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TRANSIENT MHD FREE CONVECTION FLOW OF AN INCOMPRESSIBLE VISCOUS DISSIPATIVE FLUID

S. SREEKANTH*, A. S. NAGARAJAN** AND S. VENKATA RAMANA***

*&*** *Department of Mathematics, S. V. University College, Tirupati* 517 502, *Andhra Pradesh, India*

** *Department of Mathematics, N.B.K.R. Institute of Science and Technology, Vidyanagar* 524 413, *Andhra Pradesh, India*

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The unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate in considered on taking into account viscous dissipative heat, under the influence of a uniform transverse magnetic field. The velocity distribution, the temperature distribution, coefficient of skin-friction and rate of heat transfer have been investigated. The problem is governed by a coupled non-linear system of partial differential equation. Exact solutions are not possible, have be explicit finite difference method is employed.

Key Words : Transient MHD Free Conn Vection Flow; Incompressible Viscous Dissipative Fluid; Non-linear Systems; Partial Differential Equation

1. INTRODUCTION

Engineering processes in which a fluid supports an exothermic chemical or nuclear reaction are very common today and the correct process design requires accurate correlation for the heat transfer coefficients at the boundary surfaces. Despite its increasing importance in technological and physical problems, the unsteady MHD free convection flows of dissipative fluids past an infinite plate have received much attention because of nonlinearity of the governing equations. Without taking into account viscous dissipative heat and MHD, this problem was first solved by Siegal[10] by integral method. The experimental conformation of these results were presented by Goldstein and Eckert[6]. Other papers in this field are by Gebhart[4], Schetz and Eichhorn[9], Monold and Yang[7], Sparrow and Gregg[14], Chung and Anderson[2], Goldstein and Briggs[5] etc. In all these papers, the effects of viscous dissipative heat and MHD was assumed to be neglected. However, Gebhart[3] has shown that when the temperature difference is small or in high Prandtl number fluids or when the gravitational field is of high intensity, viscous dissipative heat should be taken into account in steady free convection flow past a semi-infinite vertical plate. Following this assumption, Soundalgekar, Bhat and Mohiud-din[11] studied the effects of free convection currents on the flow past an impulsively started infinite isothermal vertical plate. Reaptis[8] has studied free convection and mass transfer effects on the flow past an infinite moving vertical porous plate with constant suction and heat sources when free stream velocity is an oscillatory function of time. Agarwal *et al.*[1] have discussed the combined buoyancy effects of thermal and mass diffusion on MHD natural convection flows. Vajravelu[15] has studied the problem of free convection heat transfer between two long vertical plates moving in opposite directions. Recently, Soundalgekar *et al.*[12] have studied free convection flow of an incompressible viscous dissipative fluid. Here, the problem is governed by a coupled non-linear system of partial differential equations, this problem was solved by finite-difference technique. It is now proposed to solve the unsteady free convection flow of a dissipative viscous fluid past an infinite vertical plate, on taking into account, the viscous dissipative heat, under the influence of uniform transverse magnetic field. The problem is governed now by coupled non-linear system of partial differential equations whose exact solution is not possible. So we employ explicit finite-difference method for its solution.

In this paper, we consider the unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate on taking into account viscous dissipative heat, under the influence of a uniform transverse magnetic field. We have computed the velocity, the temperature, coefficient of skin friction and the rate of heat transfer and the effects of the magnetic field parameter M, the Prandtl number P_r and the Eckert number E are discussed.

2. Formulation and Solution of the Problem

We consider the unsteady hydromagnetic free convective flow of an incompressible, viscous fluid past an infinite vertical plate. Initially the temperature of the plate and the fluid are assumed to be the same. At time $t' > 0$, the plate temperature is raised to T_w' which is then maintained constant. Under these conditions, the flow-variables are functions of t' and y' alone. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to it. A uniform magnetic field of intensity H_0 is applied in the y-direction. Therefore the velocity and the magnetic field are given by $\bar{q} = (u, 0)$ and $\bar{H} = (O, H_0)$. The fluid being slightly conducting the magnetic Reynolds number is much less than unity and hence the induced magnetic field can be neglected in compression with the applied magnetic field (Sparrow and Cess[13]) in the absence of any input electric field, the equations governing the flow are

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u}{\partial y'^2} + g\,\beta (T' - T'_\infty) - \frac{\sigma \mu_e^2 H_0^2}{\rho} u' \qquad \text{... (2.1)}$$

and

$$\rho C_p \frac{\partial T'}{\partial t'} = K \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'} \right)^2, \qquad \text{... (2.2)}$$

where u' is the velocity of the fluid, g is the acceleration due to gravity, K is the thermal conductivity of the fluid, ρ is the density of the fluid, ν is the kinematic viscosity, β is the coefficient of volume expansion, T_w is the temperature of the plate, T_∞ is the temperature of the fluid far away from plate, μ is the viscosity of the fluid, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, C_p is the specific heat at constant pressure, U_0, T_R, L are reference velocity, reference time and reference length respectively.

