident, for short) with B, written as $A \, q \, B$, if for at least one point $x \in X$, $A(x) + B(x) > 1$[9]. The negation of the latter statement is denoted by $A \, \not{q} \, B$.

Having furnished the prerequisites so far, we now append the definition of the fuzzy operator system, the generalized premise which we shall work upon throughout.

Definition 1[8]— Let X be a non-void set, and let $\Gamma : I^X \to I^X$ be an operator such that $\Gamma(1_X) = 1_X$ and that Γ is isotone, i.e., $\Gamma(A) \leq \Gamma(B)$ for any $A, B \in I^X$ with $A \leq B$.

Let us call the pair (X, Γ) a *fuzzy operator system.*

2. *Γ-SEPARATED* FUZZY SETS

We propose to introduce the concepts of separated fuzzy sets in a fuzzy operator system as follows.

Definition 2.1 — Two non-null fuzzy sets A and B in a fuzzy operator system (X, Γ) are said to be *Γ-separated* if $A \, \not{q} \, \Gamma(B)$ and $B \, \not{q} \, \Gamma(A)$.

The following result is immediate from the above definition :

Result 2.2 — Let (X, Γ) be a fuzzy operator system. If A, B are Γ-separated fuzzy sets in X and A_1, B_1 are non-null fuzzy subsets of A and B respectively, then A_1 and B_1 are also Γ-separated sets.

Separation of fuzzy sets in an fts has been dealt with by different researches in terms of different operators like fuzzy closure operator, semiclosure operator[6] etc. These are seen to be special cases of our definition above. Some such cases are given by Table - I.

TABLE I

	Γ-separated	Γ	Reference
1	Fuzzy weakly separated	Closure	Def. 3.1 of [10]
2	Fuzzy semi separated	semiclosure	Def. 4.1 of [5]
3	Fuzzy θ-separated	θ-closure	Def. 2.2 of [11]
4	Fuzzy δ-separated	δ-closure	Def. 3.1 of [6]
5	Fuzzy α-separated	α-closure	Def. 5.2.1 of [2]
6	Fuzzy β-separated	β-closure	Def. 5.5.1 of [2]
7	Fuzzy pre-separated	pre-closure	Def. 5.4.1 of [2]
8	Fuzzy semi pre-separated	semi-preclosure	Def. 3.1 of [3]

Definition 2.3[8] — A fuzzy set A in a fuzzy operator system (X, Γ) is said to be a *Γ-set* if $\Gamma(A) = A$; A is said to be a *Γ_*-set* if $(1 - A)$ is a Γ-set.

Theorem 2.4 — *Let A, B be a two non-null fuzzy sets in a fuzzy operator system (X, Γ) such that $A \not q B$, and A, B are either both Γ-sets or both Γ_*-sets. Then A, B are Γ-separated sets.*

PROOF : If A, B are Γ-sets and $A\, q\, B$, then clearly A and B are Γ-separated sets.

Let now A and B be Γ_*-sets and $A \not q B$. Then $A \leq 1 - B$ so that $\Gamma(A) \leq \Gamma(1 - B) = 1 - B$, i.e., $(\Gamma(A))(x) + B(x) \leq 1$, for each $x \in X$. Hence, $B \not q \Gamma(A)$. Similarly, $A \not q \Gamma(B)$. Thus A and B are Γ-separated.

Remark 2.5 : The above theorem unifies some results already existing in literature, found in an fts. For instance, if A, B are non-null fuzzy sets in an fts X such that $A \not q B$, and A, B are both fuzzy semiopen[1] (α-open[7], β-open[7], preopen[7]) or both fuzzy semiclosed (resp. α-closed, β-closed, preclosed), then A and B become fuzzy semi separated, (resp. α-separated, β-separated, pre-separated), as found in Theorem 4.2 (b) of [6] (resp. Theorems 5.2.4 (*b*), 5.5.4 (*b*) and 5.4.4 (*b*) of [2]).

