Indian J. pure appl. Math., **32**(8) : 1127-1144, August 2001

EIGEN VALUE APPROACH TO MICROPOLAR ELASTIC MEDIUM DUE TO IMPULSIVE FORCE AT THE ORIGIN

RAJNEESH KUMAR

Department of Mathematics, K. U. Kurukshetra, India

RANJIT SINGH

Department of Mathematics, Govt. College, Patti, Amritsar, India

AND

T. K. CHADHA

Department of Mathematics, G.N.D.U., Amritsar, India

(*Received* 1 *August* 1999; *after revision* 27 *February* 2001; *accepted* 14 *March* 2001)

The dynamic problem in micropolar elastic medium has been investigated by employing eigen-value approach after applying the Laplace and the Hankel transformations. An example of infinite space with concentrated force at the origin has been presented to illustrate the application of the approach. The integral transforms have been inversed by using a numerical technique to obtain the radial displacement, normal displacement, normal force stress, shear stress, couple stress and microrotation in the physical domain. The results for these quantities are given and illustrated graphically.

Key Words : Eigen Value; Micropolar Elastic Medum; Impulsive Force; Displacement - Normal and Radial; Normal Force Stress; Shear Stress; Couple Stress

1. INTRODUCTION

Eringen and Suhubi[3 &4] introduced the theory of micro-elastic solids in which the microdeformation and microrotation of the material particles contained in a microvolume element with respect to its centroid are taken into account in an average sense. Materials affected by micromotions and microdeformation are known as micromorphic materials. Later, Eringen[5] developed a theory for a subclass of micromorphic materials which are called micropolar solids and these materials show microrotation effect and microrotational inertia. Here, the material's particle in a volume element can undergo only rigid rotational motions about its centre of mass. Micropolar solids may represent the material that are made up of dipole atoms or dumbbell type molecules and are subjected to surface and body couples. Solid propellent grains, rocks, polymeric materials, wood and fiber glass are few example of such materials. The deformation in these materials is characterized not only by classical translocational degree of freedom represented by the displacement vector field $\boldsymbol{u}(x, t)$ but also by rotation vector $\boldsymbol{\phi}(x, t)$.

Das *et al.*[1] discussed a one-dimensional problem in coupled thermoelasticity using an eigen-value approach. Mahalanabis and Manna[8] discussed eigen value approach to linear micropolar elasticity by arranging basic equations of linear micropolar elasticity in the form of matrix differential equation in the Hankel transform domain. Saxena and Dhaliwal[11] discussed two dimensional problems in axisymmetric and plane strain cases in the context of coupled thermoelasticity employing the eigen value approach. The two-dimensional axisymmetric and plane strain problems in homogeneous and isotropic media are investigated by Sharma and Chand[12] using eigen-value approach. Sharma and Kumar[13] discussed the axisymmetric problem of generalized anisotropic thermoelasticity by using an eigen-value approach after employing integral transform technique. By using eigen-value approach Das, Lahiri and Giri[2] investigated a one dimensional problem with heat sources distributed over a plane area in an infinite isotropic elastic solids and a two-dimensional problem with instantaneous heat sources in an infinite transversely isotropic elastic medium. Recently, Mahalanabis and Manna[9] discussed the problem of linear micropolar thermoelasticity by using eigen-value approach.

In this paper, we consider a two-dimensional axisymmetric problem in a homogeneous isotropic micropolar elastic medium. The solutions are obtained using an eigen-value approach after employing integral transformation technique. The integral transforms are inverted using a numerical approach.

2. Basic Equations

Following Eringen[5] the constitutive relations and the field equations in micropolar elastic solid without body forces and body couples can be written as

$$t_{kl} = \lambda\, u_{r,r}\, \delta_{kl} + \mu\,(u_{k,l} + u_{l,k}) + K\,(u_{l,k} - \varepsilon_{klr}\,\phi_r), \quad \dots (1)$$

$$m_{kl} = \alpha\,\phi_{r,r}\,\delta_{kl} + \beta\,\phi_{k,l} + \gamma\,\phi_{l,k}, \quad \dots (2)$$

$$(\lambda + 2\mu + K)\,\nabla\nabla\cdot\boldsymbol{u} - (\mu + K)\,\nabla\times\nabla\times\boldsymbol{u} + K\,\nabla\times\boldsymbol{\phi} = \rho\frac{\partial^2 \boldsymbol{u}}{\partial t^2} \quad \dots (3)$$

and

$$(\alpha + \beta + \gamma)\,\nabla\nabla\cdot\boldsymbol{\phi} - \gamma\,\nabla\times\nabla\times\boldsymbol{\phi} + K\,\nabla\times\boldsymbol{u} - 2K\,\boldsymbol{\phi} = \rho j\frac{\partial^2 \boldsymbol{\phi}}{\partial t^2}, \quad \dots (4)$$

where $\lambda, \mu, \alpha, \beta, \gamma, K$ are material constants, ρ the density, j the micro-inertia, $\boldsymbol{u}$ the displacement vector, $\boldsymbol{\phi}$ the rotation vector, t_{kl} the force stress tensor, m_{kl} the couple stress tensor.