The initial and boundary conditions are

$$\left.\begin{array}{ll} t' \le 0,\ u' = 0,\ T' = T'_\infty & \text{for all } y' \\ t' > 0,\ u' = 0,\ T' = T'_w & \text{at } y' = 0 \\ \qquad\quad u' = 0,\ T' \to T'_\infty & \text{at } y' \to \infty \end{array}\right] \qquad \text{... (2.3)}$$

The following non-dimensional quantities are introduced :

$$\left.\begin{array}{l} \Delta^{T} = T_i - T_\infty', U_0 = (\nu g\beta DT)^{1/3}, L = (g\beta DT/^{-1/3)}, T_R = (g\beta DT)^{-2/3)}/\nu^{-1/3} \\ t = t'/T_R,\ \ y = y'/L,\ u = u'/U_0, \\ \theta = (T' - T'_\infty)/(T'_w - T'_\infty),\ P_r = \mu\, C_p/K \text{ (the Prandtl number)} \\ E = U_0^2/C_p\, \Delta T \quad \text{(the Eckert number)} \\ M = \sigma\, \mu_e^2\, H_0^2\, T_R/\rho \quad \text{(the magnetic parameter)} \end{array}\right] \quad \dots (2.4)$$

where $U_0 = (\nu\, g\, \beta\, \Delta\, T)^{1/3},\ L = (g\, \beta\, \Delta\, T/\nu^2)^{-1/3},\ T_R = (g\, \beta\, \Delta\, T)^{-2/3}/\nu^{-1/3}$

and $\Delta\, T = T'_w - T'_\infty,$

In view of eq. (2.4), eqs. (2.1) and (2.2) reduceds to (droping superscripts ')

$$\frac{\partial u}{\partial t} = \theta + \frac{\partial^2 u}{\partial y^2} - Mu \quad \dots (2.5)$$

and

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + P_r E \left(\frac{\partial u}{\partial y} \right)^2. \quad \dots (2.6)$$

The non dimensional initial and boundary conditions are

$$\left.\begin{array}{llll} t \le 0, & u = 0, & \theta = 0 & \text{for all } y \\ t > 0, & u = 0, & \theta = 1 & \text{at } y = 0 \\ & u = 0, & \theta \to 0 & \text{at } y \to \infty \end{array}\right] \quad \dots (2.7)$$

3. Method of Solution

Define a new dimensionless variable $\eta = \dfrac{y}{1+y}$ ... (3.1)

In view of (3.1), the eqs. (2.5)-(2.7) take the form

$$\frac{\partial u}{\partial t} = \theta + (1-\eta)^4 \frac{\partial^2 u}{\partial \eta^2} - 2\,(1-\eta)^3 \frac{\partial u}{\partial \eta} - Mu \quad \dots (3.2)$$

$$P_r \frac{\partial \theta}{\partial t} = (1-\eta)^4 \frac{\partial^2 \theta}{\partial \eta^2} - 2\,(1-\eta)^3 \frac{\partial \theta}{\partial \eta} + (1-\eta)^4 P_r E \left(\frac{\partial u}{\partial \eta} \right)^2 \quad \dots (3.3)$$

$$\left.\begin{array}{llll} t \le 0, & u = 0, & \theta = 0 & \text{for all } \eta \\ t > 0, & u = 0, & \theta = 1 & \text{at } \eta = 0 \\ & u = 0, & \theta \to 0 & \text{at } \eta = 1 \end{array}\right] \quad \dots (3.4)$$

Eqs. (3.2) and (3.3) are coupled non-linear partial differential equations, and are to be solved by using the initial and boundary conditions (3.4). However, exact or approximate solutions are not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference scheme of equations for (3.2) to (3.4) are as follows :

$$\frac{U_{i,j+1}-U_{i,j}}{\Delta t}=\theta_{i,j}+(1-\eta_{i,j})^4\left(\frac{U_{i+1,j}-2U_{i,j}+U_{i-1,j}}{(\Delta\eta)^2}\right)$$

$$-\,2(1-\eta_{i,j})^3\left(\frac{U_{i+1,j}-U_{i,j}}{\Delta\eta}\right)-MU_{i,j} \qquad \text{... (3.5)}$$

$$P_r\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}=(1-\eta_{i,j})^4\left(\frac{\theta_{i+1,j}-2\,\theta_{i,j}+\theta_{i-1,j}}{(\Delta\eta)^2}\right)-2\,(1-\eta_{i,j})^3\left(\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta\eta}\right)$$

$$+\;(1-\eta_{i,j})^4\,P_r\,E\left(\frac{U_{i+1,j}-U_{i,j}}{\Delta\eta}\right)^2. \qquad \text{... (3.6)}$$