Theorem 2.6 — *Suppose A, B are two non-null fuzzy sets in a fuzzy operator system (X, Γ) such that either both are Γ-sets or both are Γ_*-sets. If $C = A \cap (1 - B)$ and $D = B \cap (1 - A)$, then C and D are Γ-separated provided they are non-null.*

PROOF : First suppose A and B are both Γ_*-sets. Now, $D = B \cap (1 - A) \Rightarrow D \leq 1 - A \Rightarrow \Gamma(D) \leq \Gamma(1 - A) = 1 - A \Rightarrow \Gamma(D) \not q A \Rightarrow \Gamma(D) \not q C$. Similarly, $\Gamma(C)\, q\, D$. Next suppose A and B are both Γ-sets. Then $C = A \cap (1 - B) \Rightarrow C \leq A \Rightarrow \Gamma(C) \leq \Gamma(A) = A \Rightarrow \Gamma(C) \not q D$. Similarly, $\Gamma(D) \not q C$. Thus in both the cases, C and D are Γ-separated.

Remark 2.7 : We note as in Remark 2.5 that certain particular cases of the last result are obtained in [6] (resp. in [2]) for fuzzy semi-separated (resp. α-separated, β-separated, pre-separated) sets for an fts in Theorem 4.2 (*c*) (resp. in Theorems 5.2.4 (*c*), 5.5.4 (*c*) and 5.4.4(*c*)). Moreover, the case of fuzzy semi pre-separated sets has been done in [3].

Theorem 2.8 — *Let (X, Γ) be a fuzzy operator system, where Γ is an idempotent operator and also, $A \leq \Gamma(A)$ for each $A \in I^X$. Then two non-null fuzzy sets A and B in X are Γ-separated iff there exist to Γ_*-sets U and V such that $A \leq U, B \leq V$ and $A \not q V, B \not q U$.*

PROOF : Let A and B be two Γ-separatwed fuzzy sets. Now, $A\, q\, \Gamma(B) \Rightarrow A \leq 1 - \Gamma(B) = U$ (say); and $\Gamma(1 - U) = \Gamma(\Gamma(B)) = \Gamma(B)$ (Γ being idempotent) $= 1 - U$ so that $(1 - U)$ is a Γ-set, i.e., U is a Γ_*-set. Similarly, $B \leq 1 - \Gamma(A) = V$ (say) and V is a Γ_*-set. Now, $A \leq \Gamma(A) = 1 - V$ so that $A \not q V$. Similarly, $B \not q U$.

Conversely, let U and V be two Γ_*-sets such that $A \leq U, B \leq V, A \not q V$ and $B \not q U$. Then $(1 - U)$ and $(1 - V)$ are Γ-sets. Also, $A \not q V \Rightarrow A \leq 1 - V \Rightarrow \Gamma(A) \leq \Gamma(1 - V) = 1 - V \Rightarrow V \not q \Gamma(A) \Rightarrow B \not q (A)$. Similarly, $U \not q \Gamma(B)$ so that $A \not q \Gamma(B)$. Thus, A and B are Γ-separated sets.

Remark 2.9 : We observe once again that Theorems 5.2.5, 5.5.5 and 5.4.5 of [2] (resp. Theorem 4.3 of [6], Theorem 3.3 of [3]) obtained in connection with respectively the fuzzy α-separated, β-separated and pre-separated sets (resp. fuzzy semi-separated sets and semi pre-separated sets) in an fts X, where U, V have been taken to be respectively fuzzy α-open, β-open and preopen sets (resp. fuzzy semiopen, semi-preopen sets), are special cases of our general result above.

3. Γ-CONNECTEDNESS

Definition 3.1 — A fuzzy set A in a fuzzy operator system (X, Γ) is said to be a *Γ-connected set relative to X* or simply *Γ-connected* if there do not exist two Γ-separated sets B and C in X such that $A = B \cup C$. If, in addition, $A = 1_X$, then X is called a *Γ-connected space.*

Different types of fuzzy connectedness studied so far for fts's turn out to be particular cases of Γ-connectedness, as is displayed in Table II.

TABLE II

	A fuzzy set in an fts is	If it cannot be expressed as the union of two	Reference
1	Fuzzy connected	fuzzy weakly separated sets	Def. 3.4 of [10]
2	Fuzzy semi connected	fuzzy semi separated sets	Def. 4.4 of [5]
3	Fuzzy θ-connected	fuzzy θ-separated sets	Def. 2.3 of [11]
4	Fuzzy δ-connected	fuzzy δ-separated sets	Def. 3.3 of [6]
5	Fuzzy α-connected	fuzzy α-separated sets	Def. 5.2.6 of [2]
6	Fuzzy β-connected	fuzzy β-separated sets	Def. 5.5.6 of [2]
7	Fuzzy pre-connected	fuzzy pre-separated sets	Def. 5.4.6 of [2]

Theorem 3.2 — *A non-null fuzzy set C in a fuzzy operator system (X, Γ) is Γ-connected iff for every pair of Γ-separated sets A, B in X with $C \leq A \cup B$, exactly one of the possibilities (a) and (b) holds:*

(a) $C \leq A$ and $C \cap B = 0_X$, (b) $C \leq B$ and $C \cap A = 0_X$.

