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DECAY OF MHD TURBULENCE BEFORE THE FINAL PERIOD FOR THE CASE OF MULTI-POINT AND MULTI-TIME

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Following Deissler's approach, the multi-point, multi-time correlation equation for the decay of magnetic field fluctuation in MHD turbulence at time before the final period have been studied. Two and three point correlation equations have been obtained to make the set of equations determinate, the terms containing quadruple correlation are neglected in comparison to the second and third order correlation terms. Finally, by integrating the energy spectrum over all wave numbers, the decay law for magnetic energy fluctuation is obtained.

Key Words : Deissler's Method; Multi-Point; Multi-Time; Decay of MHD Turbulence

1. INTRODUCTION

In [4 & 5], Dessler developed a theory for homogeneous turbulence for time before the final period in its which Loeffer and Deissler[8] studied temperature fluctuation in homogeneous turbulence. In their study, they considered the two and three point correlation equations and solved these equations after neglecting the fourth-order correlation terms in comparison to the second and third order correlation terms. By considering the Deissler's same theory, Kumar and Patel[6] studied the first order reactant in homogeneous turbulence before the final period for the case of multi-point and single time consideration. The problem[6] is extended to the case of multi-point and multi-time concentration correlation by Kumar and Patel[7], and numerical result of [7] carried-out by Patel[9].

Following Deissler's approach, Alan and Kishore[1] also studied the decay of MHD turbulence before the final period for the case of multi-point and single time.

In this problem, the decary of MHD turbulence before the final period for the case of multi-point and multi-time has been studied. Finally we obtained the decay law of magnetic energy fluctuation before the final period in the same case.

2. FUNDAMENTAL EQUATIONS

The equations of motion for viscous, incompressible MHD turbulent flow are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k)=-\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}, \qquad \ldots (2.1)$$

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$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k)=\lambda\frac{\partial^2 h_i}{\partial x_k x_k} \qquad \text{... (2.2)}$$

with $$\frac{\partial u_i}{\partial x_i}=\frac{\partial h_i}{\partial x_i}, \qquad \text{... (2.3)}$$

where $u_i(x, t)$, ith-component of turbulent velocity; $h_i(x, t)$, ith-component of magnetic field fluctuation; $w(x, t)=p/\rho+\frac{1}{2}\langle h\rangle^2$, total MHD pressure; $p(x, t)$, hydrodynamic pressure; ρ, fluid density; ν, kinematic viscosity; $\lambda=\nu/P_M$, magnetic diffusivity; P_M, magnetic prandtle number; x_k, space coordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation.

Eqs. (2.1) through (2.3) are derived by Chandrasekhar[2], the basis of Batchelor's discussion by coupling Maxwell's equation for the electromagnetic field and the Navier-Stokes equations for the velocity field. The Maxwell equations are modified to include the induced electric field due to the fluid motion, and the Navier-Stokes equations are modified to include to the Lorentz force on fluid elements due to the magnetic field.

3. Two-Point, Two-Time Correlation and Spectral Equations

Induction equation of a magnetic field at the points p and p' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t}+u_k\frac{\partial h_i}{\partial x_k}-h_k\frac{\partial u_i}{\partial x_k}=\lambda\frac{\partial^2 h_i}{\partial x_k\,\partial x_k} \qquad \text{... (3.1)}$$

and $$\frac{\partial h_j'}{\partial t'}+u_k'\frac{\partial h_j'}{\partial x_k'}-h_k'\frac{\partial u_j'}{\partial x_k'}=\lambda\frac{\partial^2 h_j'}{\partial x_k'\,\partial x_k'}. \qquad \text{... (3.2)}$$

Multiplying eq. (3.1) by h_j' and eq. (3.2) by h_i and taking ensemble average, we get

$$\frac{\partial\langle h_i h_j'\rangle}{\partial t}+\frac{\partial}{\partial x_k}[\langle u_k h_i h_j'\rangle-\langle u_i h_k h_j'\rangle]=\lambda\frac{\partial^2\langle h_i h_j\rangle}{\partial x_k\,\partial x_k} \qquad \text{... (3.3)}$$

and $$\frac{\partial\langle h_i h_j'\rangle}{\partial t'}+\frac{\partial}{\partial x_k'}[\langle u_k' h_i h_j'\rangle-\langle u_j' h_i h_j'\rangle]=\lambda\frac{\partial^2\langle h_i h_j'\rangle}{\partial x_k'\,\partial x_k'}. \qquad \text{... (3.4)}$$

Angular bracket $\langle\ldots\ldots\rangle$ which is used to denote an ensemble average.