3. Formulation and Solution

We consider a homogeneous, isotropic micropolar elastic solid. We take a cylindrical polar co-ordinates system (r, θ, z) and z-axis is pointing into the medium. Due to the symmetry about the z-axis, all quantities are independent of θ. Since we are discussing a two-dimensional problem, we have

$$\boldsymbol{u} = (u_r, 0, u_z) \text{ and } \boldsymbol{\phi} = (0, \phi_\theta, 0). \quad \dots (5)$$

Using (5) the set of eqs. (3) and (4) reduce to

$$(\lambda+\mu)\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}+\frac{\partial^2 u_z}{\partial r\,\partial z}\right]$$

$$+\ (\mu+K)\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}+\frac{\partial^2 u_r}{\partial z^2}\right]-K\frac{\partial \phi_\theta}{\partial z}=\rho\frac{\partial^2 u_r}{\partial t^2}, \quad \text{... (6)}$$

$$(\lambda+\mu)\left[\frac{\partial^2 u_r}{\partial z\,\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial z}+\frac{\partial^2 u_z}{\partial z^2}\right]+(\mu+K)\left[\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{\partial^2 u_z}{\partial z^2}\right]$$

$$+\ \frac{K}{r}\frac{\partial}{\partial r}(r\,\phi_\theta)=\rho\frac{\partial^2 u_z}{\partial t^2} \quad \text{... (7)}$$

and

$$\gamma\left[\frac{\partial^2 \phi_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial \phi_\theta}{\partial r}+\frac{\partial^2 \phi_\theta}{\partial z^2}-\frac{\phi_\theta}{r^2}\right]-2K\phi_\theta+K\left[\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}\right]=\rho j\frac{\partial^2 \phi_\theta}{\partial t^2}. \quad \text{... (8)}$$

Introducing dimensionless quantities :

$$r'=\frac{r}{h},\ z'=\frac{z}{h},\ u_r'=\frac{\rho h \omega^{*2} u_r}{\mu},\ u_z'=\frac{\rho h \omega^{*2} u_z}{\mu},$$

$$\phi_\theta'=\frac{\rho h^2 \omega^{*2}}{\mu}\phi_\theta,\ t'=\frac{\mu}{\rho h^2 \omega^*}t,\ t_{zz}'=\frac{1}{K}t_{zz},\ t_{zr}'=\frac{1}{K}t_{zr}$$

$$m_{z\theta}'=\frac{1}{Kh}m_{z\theta},\ \omega^{*2}=\frac{K}{\rho j}$$

where h is the standard length.

Eqs. (6) to (8) reduce to (on suppressing the dashes)

$$\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right]u_r$$

$$+\ (1-a^2)\frac{\partial^2 u_z}{\partial r\,\partial z}+a^2\frac{\partial^2 u_r}{\partial z^2}-s_4^*\frac{\partial \phi_\theta}{\partial z}=\frac{1}{(s_1+s_2)}\frac{\partial^2 u_r}{\partial t^2}, \quad \text{... (10)}$$

$$\frac{\partial^2 u_z}{\partial z^2}+(1-a^2)\left[\frac{\partial^2 u_r}{\partial r\,\partial z}+\frac{1}{r}\frac{\partial u_r}{\partial z}\right]$$

$$+\ a^2\left[\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right]+s_4^*\frac{1}{r}\frac{\partial}{\partial r}(r\,\phi_\theta)=\frac{1}{(s_1+s_2)}\frac{\partial^2 u_z}{\partial t^2} \quad \text{... (11)}$$

and $$s_4\left[\frac{\partial^2\phi_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_\theta}{\partial r}+\frac{\partial^2\phi_\theta}{\partial z^2}-\frac{\phi_\theta}{r^2}\right]-\frac{2h^2 s_3\phi_\theta}{j}+\frac{h^2 s_3}{j}\left(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r}\right)=\frac{\partial^2\phi_\theta}{\partial t^2}, \quad \ldots (12)$$

where $$s_1=\frac{\rho h^2(\lambda+\mu)\omega^{*^2}}{\mu^2},\ s_2=\frac{\rho h^2(\mu+K)\omega^{*^2}}{\mu^2},\ s_3=\frac{\rho h^2 K\omega^{*^2}}{\mu^2}$$

$$s_4=\frac{\gamma\rho h^2\omega^{*^2}}{j\mu^2},\ a^2=\frac{s_2}{s_1+s_2},\ s_4^*=\frac{s_3}{s_1+s_2}. \quad \ldots (13)$$

Applying Laplace transform w.r.t. time, defined by

$$(\bar{u}_r(r,z,p),\bar{u}_z(r,z,p),\bar{\phi}_\theta(r,z,p)\}$$

$$=\int_0^\infty \{(u_r(r,z,t),u_z(r,z,t),\phi_\theta(r,z,t)\}\ e^{-pt}\,dt \quad \ldots (14)$$

and then Hankel transform w.r.t. 'r' defined by

$$\{\tilde{u}_z(\xi,z,p),\tilde{\phi}_\theta(\xi,z,p)\}=\int_0^\infty \{\bar{u}_z(r,z,p),\bar{\phi}_\theta(r,z,p)\}\ rJ_0(\xi r)\,dr \quad \ldots (15)$$

$$\tilde{u}_r(\xi,z,p)=\int_0^\infty \bar{u}_r(r,z,p)\,r\,J_1(\xi r)\,dr \quad \ldots (16)$$

to eqs. (10) to (12) we obtain

$$\tilde{u}_r''=\frac{1}{a^2}\left[\xi^2+\frac{p^2}{s_1+s_2}\right]\tilde{u}_r+\frac{(1-a^2)\xi}{a^2}\tilde{u}_z'+\frac{s_4^*}{a^2}\tilde{\phi}_\theta', \quad \ldots (17)$$

$$\tilde{u}_z''=\left[a^2\xi^2+\frac{p^2}{s_1+s_2}\right]\tilde{u}_z-(1-a^2)\,\xi\,\tilde{u}_r'-\xi s_4^*\tilde{\phi}_\theta \quad \ldots (18)$$

and $$\tilde{\phi}_\theta''=-\frac{Kh^2}{\gamma}\tilde{u}_r'-\frac{Kh^2\xi}{\gamma}\tilde{u}_z+\left[\xi^2+\frac{2Kh^2}{\gamma}+\frac{p^2}{s_4}\right]\tilde{\phi}_\theta.. \quad \ldots (19)$$