PROOF : Let C be Γ-connected. Since $C \leq A \cup B$, both of $C \cap A = 0_X$ and $C \cap B = 0_X$ cannot hold simultaneously. If $C \cap A \neq 0_X$ and $C \cap B \neq 0_X$, then by Result 2.2, they are also Γ-separated sets and $C = (C \cap A) \cup (C \cap B)$ which goes against the Γ-connectedness of C. Now, if $C \cap A = 0_X$ then $C \leq B$, while $C \leq A$ holds if $C \cap B = 0_X$.

Conversely, let the given condition hold. If possible, let C be not Γ-connected. Then there exist two Γ-separated sets A, B in X such that $C = A \cup B$. By hypothesis, either $C \cap A = 0_X$ or $C \cap B = 0_X$ so that either $A = 0_X$ or $B = 0_X$, none of which is true. Thus C is Γ-connected.

Remark 3.3 : The above theorem unifies the corresponding results on fuzzy connectedness and some of its variant forms found for an fts, as are given below :

(a) Theorem 2.8 on [11] for fuzzy θ-connectedness.

(b) Theorem 4.7 of [6] for fuzzy semi-connectedness.

(c) Theorem 3.7 of [5] for fuzzy δ-connectedness.

(d) Theorem 5.2.11 of [2] of fuzzy α-connectedness.

(e) Theorem 5.4.11 of [2] for fuzzy pre-connectedness.

(f) The result observed in Remark 5.5.12 of [2] for fuzzy β-connectedness.

(a) Lemma 3.7 of [10] for fuzzy connectedness.

Theorem 3.4 — *The union of any aggregate of Γ-connected sets in a fuzzy operator system, no two of which are Γ-separated, is a Γ-connected set.*

PROOF : Let $\{A_\alpha : \alpha \in \Lambda\}$ be a family of Γ-connected sets in a fuzzy operator system (X, Γ), no two of which are Γ-separated in X, and let $C = \bigcup \{A_\alpha : \alpha \in \Lambda\}$ (Λ being an indexing set). If possible, let C be not Γ-connected. Then there exist two non-null fuzzy sets A and B which are Γ-separated in X and $C = A \bigcup B$.

For each $\alpha \in \Lambda, A_\alpha$ is Γ-connected and $A_\alpha \leq A \bigcup B$. Then by Theorem 3.2, either $A_\alpha \leq A$ and $A_\alpha \bigcap B = 0_X$, or else $A_\alpha \leq B$ and $A_\alpha \bigcap A = 0_X$. If possible, let for some $\alpha, \beta \in \Lambda$ with $\alpha \neq \beta, A_\alpha \leq A$ and $A_\beta \leq B$. Then A_α, A_β being non-null subsets of Γ-separated sets, are also Γ-separated, which is not the case. Thus either $A_\alpha \leq A$ with $A_\alpha \bigcap B = 0_X$ for each $\alpha \in \Lambda$, or else $A_\alpha \leq B$ with $A_\alpha \bigcap A = 0_X$ for each $\alpha \in \Lambda$. In the first case $B = 0_X$ (since $B \leq C$), and in the second case $A = 0_X$, none of which is true. Thus C is Γ-connected.

Theorem 3.5 — *Let $\{A_\alpha : \alpha \in \Lambda\}$ be a family of Γ-connected sets in a fuzzy operator system (X, Γ) such that $A_\alpha \bigcap A_\beta \neq 0_X$ for any $\alpha, \beta \in \Lambda$. Then $\bigcup_{\alpha \in \Lambda} A_\alpha = C$ (say) is Γ-connected too.*

PROOF : If possible, let C be not Γ-connected. Then there exist two Γ-separated sets A, B in X such that $C = A \bigcup B$.

Now, for each $\alpha \in \Lambda, A_\alpha$ is Γ-connected and $A_\alpha \leq A \bigcup B$, where A, B are Γ-separated.