Using the transformation

$$\frac{\partial}{\partial x_k}=-\frac{\partial}{\partial r_k},\ \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k} \qquad \text{... (3.5)}$$

$$\left(\frac{\partial}{\partial t}\right)_{r'} = \left(\frac{\partial}{\partial t}\right)_{\Delta t} - \frac{\partial}{\partial \Delta t}, \quad \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t}.$$

Eqs. (3.3) and (3.4) can be written as

$$\frac{\partial \langle h_i h_j' \rangle}{\partial t} + \frac{\partial}{\partial r_k}[\langle u_k' h_i h_j' \rangle - \langle u_j' h_i h_k' \rangle](\hat{r}, \Delta t, t) - \frac{\partial}{\partial r_k}[\langle u_k h_i h_j' \rangle - \langle u_i h_k h_j' \rangle]$$

$$(\hat{r}, \Delta t, t) = 2\lambda \frac{\partial^2 \langle h_i h_j' \rangle}{\partial r_k \partial r_k} \quad \ldots (3.6)$$

and

$$\frac{\partial \langle h_i h_j' \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}[\langle u_k' h_i h_j' \rangle - \langle u_j' h_i h_k' \rangle](r, \Delta t) = \lambda \frac{\partial^2 \langle h_{ij}' \rangle}{\partial r_k \partial r_k}. \quad \ldots (3.7)$$

Using the relations (cf. Chandrasekhar[2])

$$\langle u_k h_i h_j' \rangle = -\langle u_k' h_i h_j' \rangle, \ \langle u_j' h_i h_k' \rangle = -\langle u_i h_k h_j' \rangle.$$

Eqs. (3.6) and (3.7) become

$$\frac{\partial \langle h_i h_j' \rangle}{\partial t} + 2\frac{\partial}{\partial r_k}[\langle u_k' h_i h_j' \rangle - \langle u_i h_k h_j' \rangle] = 2\lambda \frac{\partial^2 \langle h_i h_j' \rangle}{\partial r_k \partial r_k} \quad \ldots (3.8)$$

and

$$\frac{\partial \langle h_i h_j' \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}[\langle u_k' h_i h_j' \rangle - \langle u_i h_k h_j' \rangle] = \lambda \frac{\partial^2 \langle h_i h_j' \rangle}{\partial r_k \partial r_k}. \quad \ldots (3.9)$$

In order to convert eqs. (3.8) and (3.9) to spectral form, we define the following three dimensional Fourier transforms :

$$\langle h_i h_j' \rangle (\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \psi_i \psi_j' \rangle (\hat{K}, \Delta t, t) \exp[\hat{i}(\hat{K}, \hat{r})] \, d\hat{K}. \quad \ldots (3.10)$$

$$\langle u_i h_k h_j' \rangle (\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \alpha_i \psi_k \psi_j' \rangle (\hat{K}, \Delta t, t) \exp[\hat{i}(\hat{K}\hat{r})] \, d\hat{K} \quad \ldots (3.11)$$

interchanging the subscripts i and j and then interchanging the points p and p' gives

$$\langle u_k' h_i h_j' \rangle (\hat{r}, \Delta t, t) = \langle u_k h_i h_j' \rangle (-\hat{r}, -\Delta t, t + \Delta t)$$

$$= \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi_j' \rangle (-\hat{K}, -\Delta t, t + \Delta t) \exp[\hat{i}(\hat{K} \cdot \hat{r})] \, d\hat{K}, \quad \ldots (3.12)$$