The system of eqs. (17) to (19) can be written as

$$\frac{d}{dz}W(\xi,z,p)=A(\xi,p)\,W(\xi,z,p), \quad \ldots (20)$$

where $$W=\begin{bmatrix}U\\U'\end{bmatrix},\ A=\begin{bmatrix}0 & I\\A_2 & A_1\end{bmatrix},\ U=\begin{bmatrix}\tilde{u}_r\\\tilde{u}_z\\\tilde{\phi}_\theta\end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & \dfrac{(1-a^2)\,\xi}{a^2} & \dfrac{s_4^*}{a^2} \\ -(1-a^2)\,\xi & 0 & 0 \\ -\dfrac{Kh^2}{\gamma} & 0 & 0 \end{bmatrix} \quad \dots (21)$$

$$A_2 = \begin{bmatrix} \dfrac{1}{a^2}\left(\xi^2 + \dfrac{p^2}{s_1+s_2}\right) & 0 & 0 \\ 0 & a^2\,\xi^2 + \dfrac{p^2}{s_1+s_2} & -s_4^*\,\xi \\ 0 & -\dfrac{K\,\xi\,h^2}{\gamma} & \xi^2 + \dfrac{2Kh^2}{\gamma} + \dfrac{p^2}{s_4} \end{bmatrix}$$

To solve eq. (20), we take

$$W(\xi, z, p) = X(\xi, p)\, e^{lz} \quad \dots (22)$$

so that

$$A(\xi, p)\, W(\xi, z, p) = lw(\xi, z, p) \quad \dots (23)$$

which leads to eigen value problem. The characteristic equation corresponding to the matrix A is given by

$$\det\,(A - lI) = 0 \quad \dots (24)$$

which on expansion provides us

$$l^6 - \lambda_1\, l^4 + \lambda_2\, l^2 - \lambda_3 = 0, \quad \dots (25)$$

where

$$\lambda_1 = \left(1 + \frac{1}{a^2}\right)\frac{p^2}{s_1+s_2} + \left[3\,\xi^2 + \frac{2Kh^2}{\gamma} + \frac{p^2}{s_4} - \frac{Kh^2\, s_4^*}{\gamma a^2}\right], \quad \dots (26)$$

$$\lambda_2 = \left[\xi^2 + \frac{2Kh^2}{\gamma} + \frac{p^2}{s_4}\right]\left[\frac{p^2}{s_1+s_2}\left(1+\frac{1}{a^2}\right) + 2\,\xi^2\right]$$
$$- s_4^*\,\frac{Kh^2}{\gamma a^2}\left[2\,\xi^2 + \frac{p^2}{s_1+s_2}\right] + \frac{1}{a^2}\left(\xi^2 + \frac{p^2}{s_1+s_2}\right)\left[a^2\,\xi^2 + \frac{p^2}{s_1+s_2}\right] \quad \dots (27)$$

and

$$\lambda_3 = \frac{1}{a^2}\left[\xi^2 + \frac{p^2}{s_1+s_2}\right]\left[a^2\,\xi^2 + \frac{p^2}{s_1+s_2}\right]\left[\xi^2 + \frac{2Kh^2}{\gamma} + \frac{p^2}{s_4}\right]$$
$$- \frac{1}{\gamma a^2}\left[\xi^2 + \frac{p^2}{s_1+s_2}\right] s_4^*\, Kh^2\, \xi^2. \quad \dots (28)$$

The eigen values of the matrix A are characteristic roots of the eq. (25) we assume that real parts of l_i are positive. The vector $\bar{X}(\xi, p)$ corresponding to the eigen values l_i can be determined by solving the homogeneous equation.

$$[A - lI]\, X(\xi, p) = 0. \qquad \text{... (29)}$$

The set of eigen vectors $X_i(\xi, p)$, $(i = 1, 2, 3, 4, 5, 6)$ may be obtained as

$$X_i(\xi, p) = \begin{bmatrix} X_{i1} & (\xi, p) \\ X_{i2} & (\xi, p) \end{bmatrix}, \qquad \text{... (30)}$$

where $$X_{i1}(\xi, p) = \begin{bmatrix} a_i l_i \\ b_i \\ -\xi \end{bmatrix}, \quad X_{i2}(\xi, p) = \begin{bmatrix} a_i l_i^2 \\ -b_i l_i \\ \xi l_i \end{bmatrix} \qquad \text{... (31)}$$

$l = l_i;\ i = 1, 2, 3$

$$X_{j1}(\xi, p) = \begin{bmatrix} -a_i l_i \\ b_i \\ -\xi \end{bmatrix}, \quad X_{j2}(\xi, p) = \begin{bmatrix} a_i l_i^2 \\ -b_i l_i \\ \xi l_i \end{bmatrix} \qquad \text{... (32)}$$