Here, index i refers to η and j to time. The mesh system is divided by taking $\Delta\eta = 0.1$. From the initial condition in (3.4), we have the following equivalent.

$$u(0,\,0)\;\;0,\;\theta(0,0)=1,\;u(i,\,0)=0,\;\theta\,(i,\,0)=0\text{ for all } i \text{ except } i=0 \qquad \text{... (3.7)}$$

The boundary conditions from (3.4) are expressed in finite-difference form as follows

$$\left.\begin{array}{lll} u\,(0,j)=0 & \theta\,(0,j)=1 & \text{for all } j \\ u\,(1,j)=0 & \theta\,(1,j)=0 & \text{for all } j \end{array}\right]. \qquad \text{... (3.8)}$$

First the velocity at the end of time step viz, $u(i,\ j+1)$ $(i = 1,\ 10)$ is computed from (3, 5) in terms of velocities and temperatures at points on the earlier time-step. Then θ $(i,\ j+1)$ is computed from (3.6). The procedure is repeated until $t = 1$ (i.e., $j = 800$). During computation Δt was chosen as 0.00125.

These computations were carried out for p_r 0.71, 7 and 100 and $E = 0$, 0.1, 0.3, 0.5. To judge the accuracy of the convergence and stability of finite-difference scheme, the same program was run with smaller values of Δt i.e., $\Delta t = 0.0009$, 0.001 and no significant change was observed. Hence, we conclude that the finite-difference scheme is stable and convergent.

Skin-Friction

We now calculate from the velocity field the skin-friction. It is given in non dimensional form as

$$\tau=-\left(\frac{du}{d\eta}\right)_{\eta=0},\quad\text{where } \tau=\tau'/\rho\,U_0^2$$

Numerical values of τ are calculated by applying the Newton's interpolation formula for four points.

In non-dimensional form; the rate of heat transfer can be shown to be given by

$$q=-\left(\frac{d\,\theta}{d\,\eta}\right)_{\eta=0}$$

and we have computed q by following the above procedure. These values of q are listed in Table I.

CONCLUSIONS

From Fig. 1 and 2, we observe that the velocity distribution u is drawn against η for different values of magnetic field parameter M and time t respectively for the cases of the Prandtl number $P_r = 0.71$ and 7. We noticed that velocity distribution u increases with the increase in t, where as u decreases with the increase in M, in all the cases $P_r = 0.71$ and 7. Further, we observe that u decreases with the increase in Prandtl number. Also, we noticed that u increases first near the plate