where $\hat{K}$ is known as a wave-number vector and $d\hat{K} = dK_1, dK_2 \, dK_3$. The magnitude of $\hat{K}$ has the

dimension 1/length and can be considered to be the reciprocal of an eddy size. Substitution of eqs. (3.10) through (3.12) into eqs. (3.8) and (3.9) leads to the spectral equations

$$\frac{\partial \langle \psi_i \psi_j' \rangle}{\partial t} + 2\lambda k^2 \langle \psi_i \psi_j' \rangle$$

$$= 2ik_k [\langle \alpha_i \psi_k \psi_j' \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi_j' (-\hat{K} - \Delta t, t + \Delta t)] \quad \ldots (3.13)$$

and

$$\frac{\partial \langle \psi_i \psi_j' \rangle}{\partial \Delta t} + \lambda k^2 \langle \psi_i \psi_j' \rangle = ik_k [\langle \alpha_i, \psi_k \psi_j' \rangle$$

$$(\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi_j' (-\hat{K}, -\Delta t, t + \Delta t)]. \quad \ldots (3.14)$$

The tensors eqs. (3.13) and (3.14) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial \langle \psi_i \psi_j' \rangle}{\partial \Delta t} + 2\lambda k^2 \langle \psi_i \psi_i' \rangle = 2ik_k [\langle \alpha_i, \psi_k \psi_i' \rangle$$

$$(\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi_i' (-\hat{K}, -\Delta t, t + \Delta t)] \quad \ldots (3.15)$$

and

$$\frac{\partial \langle \psi_i \psi_i' \rangle}{\partial \Delta t} + \lambda k^2 \langle \psi_i \psi_i' \rangle = ik_k [\langle \alpha_i, \psi_k \psi_i' \rangle$$

$$(\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi_i' \rangle_i (-\hat{K}, -\Delta t, t + \Delta t)]. \quad \ldots (3.16)$$

The terms on the right side of eqs. (3.15) and (3.16) are collectively proportional to what is known as the magnetic energy transfer term. They account for the transfer of energy from one wave number to another or form one eddy size to another.

4. Three-Point, Three-Time Correlation Equations and Solution for Times Before the Final Period

In the present investigation, under the same assumption as before it is proposed to obtain an expression for the transfer term applicable at time before the final period from the three-point correlation or spectral equations. To obtain the three-point equation, we take the momentum equation of MHD turbulence at the point p and induction equations of magnetic field fluctuation at p' and p'' separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial W}{\partial x_l} + \nu \frac{\partial^2 u_l}{\partial x_k \partial x_k} \quad \ldots (4.1)$$

$$\frac{\partial h_i'}{\partial t'} + u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial u_i'}{\partial x_k'} = \lambda \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} \quad \ldots (4.2)$$

$$\frac{\partial h_j''}{\partial t''} + u_k'' \frac{\partial_j''}{\partial x_k''} - h_k'' \frac{\partial u_j''}{\partial x_k''} = \lambda \frac{\partial^2 h_j''}{\partial x_k'' \partial x_k''}. \qquad \text{... (4.3)}$$

Multiplying eqs. (4.1) through (4.3) by $h_i' h_j''$, $u_l h_j'$ and $u_l h_i'$ and averaging, one obtains

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t} + \frac{\partial}{\partial x_k} [\langle u_k u_l h_i' h_j'' \rangle - \langle h_k h_l h_i' h_j'' \rangle]$$

$$= - \frac{\partial \langle W h_i' h_j'' \rangle}{\partial x_l} + \nu \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial x_k \partial x_k} \qquad \text{... (4.4)}$$

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t'} + \frac{\partial}{\partial x_k'} [\langle u_l u_k' h_i' h_j'' \rangle - \langle u_l u_i' h_k' h_j'' \rangle] = \lambda \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial x_k \partial x_k} \qquad \text{... (4.5)}$$

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t''} + \frac{\partial}{\partial x_k''} [\langle u_l u_k'' h_i' h_j'' \rangle - \langle u_l u_j' h_i' h_k'' \rangle] = \lambda \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial x_k'' \partial x_k''} \qquad \text{... (4.6)}$$