Then by Theorem 3.2, either $A_\alpha \leq A$ with $A_\alpha \bigcap B = 0_X$ or $A_\alpha \leq B$ with $A_\alpha \bigcap A = 0_X$. Suppose for some $\alpha, \beta \in \Lambda$ with $\alpha \neq \beta, A_\alpha \leq A$ and $A_\beta \leq B, A_\alpha \bigcap B = 0_X$ and $A_\beta \bigcap A = 0_X$. Then $A_\alpha \bigcap B = 0_X$ and $A_\beta \leq B \Rightarrow A_\alpha \bigcap A_\beta = 0_X$, a contradiction to the hypothesis. Hence either $A_\alpha \leq A$ with $A_\alpha \bigcap B = 0_X$ for each $\alpha \in \Lambda$, in which case $B = 0_X$, or else $A_\alpha \leq B$ with $A_\alpha \bigcap A = 0_X$ for each $\alpha \in \Lambda$ implying that $A = 0_X$. In any case, we arrive at a contradiction. Hence, C is Γ-connected.

Remark 3.6 : From our unified results in Theorems 3.4 and 3.5, many results in respect of fuzzy connectedness and some of its allied forms in an fts can be obtained. For example, Theorems 2.9 and 3.10 of [11] and [5] respectively for fuzzy θ-connectedness and δ-connectedness are special cases of Theorem 3.4, while Theorems 5.2.12, 5.4.12 and 5.513 of [2] for fuzzy α-connectedness, pre-connectedness and β-connectedness respectively and Theorem 3.8 of [10] for fuzzy connectedness are particular forms of Theorem 3.5.

Theorem 3.7 — *Let $\mathcal{J}$ be a family of Γ-connected sets in a fuzzy operator system (X, Γ) such that there is a non-null member A_0 of $\mathcal{J}$ with the property that A_0 and A are not Γ-separated, for each $A \in \mathcal{J}$. Then $\bigcup \{A : A \in \mathcal{J}\}$ is Γ-connected.*

PROOF : If possible, let $\bigcup \{A : A \in \mathcal{J}\}$ be not Γ-connected. Then for two Γ-separated sets C, D in X we have $A_0 \leq \bigcup \{A : A \in \mathcal{J}\} = C \bigcup D$. Thus by Theorem 3.2, either $A_0 \leq C$ with $A_0 \bigcap D = 0_X$, or else $A_0 \leq D$ with $A_0 \bigcap C = 0_X$. Let $A_0 \leq C$, then as shown in Theorem 3.4, we have $\bigcup \{A : A \in \mathcal{J}\} \leq C$ and $D = 0_X$, a contradiction. In the case when $A_0 \leq D$, a similar contradiction is arrived at.

Theorem 3.8 — *A non-null fuzzy set A in a fuzzy operator system* (X, Γ) *is* Γ*-connected iff for any two fuzzy points* x_α, y_β *in A there is a* Γ*-connected fuzzy set B such that* $B \leq A$ *and* $x_\alpha, y_\beta \leq B$.

PROOF : That the condition is necessary is obvious. Conversely, suppose that A is not Γ-connected so that there exist two Γ-separated fuzzy sets P, Q in X such that $A = P \bigcup Q$. As P, Q are non-null, choose two fuzzy points x_α and y_β such that $x_\alpha \leq P$ and $y_\beta \leq Q$. Then $x_\alpha, y_\beta \leq A$ and hence by hypothesis, there exists a Γ-connected set B contained in A such that $x_\alpha, y_\beta \leq B$. Thus $B \bigcap P$ and $B \bigcap Q$ are non-null fuzzy sets, and are Γ-separated such that $B = (P \bigcap B) \bigcup (Q \bigcap B)$, a contradiction to the Γ-connectedness of B.

Remark 3.9 : The last two theorems proved in our chosen generalized setting naturally have their counterparts in the particular contexts of fuzzy connectedness and the like in an fts. In fact, Theorem 2.10 of [11], Corollary 3.11 of [5] and Theorem 4.9 of [6] corroborate our contention in connection with fuzzy θ-connectedness, δ-connectedness and semi-connectedness respectively for Theorem 3.7, whereas Theorem 2.11 of [11] for fuzzy θ-connectedness in an fts particularizes Theorem 3.8.

Definition 3.10[8] — Let (X, Γ) and (Y, Γ') be two fuzzy operator systems. A function $f : (X, \Gamma) \to (Y, \Gamma')$ is said to be *(Γ, Γ')-continuous* if $f(\Gamma(A)) \leq \Gamma'(f(A))$, for each $A \in I^X$.