$j = i + 3,\ l = -l_i;\ i = 1, 2, 3$

$$a_i = \frac{\left[\xi(a^2 - 1)\left\{\left(\xi^2 + \frac{2Kh^2}{\gamma} + \frac{p^2}{s_4}\right) - l_i^2\right\} - \frac{K\xi h^2 s_4^*}{\gamma} \right]}{\Delta} \qquad \text{... (33)}$$

$$b_i = \frac{\left[\left(\xi^2 + \frac{p^2}{s_1 + s_2}\right)\left(\xi^2 + \frac{2Kh^2}{\gamma} + \frac{p^2}{s_4}\right) + a^2 l_i^2 \left\{ l_i^2 - \left(\xi^2 + \frac{2Kh^2}{\gamma} + \frac{p^2}{s_4}\right)\right\} - \left(\xi^2 + \frac{p^2}{s_1 + s_2}\right) l_i^2 + + \frac{K s_4^* h^2 l_i^2}{\gamma} \right]}{\Delta} \qquad \text{... (34)}$$

$$\Delta = \frac{Kh^2}{\gamma}\left[l_i^2 - \left(\xi^2 + \frac{p^2}{s_1 + s_2}\right)\right]. \qquad \text{... (35)}$$

The solution of eq. (20) is given by c.f. Ref. [12]

$$W(\xi, z, p) = \sum_{i=1}^{3} \left[B_i X_i(\xi, p) \exp(l_i z) + B_{i+3} X_{i+3}(\xi, p) \exp(-l_i z)\right] \qquad \text{... (36)}$$

where $B_i = (i = 1, 2, 3, 4, 5, 6)$ are arbitrary constants.

Eq. (36) represents the solution of the general problem in the axisymmetric case of homogeneous isotropic, micropolar elasticity by employing the eigen value approach and therefore can be applied to a broad class of problem in the domains of Laplace and Hankel transforms.

4. APPLICATION

We consider an infinite micropolar space in which a concentrated force of magnitude $F = \frac{-F_0\,\delta(r)\,\delta(t)}{2\pi r}$ acting in the direction of the z-axis at the origin of the cylindrical co-ordinate system. The problem is axisymmetric w.r.t. the z-axis. The boundary conditions at the plane $z = 0$ are given by

$$u_r(r, 0^+, t) - u_r(r, 0^-, t) = 0,\ u_z(r, 0^+, t) - u_z(r, 0^-, t) = 0, \quad \text{... (37)}$$

$$\phi_\theta(r, 0^+, t) - \phi_\theta(r, 0^-, t) = 0, \quad \text{... (38)}$$

$$m_{z\theta}(r, 0^+, t) - m_{z\theta}(r, 0^-, t) = 0,\ t_{zr}(r, 0^+, t) - t_{zr}(r, 0^-, t) = 0, \quad \text{... (39)}$$

$$t_{zz}(r, 0^+, t) - t_{zz}(r, 0^-, t) = -\frac{F_0\,\delta(r)\,\delta(t)}{2\pi r}. \quad \text{... (40)}$$

Applying the Laplace and Hankel transform to eqs. (37) to (40), we get :

$$\tilde{u}_r(\xi, 0^+, p) - \tilde{u}_r(\xi, 0^-, p) = 0,\ \tilde{u}_z(\xi, 0^+, p) - \tilde{u}_z(\xi, 0^-, p) = 0, \quad \text{... (41)}$$

$$\tilde{\phi}_\theta(\xi, 0^+, p) - \tilde{\phi}_\theta(\xi, 0^-, p) = 0, \quad \text{... (42)}$$

$$\tilde{m}_{z\theta}(\xi, 0^+, p) - \tilde{m}_{z\theta}(\xi, 0^-, p) = 0,\ \tilde{t}_{zr}(\xi, 0^+, p) - \tilde{t}_{zr}(\xi, 0^-, p) = 0, \quad \text{... (43)}$$

$$\tilde{t}_{zz}(\xi, 0^+, p) - \tilde{t}_{zz}(\xi, 0^-, p) = -\frac{\xi F_0}{2\pi}. \quad \text{... (44)}$$

The transformed displacements, microrotation, couple stress and stresses are given for $z \geq 0$ by

$$\tilde{u}_r(\xi, z, p) = -\{a_1 l_1 B_4 \exp(-l_1 z) + a_2 l_2 B_5 \exp(-l_2 z) + a_3 l_3 B_6 \exp(-l_3 z)\}, \quad \text{... (45)}$$

$$\tilde{u}_z(\xi, z, p) = b_1 B_4 \exp(-l_1 z) + b_2 B_5 \exp(-l_2 z) + b_3 B_6 \exp(-l_3 z), \quad \text{... (46)}$$

$$\tilde{\phi}_\theta(\xi, z, p) = -\xi\{B_4 \exp(-l_1 z) + B_5 \exp(-l_2 z) + B_6 \exp(-l_3 z)\}, \quad \text{... (47)}$$

$$\tilde{\phi}'_\theta(\xi, z, p) = \xi\{l_1 B_4 \exp(-l_1 z) + l_2 B_5 \exp(-l_2 z) + l_3 B_6 \exp(-l_3 z)\}, \quad \text{... (48)}$$