If we use the transformations

$$\frac{\partial}{\partial x_k} = -\left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'} \right), \frac{\partial}{\partial x_k'} = \frac{\partial}{\partial r_k}, \frac{\partial}{\partial x_k''} = \frac{\partial}{\partial r_k'}$$

$$\left(\frac{\partial}{\partial t} \right)_{t', t''} = \left(\frac{\partial}{\partial t} \right)_{\Delta r, \Delta t'} - \frac{\partial}{\partial \Delta t} - \frac{\partial}{\partial \Delta t'}$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t}, \frac{\partial}{\partial t''} = \frac{\partial}{\partial \Delta t'}$$

into eqs. (4.4) to (4.6), we have

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial t} - \left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'} \right) [\langle u_k u_l h_i' h_j'' \rangle - \langle h_k h_l h_i' h_j'' \rangle]$$

$$+ \frac{\partial}{\partial r_k} [\langle u_l u_k' h_i' h_j'' \rangle - \langle u_l u_i' h_k' h_j'' \rangle] + \frac{\partial}{\partial r_k'} [\langle u_l u_k'' h_i' h_j'' \rangle - \langle u_l u_j'' h_i' h_k'' \rangle$$

$$= - \left(\frac{\partial}{\partial r_l} + \frac{\partial}{\partial r_l'} \right) \langle W h_i' h_j'' \rangle + \nu \left(\frac{\partial}{\partial r_k} + \frac{\partial}{\partial r_k'} \right)^2 \langle u_l h_i' h_j'' \rangle$$

$$+ \lambda \left[\frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k \partial r_k} + \frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k' \partial r_k'} \right], \qquad \text{... (4.7)}$$

$$\frac{\partial \langle u_l h_i' h_j'' \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k} [\langle u_l u_k' h_i' h_j'' \rangle - \langle u_l u_i' h_k' h_j'' \rangle] = \lambda \left[\frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k \partial r_k} \right. \quad \text{... (4.8)}$$

and

$$\frac{\partial (u_l h_i' h_j'' \rangle}{\partial \Delta t'} + \frac{\partial}{\partial r_k'} [\langle u_l u_k'' h_i' h_j'' \rangle - \langle u_l u_j'' h_i' h_k'' \rangle] = \lambda \left[\frac{\partial^2 \langle u_l h_i' h_j'' \rangle}{\partial r_k' \partial r_k'} \right. \quad \text{... (4.9)}$$

Using the six dimensional Fourier transforms of the type

$$\langle u_l h_i' h_j'' \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_j'' \rangle$$

$$(\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp [\hat{i} (\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] \, d\hat{K} \, d\hat{K}' \quad \text{... (4.10)}$$

$$\langle u_l u_k' h_i' h_j'' \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_l \phi_k' \beta_i' \beta_j'' \rangle$$

$$(\hat{K}, \hat{K}', \Delta t, \Delta t', t) \exp [\hat{i} (\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] \, d\hat{K} \, d\hat{K}' \quad \text{... (4.11)}$$

$$\langle W h_i' h_j'' \rangle (\hat{r}, \hat{r}', \Delta t, \Delta t', t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \gamma \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}, \Delta t, \Delta t', t)$$

$$\exp [\hat{i} (\hat{K} \cdot \hat{r} + \hat{K}' \cdot \hat{r}')] \, d\hat{K} \, d\hat{K}' \quad \text{... (4.12)}$$

into eqs. (4.7) to (4.9) and after neglecting the quadruple correlation terms (as they decay faster than the lower-border correlation terms), one obtains

$$\frac{\partial}{\partial t} \langle \phi_l \beta_i'' \beta_j'' \rangle (\hat{K}, \hat{K}, \Delta t, \Delta t', t) + \lambda [(1 + P_M)(k^2 + k'^2) + 2P_M kk']$$

$$\langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}, \Delta t \, \Delta t', t) = 0, \quad \text{... (4.13)}$$

$$\frac{\partial}{\partial \Delta t} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}, \Delta t, \Delta t', t) + \lambda k^2 \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}, \Delta t, \Delta t', t) = 0 \quad \text{... (4.14)}$$

and

$$\frac{\partial}{\partial \Delta t'} \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}, \Delta t, \Delta t', t) + \lambda k^2 \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}, \Delta t, \Delta t', t) = 0. \quad \text{... (4.15)}$$

The term $\langle \gamma \beta_i' \beta_j'' \rangle$ associated with the pressure correlation are also neglected because it is related to the quadruple correlation's (eq. (3.16) of Sarkar and Kishore[1]).