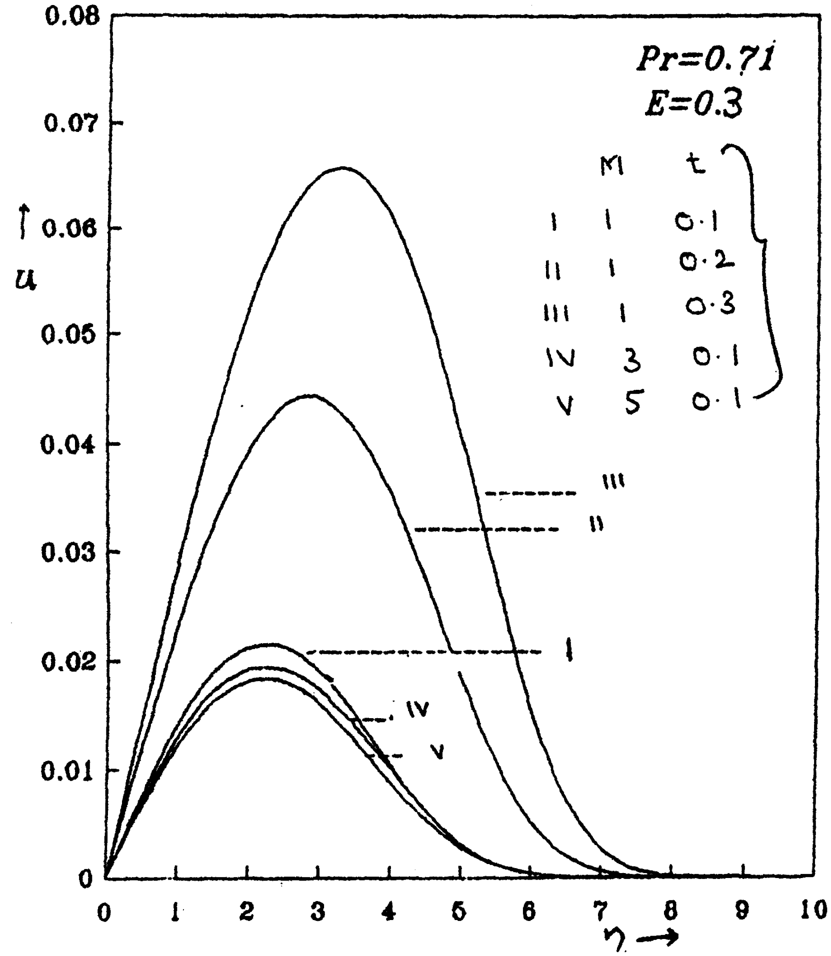

FIG. 1. Velocity profiles

and then the trend gets reversed as η increases. From Table I, we observe that u is drawn against η for different values of Eckert Number E. We noticed that u increases with increase in E for all the cases of $P_r = 0.71$ and 7 respectively. In Fig. 4, the temperature distribution θ is drawn against η for different values of P_r and t. We noticed that θ decreases with the increase in P_r (I, IV, V). Further, we observe that θ increases with increase in t (I, II, III). Also, we conclude that θ decreases with increase in η. On Fig. 4, the skin-friction is shown against t for different values of M in the cases of $P_r = 0.71$ and 7 respectively. We noticed that τ decreases with the increase in M or P_r where as it increases with the increase in t. From Table I, we observe that q increases with the increase in M whereas it decreases with increase in t. Further we noticed that q increases with the increase in P_r.