$$\tilde{m}_{z\theta}(\xi, z, p) = s_7\,\xi\,\{l_1 B_4 \exp(-l_1 z) + l_2 B_5 \exp(-l_2 z) + l_3 B_6 \exp(-l_3 z), \quad \text{... (49)}$$

$$\tilde{t}_{zr}(\xi, z, p) = \{a_1 l_1^2 s_8 - \xi b_1 s_9 + \xi s_{10}\} B_4 \exp(-l_1 z)$$
$$+ \{a_2 l_2^2 s_8 - \xi b_2 s_9 + \xi s_{10}\} B_5 \exp(-l_2 z)$$
$$+ \{a_3 l_3^2 s_8 - \xi b_3 s_9 + \xi s_{10}\} B_6 \exp(-l_3 z), \quad \ldots (50)$$

$$\tilde{t}_{zz}(\xi, z, p) = -[l_1 (b_1 s_6 + \xi a_1 s_5) B_4 \exp(-l_1 z)$$
$$+ l_2 (b_2 s_6 + \xi a_2 s_5) B_5 \exp(-l_2 z)$$
$$+ l_3 (b_3 s_6 + \xi a_3 s_5) B_6 \exp(-l_3 z)], \quad \ldots (51)$$

and for $z \leq 0$ by

$$\tilde{u}_r(\xi, z, p) = a_1 l_1 B_1 \exp(l_1 z) + a_2 l_2 B_2 \exp(l_2 z) + a_3 l_3 B_3 \exp(l_3 z), \quad \ldots (52)$$

$$\tilde{u}_z(\xi, z, p) = b_1 B_1 \exp(l_1 z) + b_2 B_2 \exp(l_2 z) + b_3 B_3 \exp(l_3 z), \quad \ldots (53)$$

$$\tilde{\phi}_\theta(\xi, z, p) = -\xi \{B_1 \exp(l_1 z) + B_2 \exp(l_2 z) + B_3 \exp(l_3 z)\}, \quad \ldots (54)$$

$$\tilde{\phi}'_\theta(\xi, z, p) = -\xi \{l_1 B_1 \exp(l_1 z) + l_2 B_2 \exp(l_2 z) + l_3 B_3 \exp(l_3 z)\}, \quad \ldots (55)$$

$$\tilde{m}_{z\theta}(\xi, z, p) = -s_7 \xi \{l_1 B_1 \exp(l_1 z) + l_2 B_2 \exp(l_2 z) + l_3 B_3 \exp(l_3 z)\}, \quad \ldots (56)$$

$$\tilde{t}_{zr}(\xi, z, p) = \{a_1 l_1^2 s_8 - \xi b_1 s_9 + \xi s_{10}\} B_1 \exp(l_1 z)$$
$$+ \{a_2 l_2^2 s_8 - \xi b_2 s_9 + \xi s_{10}\} B_2 \exp(l_2 z)$$
$$+ \{a_3 l_3^2 s_8 - \xi b_3 s_9 + \xi s_{10}\} B_3 \exp(l_3 z), \quad \ldots (57)$$

$$\tilde{t}_{zz}(\xi, z, p) = [l_1 (b_1 s_6 + \xi a_1 s_5) B_1 \exp(l_1 z)$$
$$+ l_2 (b_2 s_6 + \xi a_2 s_5) B_2 \exp(l_2 z)$$
$$+ l_3 (b_3 s_6 + \xi a_3 s_5) B_3 \exp(l_3 z)], \quad \ldots (58)$$

where

$$s_5 = \frac{\lambda \mu}{\rho K h^2 \omega^{*2}}, \; s_6 = \frac{(\lambda + 2\mu + K)\mu}{\rho K h^2 \omega^{*2}}, \; s_7 = \frac{\gamma \mu}{\rho K h^2 \omega^{*2}}, \quad \ldots (59)$$

$$s_8 = \frac{(\mu + K)\mu}{\rho K h^2 \omega^{*2}}, \; s_9 = \frac{\mu^2}{\rho K h^2 \omega^{*2}}, \; s_{10} = \frac{\mu}{\rho h^2 \omega^{*2}}.$$

Using conditions (41) to (44) in eqs. (45) to (58) we obtain

$$:a_1 l_1 (B_1 + B_4) + a_2 l_2 (B_2 + B_5) + a_3 l_3 (B_3 + B_6) = 0, \quad \dots (60)$$

$$(B_1 - B_4) + (B_2 - B_5) + (B_3 - B_6) = 0, \quad \dots (61)$$

$$b_1 (B_1 - B_4) + b_2 (B_2 - B_5) + b_3 (B_3 - B_6) = 0, \quad \dots (62)$$

$$l_1 (B_1 + B_4) + l_2 (B_2 + B_5) + l_3 (B_3 + B_6) = 0, \quad \dots (63)$$

$$[a_1 l_1^2 s_8 - \xi b_1 s_9 + \xi s_{10}] (B_1 - B_4) + [a_2 l_2^2 s_8 - \xi b_2 s_9 + \xi s_{10}] (B_2 - B_5)$$

$$+ [a_3 l_3^2 s_8 - \xi b_3 s_9 + \xi s_{10}] (B_3 - B_6) = 0, \quad \dots (64)$$

$$l_1 (b_1 s_6 + \xi a_1 s_5) (B_1 + B_4) + l_2 (b_2 s_6 + \xi (b_2 s_6 + \xi a_2 s_5) (B_2 + B_5)$$

$$+ l_3 (b_3 s_6 + \xi a_3 s_5) (B_3 + B_6) = \frac{F_0}{2\pi}. \quad \dots (65)$$