The tensor eqs. (4.13) to (4.15) can be converted to scalar equations by contraction of the indices i and j and inner multiplication by k_l

$$\frac{\partial}{\partial t} k_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda [(1 + P_M)(k^2 + k'^2) + 2P_M kk']$$

$$\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0$$

$$\frac{\partial}{\partial \Delta t} k_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}, \Delta t, \Delta t', t) + \lambda k^2 \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}, \Delta', \Delta t', t) = 0 \quad \ldots(94.14a)$$

and

$$\frac{\partial}{\partial \Delta t'} k_l \langle \phi_l \beta_i' \beta_i'' \ (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda k'^2 \langle \phi_l \beta_i', \beta_i'' \ (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0. \quad \ldots \ (4.15a)$$

Integrating eqs. (4.13*a*) to (4.15*a*) between t_0 to t, we obtain

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = f_l \ \exp \ \{-\lambda [(1 + P_M) (k^2 + k'^2) + 2P_M kk' \cos \theta] (t - t_0)\},$$

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = g_l \exp [-\lambda k^2 \Delta t]$$

and

$$k_l \langle \phi_l \beta_i' \beta_i'' = q_l \exp [-\lambda k'^2 \Delta t'].$$

For these relations to be consistent, we have

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = k_l \langle \phi_l \beta_i' \eta_i'' \rangle_0 \ \exp \ \{-\lambda [(1 + P_M) (k^2 + k'^2) (t - t_0)$$

$$+ \ k^2 \Delta t + k'^2 \Delta t' + 2 P_M kk' \cos \xi (t - t_0)\}, \quad \ldots \ (4.16)$$

where the subscript 0 refers to the value of $\langle \phi \beta_i' \beta_i'' \rangle$ at $t = t_0, \Delta t = \Delta t' = 0$ and ξ is the angle between $\hat{K}$ and $\hat{K}'$.

By letting $\hat{r}' = 0, \Delta t = 0$ in the eq. (4.10) and comparing with eqs. (3.11) and (3.12), we obtain the relations

$$\langle \alpha_i \psi_k \psi_i' (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \ (\hat{K}, \hat{K}', \Delta t, 0, t) \, d\hat{K}' \quad \ldots \ (4.17)$$

and

$$\langle \alpha_i \psi_k \psi_i' \ (-\hat{K}, -\Delta t, t + \Delta t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \ (-\hat{K}, \hat{K}', \Delta t, 0, t) \, d\hat{K}'. \quad \ldots(4.18)$$

Substituting eqs. (4.16) to (4.18) into eq. (3.15), one obtains

$$\frac{\partial}{\partial t} \langle \psi, \psi_i' \rangle (K, \Delta t, t) + 2 \lambda k^2 \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t)$$

$$= \int_{-\infty}^{\infty} 2ik_l [\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}']_0$$

$$\exp \ [-\lambda \{(1 + P_M) (k^2 + k'^2) (t - t_0) + k^2 \Delta t + 2P_M (t - t_0) kk' \cos \xi \, d (\cos \xi)\}] \ dk' \quad \ldots(4.19)$$

$$\frac{\partial}{\partial t}\langle \psi, \psi_i' \rangle (\hat{\mathbf{K}}, \Delta t, t) + 2\lambda k^2 \langle \psi_i \psi_i' \rangle (\hat{\mathbf{K}}, \Delta t, t)$$