Theorem 3.11[8] — *A function* $f : (X, \Gamma) \to (Y, \Gamma')$ *(where* (X, Γ) *and* (Y, Γ') *are two given fuzzy operator systems) is* (Γ, Γ')*-continuous iff* $\Gamma(f^{-1}(B)) \leq f^{-1}(\Gamma'(B))$, *for each* $B \in I^Y$.

Lemma 3.12 — Let (X, Γ) and (Y, Γ') be two fuzzy operator systems and $f : (X, \Gamma) \to (Y, \Gamma')$ be a *(Γ, Γ')-continuous surjection.* If C and D are *Γ-separated fuzzy sets in Y, then* $f^{-1}(C)$ *and* $f^{-1}(D)$ *are* Γ*-separated fuzzy sets in X.*

PROOF : As f is a surjective, $f^{-1}(C)$ and $f^{-1}(D)$ are non-null fuzzy sets in X. If possible, let $f^{-1}(C)$ and $f^{-1}(D)$ be not Γ-separated in X. Then $f^{-1}(C)\, q\, \Gamma(f^{-1}(D)) \Rightarrow f^{-1}(C)\, q f^{-1}(\Gamma'(D))$ (by Theorem 3.11) $\Rightarrow ff^{-1}(C)\, q f(f^{-1}(\Gamma'(D)))$ [since it is known that $A\, q\, B \Rightarrow f(A)\, q\, f(B)$] $\Rightarrow C\, q\, \Gamma'(D)$ (as $ff^{-1}(C) \leq C$). Similarly, $f^{-1}(D), q\, \Gamma(f^{-1}(C)) \Rightarrow D\, q\, \Gamma'(C)$. Thus C and D are not Γ'-separated sets in Y, a contradiction.

Theorem 3.13 — *Let* (X, Γ) *and* (Y, Γ') *be two fuzzy operator systems and* $f : (X, \Gamma) \to (Y, \Gamma')$ *be a* (Γ, Γ')*-continuous bijection. If A is a* Γ*-connected set in X, then f(A) is* Γ'*-connected in Y.*

PROOF : Suppose that $f(A) = (B$ say) is not Γ'-connected in Y. Then $B = C \bigcup D$, for two Γ'-separated fuzzy sets C and D in Y. By Lemma 3.12, $f^{-1}(C)$ and $f^{-1}(D)$ are Γ-separated sets in X, and $A = f^{-1} f(A)$ (as f is an injection) $= f^{-1}(B) = f^{-1}(C) \bigcup f^{-1}(D)$, which shows that A is not Γ-connected, a contradiction. Hence $f(A)$ is Γ'-connected in Y.

Remark 3.14 : There is a long list of existing results concerning fuzzy connectedness and many of its similar versions in fts's *vis-a-vis* different fuzzy continuous-like functions between fts's, that have been bound together in view of the unification done in Theorem 3.13. Table - III provides a list of such results.

TABLE III

	f	A	$f(A)$	Reference
1	fuzzy continuous	fuzzy connected	fuzzy connected	Th. 3.4 of [10]
2	fuzzy weakly continuous	fuzzy connected	fuzzy θ-connected	Th. 3.6 (a) of [11]
3	fuzzy irresolute	fuzzy semi-connected	fuzzy semi-connected	Th. 412 of [5]
4	fuzzy θ continuous	fuzzy θ-connected	fuzzy θ-connected	Th. 3.6 (c) of [11]
5	fuzzy δ continuous	fuzzy δ-connected	fuzzy δ-connected	Th. 3.14 of [6]
6	fuzzy weakly δ-continuous	fuzzy δ-connected	fuzzy θ-connected	Th. 3.6 (b) of [11]
7	fuzzy β-continuous	fuzzy β-connected	fuzzy connected	Th. 5.510 of [2]
8	fuzzy M-β continuous	fuzzy β-connected	fuzzy β-connected	Th. 5.5.9 of [2]
9	fuzzy pre-continuous	fuzzy pre-connected	fuzzy connected	Th. 5.4.9 of [2]
10	fuzzy almost continuous	fuzzy connected	fuzzy δ-connected	Th. 3.16 of [6]
11	fuzzy M-pre continuous	fuzzy pre-connected	fuzzy pre-connected	Th. 5.4.13 of [2]
12	fuzzy α-continuous	fuzzy α-connected	fuzzy connected	Th. 5.2.9 of [2]
13	fuzzy M-α-continuous	fuzzy α-connected	fuzzy α-connected	Th. 5.2.12 of [2]

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