Solving system of eqs. (60) to (65), we obtain

$$B_1 = B_4 = \frac{F_0 (a_3 - a_2)}{4 \pi l_1 \Delta_1}, \quad \dots (66)$$

$$B_2 = B_5 = \frac{F_0 (a_1 - a_3)}{4 \pi l_2 \Delta_1}, \quad \dots (67)$$

$$B_3 = B_6 = \frac{F_0 (a_2 - a_1)}{4 \pi l_3 \Delta_1}, \quad \dots (68)$$

where

$$\Delta_1 = s_6 [(a_2 b_3 - a_3 b_2) + (a_3 b_1 - a_1 b_3) + (a_1 b_2 - a_2 b_1)]. \quad \dots (69)$$

Thus the function $\tilde{u}_r, \tilde{u}_z, \tilde{\phi}_\theta, \tilde{m}_{z\theta}, \tilde{t}_{zr}$ and $\tilde{t}_{zz}$ have been determined in the transform domain and these enable us to find the displacements, microrotation, couple stress and stress.

5. Inversion of the Transforms

The transformed displacements and stresses are functions of z, the parameters of Laplace and Hankel transforms p and ξ respectively, and hence are of the form $\tilde{f}(\xi, z, p)$. To get the function $f(r, z, t)$ in the physical domain, first we invert the Hankel transform using

$$\bar{f}(r, z, p) = \int_0^\infty \xi \tilde{f}(\xi, z, p) J_n (\xi r) \, d\xi. \quad \dots (70)$$

Thus, expression (70) gives us the Laplace $\bar{f}(r, z, p)$ of the function $f(r, z, t)$.

Now, for the fixed values of ξ r and z, the function $\bar{f}(r, z, p)$ in the expression (70) can be considered as the Laplace transform $\bar{g}(p)$ of some function $g(t)$. Following Honig and Hirdes[7], the Laplace transformed function $\bar{g}(p)$ can be inverted as given below.

The function $g(t)$ can be obtained by using

$$g(t) = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} e^{pt} + \bar{g}(p)\, dp, \qquad \dots (71)$$

where C is an arbitrary real number greater than all the real parts of the singularities of $\bar{g}(p)$. Taking $p = C + iy$, we get

$$g(t) = \frac{e^{Ct}}{2\pi} \int_{-\infty}^{\infty} e^{ity}\, \bar{g}(C + iy)\, dy, \qquad \dots (72)$$

Now, taking $e^{-Ct} g(t)$ as $h(t)$ and expanding it as Fourier series in $[0, 2L]$, we obtain approximately the formula

$$g(t) = g_{\infty}(t) + E_D, \qquad \dots (73)$$

where
$$g_{\infty}(t) = \frac{C_0}{2} + \sum_{k=1}^{\infty} C_k,\ 0 \le t \le 2L, \qquad \dots (74)$$

$$C_k = \frac{e^{Ct}}{L} \mathrm{Re}\left[e^{\frac{ik\pi t}{L}}\, \bar{g}\left(C + \frac{ik\pi}{L} \right) \right].$$

E_D is the discretization error and can be made arbitrarily small by choosing C large enough.

Since the infinite series in (74) can be summed up only to a finite number of N terms, so the approximate value of $g(t)$ becomes

$$g_N(t) = \frac{C_0}{2} + \sum_{k=1}^{N} C_k,\ 0 \le t \le 2L, \qquad \dots (75)$$

Now, we introduce a truncation error E_T that must be added to the discretization error to produce the total approximation error in evaluating $g(t)$ using the above formula. The discretization error is reduced by using the 'Korrecktur method' and then 'ε-algorithm' is used to reduce the truncation error and hence to accelerate the convergence.

The Korrecktur method formula, to evaluate the function $g(t)$ is

$$g(t) = g_{\infty}(t) - e^{-2CL} g_{\infty}(2L + t) + E_D',$$

where
$$|E_D'| \ll |E_D|.$$

Thus, the approximate value of $g(t)$ becomes

$$g_{N_k}(t) = g_N(t) - e^{-2CL} g_{N'}(2L+t), \quad \dots (76)$$

where N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm which is used to accelerate the convergence of the series in (75). Let N be a natural number and $s_m = \sum_{k=1}^{m} C_k$ be the sequence of partial sums of eq. (75). We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \ \varepsilon_{1,m} = s_m,$$

$$\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n-m}}; \ n, \ m = 1, 2, 3 \ ....,$$

The sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, ..., \varepsilon_{N,1}$ converges to $g(t) + E_D - C_0/2$ faster than the sequence of partial S_m, m = 1, 2, 3, ... The actual procedure to invert the Laplace transform consists of eq. (76) together with the ε-algorithm. The values of C and L are chosen according to the criteria outlined by Honig and Hirdes[7].