$$= 2 \int_0^\infty 2\pi i k_l [\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{\mathbf{K}}, \hat{\mathbf{K}}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{\mathbf{K}}, -\hat{\mathbf{K}}')]_0 k'^2 \cdot$$

$$\left[\int_{-1}^{1} \exp -\lambda ((1+P_M)(k^2+k'^2)(t-t_0) + k^2 \Delta t + 2P_M (t-t_0) kk' \cos\xi)] \, d(\cos\xi) \right] dk' \quad \ldots (4.20)$$

where $d\hat{\mathbf{K}}' = dK_1' \, dK_2' \, dK_3'$ is written in terms of k' and ξ as $-2\pi k'^2 \, d(\cos\xi) \, dk'$ (cf. Deissler[5]) and the quantity $[\langle \phi_l \beta_i' \beta_i'' (\hat{\mathbf{K}}, \hat{\mathbf{K}}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{\mathbf{K}}, -\hat{\mathbf{K}}')]_0$ depends on the initial condition of the turbulence.

In order to make further calculation it is necessary to assume a relation, which gives $ik_l [\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{\mathbf{K}}, \mathbf{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{\mathbf{K}} - \hat{\mathbf{K}}')]_0$ as a function of k and k'. The relation assumed here is

$$(2\pi)^2 ik_l [\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{\mathbf{K}}, \hat{\mathbf{K}}') - \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{\mathbf{K}}, \hat{\mathbf{K}}')]_0 = -\delta_0 [k^2 k'^4 - k^4 k'^2] \quad \ldots (4.21)$$

where δ_0 is a constant determined by the initial conditions.

Substituting eq. (4.21) in eq. (4.20), and multiplying both sides by k^2 and writing $\langle \psi_i \psi_j \rangle$ in terms of the magnetic energy spectrum function $E_M = 2\pi k^2 \langle \varphi_i \varphi_i' \rangle$, we get

$$\frac{\partial E_M}{\partial t} + 2\lambda k^2 E_M = M_T, \quad \ldots (4.22)$$

where M_T is the magnetic energy transfer term and is given by

$$M_T = -2\delta_0 \int_0^\infty (k^{2k,4} - k^4 k'^4) \, k^2 k'^2$$

$$\times \left[\int_{-1}^{1} \exp(-\lambda [(1+P_M)(k^2+k'^2)(t-t_0) + k^2 \Delta t + 2P_M (t-t_0) kk' \cos\xi] \, d(\cos\xi)) \right] dk' \quad \ldots (4.23)$$

Integrating eq. (4.23) with respect to $\cos\xi$ and k', we have

$$M_T = -\frac{\delta_0 P^{5/2} M\sqrt{\pi}}{4\lambda^{3/2} (t-t_0)^{3/2} (1+P_M)^{5/2}} \exp\left[-k^2 \lambda \frac{1+2P_M}{1+P_M} \left(t - t_0 + \frac{1+P_M}{1+2P_M} \Delta t \right) \right.$$

$$\left. \left[\frac{15 k^4}{4P_M^2 (t-t_0)^2 \lambda^2} \left(\frac{P_M}{1+P_M} \right) + \left\{ 5 \left(\frac{P_M}{1+P_M} \right)^2 - \frac{3}{2} \right\} \frac{k^6}{P_M \lambda (t-t_0)} \right. \right.$$

$$+\left\{\left(\frac{P_M}{1+P_M}\right)^3-\frac{P_M}{1+P_M}\right\}k^8-\frac{\delta_0 P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_0+\Delta t)^{3/2}(1+P_M)^{5/2}}$$

$$\exp\left[-k^2\lambda\left(\frac{1+2P_M}{1+P_M}\right)\left(t-t_0+\frac{P_M}{1+P_M}\Delta t\right)\right]\left[\frac{15k^4}{4\lambda^2 P_M^2(t-t_0+\Delta t)^2}\left(\frac{P_M}{1+P_M}\right)\right.$$

$$\left.+\left\{5\left(\frac{P_M}{1+P_M}\right)^2-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-t_0+\Delta t)}+\left\{\left(\frac{P_M}{1+P_M}\right)^3-\frac{P_M}{1+P_M}\right\}k^8\right]. \quad \ldots (4.24)$$

The series of eq. (4.24) contains only even power of k and the equation represents the transfer function owing to consideration of magnetic field at three-point and three-times. If we put $\Delta t=0$, eq. (4.24) reduces to eq. (4.12) of [1], which is obtained earlier Sarker and Kishore.