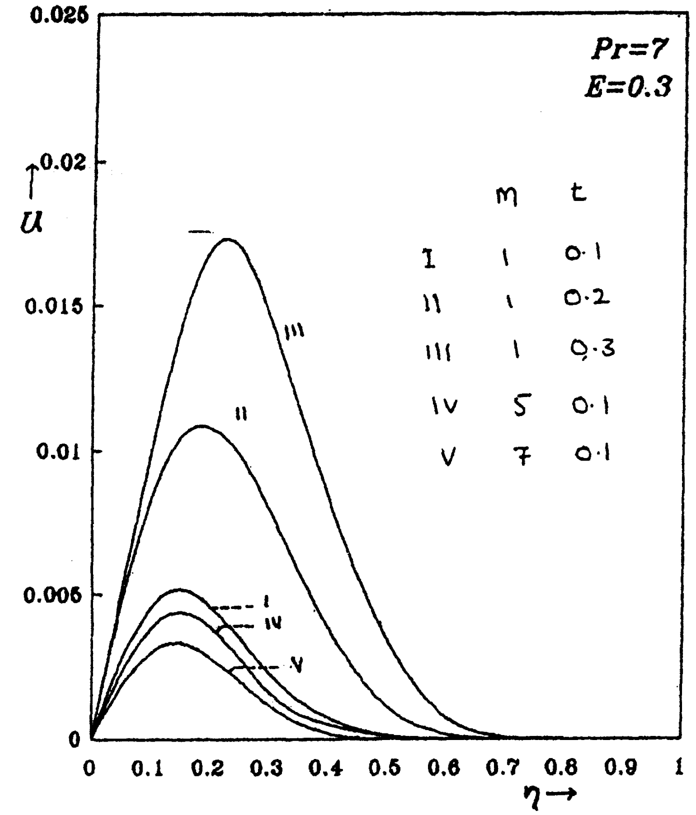

FIG. 2. Velocity profiles

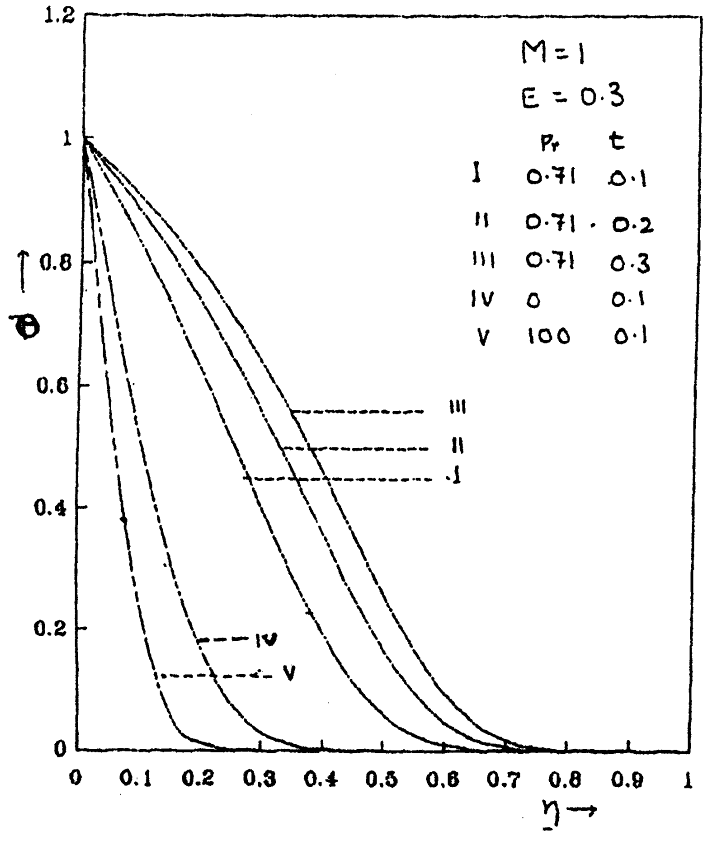

FIG. 3. The temperature profiles

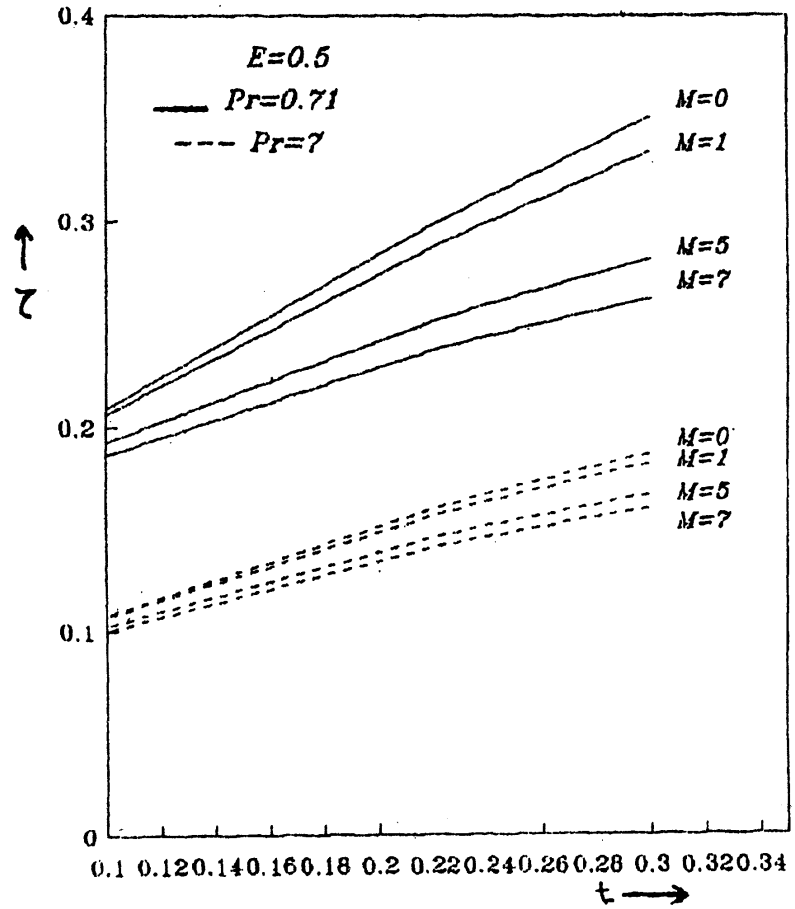

FIG. 4. τ against t for different M

TABLE I

Values of q, $E = 0.3$

Pr	t/M	1.0	5.0	7.0
0.71	0.1	1.466910452961730	1.466949752415230	1.46696525573480
	0.2	1.029097084044530	1.029443921615590	1.02956038515762
	0.3	0.829007040161925	0.829930740340081	0.83019865541012
7.0	0.1	5.738172094033400	5.738168217650010	5.73816651970487
	0.2	3.393478374472500	3.393493339249530	3.39349569647227
	0.3	2.661406215706130	2.661608014413550	2.661666225591319
100.0	0.1	16.513783373911000	16.513783359006200	16.5137833551595
	0.2	14.904670603894400	14.904669738909900	14.9046693915171
	0.3	13.499315776957400	13.499311894674200	13.4993102894712

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