The last step is to evaluate the integral in equation (70). The method for evaluating this integral by Press *et al.*[10], which involves the use of Romberg's integration with adaptive step size. This, also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Numerical Results and Discussion

Following Gauthier[6], we take the following values of relevant parameters for the case of Aluminium epoxy composite as

$$\rho = 2.19 \text{ gm/cm}^3, \ \lambda = 7.59 \times 10^{10} \text{ dyne/cm}^2,$$

$$\mu = 1.89 \times 10^{10} \text{ dyne/cm}^2, \ K = .0149 \times 10^{10} \text{ dyne/cm}^2,$$

$$\gamma = 0.0268 \times 10^{10} \text{ dyne}, \ j = .00196 \text{ cm}^2$$

Gauthier[6] has considered $\varepsilon = \frac{K}{\mu}$ as coupling coefficient. The computations were carried out for three values of time namely t = 0.025, 0.075, 0.125 for fixed ε = 0.0078, and for three values of coupling coefficients namely ε = 0.0078 0.01, 0.0125 for fixed time t = 0.075 at z = 1.0 in the range $0 \le r \le 10$. Figure 1 shows the variation of normal displacement which decreases in the range $0 \le r \le 1, 8 \le r \le 10$ and increases in the range $1.5 \le r \le 7.5$ as time increases from .025 to .125 for fixed value of ε = 0.0078. The variation of couple stress is shown in the figure 3 which increases in range $0 \le r \le 1$, decreases in the range $1.5 \le r \le 2$, oscillates in the range $2.5 \le r \le 5$, decreases as r lies between 5.5 to 7.5 and increases in $8 \le r \le 8.5$ and decreases in $9 \le r \le 10$ as time increases from .025 to .125 for fixed value of ε = .0078. Figure 57 shows the variation of normal force stress which increases in the range $0 \le r \le 1.5$, oscillates in modulus value in the range $2 \le r \le 10$ as time increases from .025 to .125.