If we integrate eq. (4.24) for $\Delta t = 0$ over all wave numbers, we find that

$$\int_0^\infty M_T \cdot dk = 0, \quad \ldots (4.25)$$

which indicating that the expression for M_T satisfying the conditions of continuity and homogeneity. Physically it was to be expected as M_T is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

For obtaining the magnetic energy spectrum function E_M, eq. (4.22) can be written in integral form as

$$E_M=\exp[-2\lambda k^2(t+t_0+\Delta t/2]\int M_T\exp[2\lambda k^2(t-t_0+\Delta t/2)]\,dt$$

$$+J(k)\exp[-2\lambda k^2(t-t_0+\Delta t/2)], \quad \ldots (4.26)$$

where $J(k)=\dfrac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin[3].

Substituting the value of M_T as given by eq. (4.24) into eq. (4.26), gives the equation

$$E_M=\frac{N_0 k^2}{\pi}\exp[-2\lambda k^2(t-t_0+\Delta t/2)]$$

$$+\frac{\delta_0 P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}\exp\left[-k^2\lambda\left(\frac{1+2P_M}{1+P_M}\right)\left(t-t_0+\frac{1+P_M}{1+2P_M}\Delta t\right)\right]$$

$$\left[\frac{3k^4}{2P_M(t-t_0)^{5/2}\lambda^2}\right.$$

$$\left.+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0)^{3/2}}-\frac{4(3P_M^2-2P_M+3)}{3(1+P_M)^2(t-t_0)^{1/2}}+\frac{8\lambda^{1/2}(3P_M^2-2P_M+3)}{3(1+P_M)^{5/2}}k^9F(\omega)\right.$$

$$\frac{\delta_0 \sqrt{\pi}\, P_M}{4\,\lambda^{3/2}(1+P_M)} \exp\left[-\lambda k^2 \left(\frac{1+2P_M}{1+P_M}\right)\left(t-t_0+\frac{P_M}{1+P_M}\Delta t\right)\right]$$

$$\left[\frac{3k^4}{2\,\lambda^2 P_M (t-t_0+\Delta t)^{5/2}} + \frac{(7P_M-6)\,k^6}{3\,\lambda(1+P_M)(t-t_0+\Delta t)^{3/2}} + \frac{4(3P_M^2-2P_M+3)}{3(1+P_M)^2 (t-t_0+\Delta t)^{\frac{1}{2}}}\right.$$

$$\left. + \frac{8\,\lambda^{1/2}(3P_M^2-2P_M+3)}{(1+P_M)^{5/2}} k^9 F(\omega)\right] \qquad \ldots (4.27)$$

where, $F(\omega) = e^{-\omega^2} \int_0^{\omega} e^{x^2}\, dx$

$$\omega = k\sqrt{\frac{\lambda(t-t_0)}{1+P_M}} \quad \text{or} \quad k\sqrt{\frac{\lambda(t-t_0+\Delta t)}{1+P_M}}$$

the expression for the magnetic energy decay is obtained from eq. (3.10) by setting $r = 0$, $j = i$

$$d\hat{K} = -2\pi k^2 d(\cos\xi)\, d\hat{k} \text{ and } E = 2\pi k^2 \langle \psi_i \psi_j' \rangle.$$

Thus $$\frac{\langle h_i h_j' \rangle}{2} = \int_0^{\infty} E_M\, dk. \qquad \ldots (4.28)$$

Substituting eq. (4.27) into eq. (4.28) and after integration, one can obtains

$$\frac{\langle h_i h_j' \rangle}{2} = \frac{N_0}{8\,\lambda^{3/2}\sqrt{2\pi}\,(T+\Delta T/2)^{3/2}} + \frac{\pi\,\delta_0}{4\,\lambda^6 (1+P_M)(1+2P_M)^{5/2}}$$

$$\left[\frac{9}{16\,T^{5/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{5/2}} + \frac{9}{16(T+\Delta T)^{5/2}\left(T+\frac{P_M}{1+P_M}\Delta T\right)^{5/2}}\right.$$

$$\left. \frac{5P_M(7P_M-6)}{16(1+2P_M)T^{3/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}} + \frac{5P_M(7P_M-6)}{16(1+2P_M)(T+\Delta T)\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{7/2}}\right.$$

$$\frac{35\,P_M(3P_M^2-2P_M+3)}{8\,(1+2P_M)\,T^{1/2}\left(T+\dfrac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}}+\frac{35\,P_M(3P_M^2-2P_M+3)}{8\,(1+2P_M)\,(T+\Delta T)^{1/2}\left(T+\dfrac{P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$+\frac{8\,P_M(3P_M^2-2P_M+3)\,(1+2P_M)^{5/2}}{3.2^{23/2}\,(1+P_M)^{11/2}}\sum_{n=0}^{\infty}\frac{1.3.5.----(2n+9)}{n\,!\,(2n+1)\,2^{2n}\,(1+P_M)^n}$$

$$\times\left\{\frac{T^{(2n+1)/2}}{(T+\Delta T)^{(2n+1)/2}}+\frac{(T+\Delta T)^{(2n+1)/2}}{(T+\Delta T/2)^{(2n+1)/2}}\right\}\Bigg], \qquad \text{... (4.29)}$$

where $T=t-t_0$

and

$$\frac{\langle h_i h_j'\rangle}{2}=\frac{N_0}{8\,\lambda^{3/2}\sqrt{2\,\pi}\,T_m^{3/2}}+\frac{\pi\,\delta_0}{4\,\lambda^6\,(1+P_M)\,(1+2P_M)^{5/2}}$$

$$\times\Bigg[\frac{9}{16\,(T_m-\Delta T/2)^{5/2}\left(T_m+\dfrac{\Delta T}{1+2P_M}\right)^{5/2}}+\frac{9}{16\,(T_m+\Delta T/2)^{5/2}\left(T_m-\dfrac{\Delta T}{2\,(1+P_M)}\right)^{5/2}}$$

$$+\frac{5\,P_M(7P_M-6)}{16\,(1+2P_M)\,T^{3/2}\left(T+\dfrac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}}$$

$$+\frac{5\,P_M(7P_M-6)}{16\,(1+2P_M)\,(T+\Delta T)\,T^{3/2}\left(T+\dfrac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}}+\text{-----} \qquad \text{... (4.30)}$$

where $T_m=T+\Delta T/2.$

This is the decay law of magnetic energy fluctuation before the final period for the case of multi-point and multi-time.

If we put $\Delta T=0$, we can easily find-out that

$$\frac{\langle h^2\rangle}{2}=\frac{N_0}{8\,\lambda^{3/2}\sqrt{2\,\pi}}+\frac{\pi\,\delta_0}{4\,\lambda^6\,(1+P_M)\,(1+2P_M)^{5/2}}T^{-5}\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+---\right\}$$

$$=\frac{N_0}{8\sqrt{2\pi}\,\lambda^{3/2}}T^{-3/2}+\frac{\delta_0 S}{2\,\lambda^6}T^{-5} \qquad \text{... (4.31)}$$

where $$S=\frac{\pi}{(1+P_M)(1+2P_M)^{5/2}}\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+----\right\},$$

which is same as obtained earlier by Kishore and Sarker[1] and is analogous to the result of Loeffer and Deissler[8].

5. CONCLUDING REMARKS

This study show that the terms associated with the higher-order correlation die out faster than those associated with the lower order ones. By neglecting the quadruple correlation in the three-point, three-time correlation equation the result (4.30) applicable to the MHD tuburlence before the final period of decay were obtained. If higher order correlation equations were considered in the analysis, it appears that more terms in higher power of T_m could be added to eq. (4.30)

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