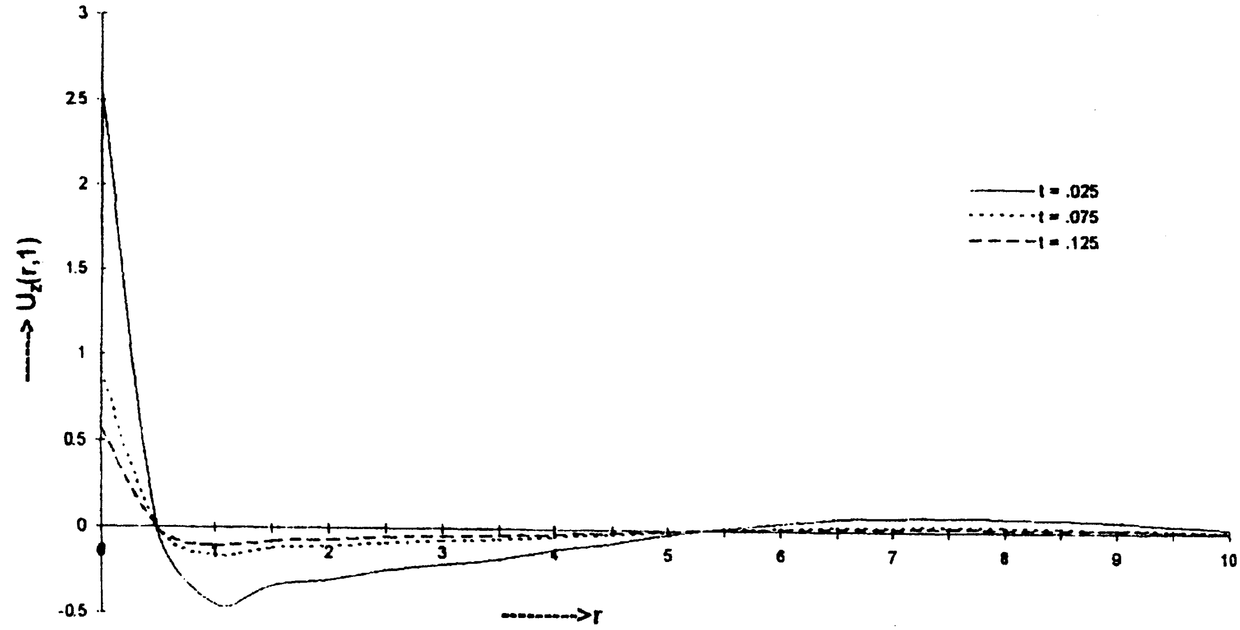

FIG. 1. Normal displacement $U_z(r, 1)$ $U_z = (4\pi/F_0)\, u_z$

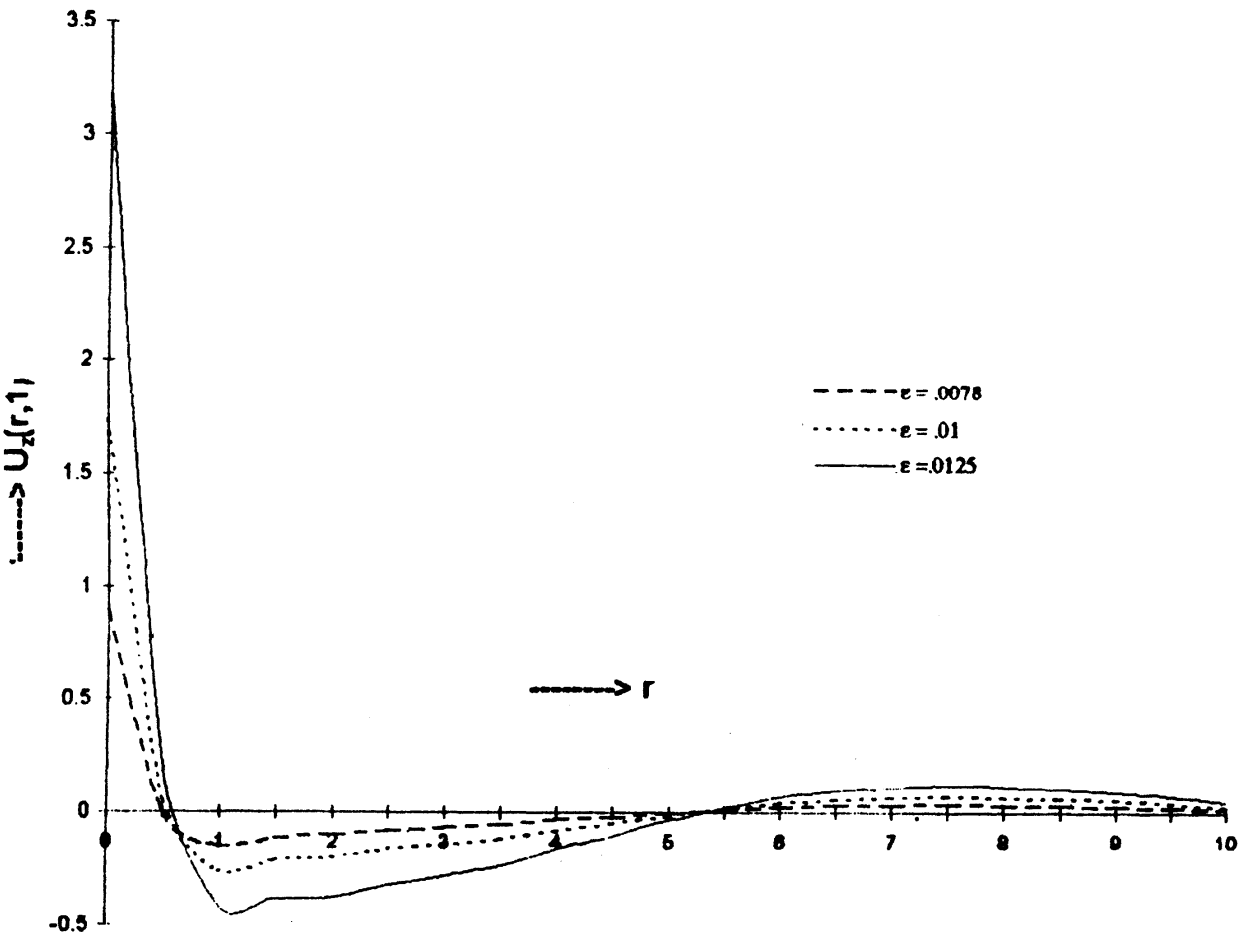

FIG. 2. Normal displacement $U_z(r, 1)$ $U_z = (4\pi/F_0)\, u_z$

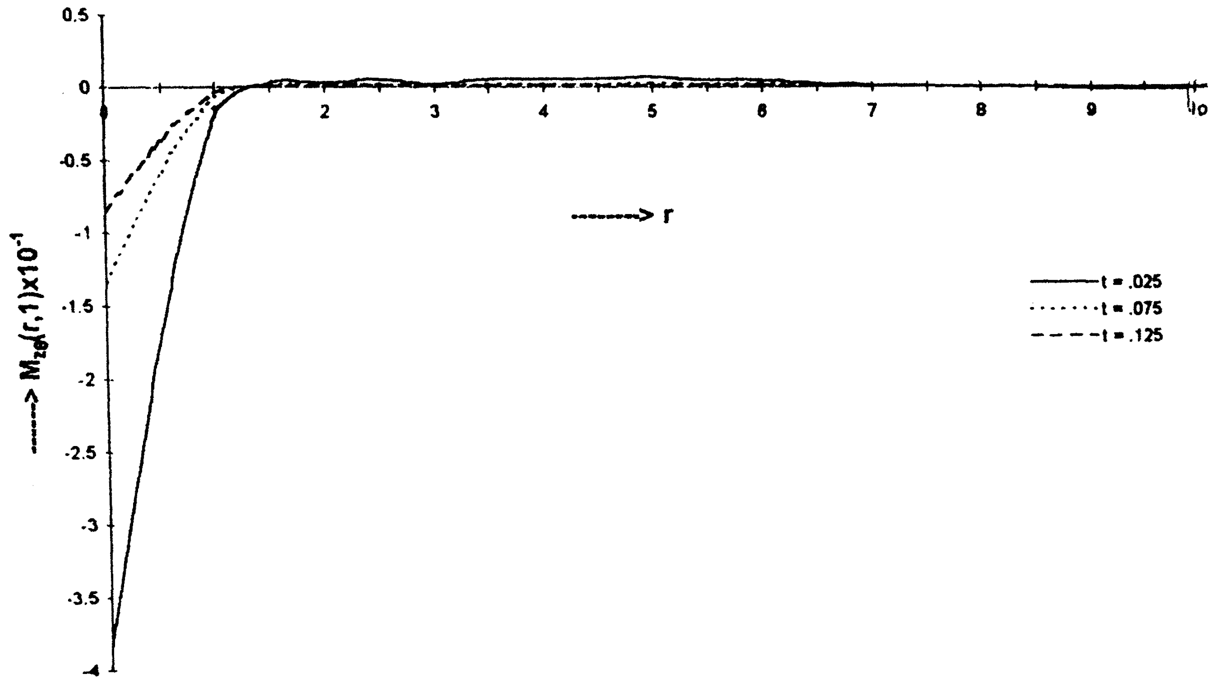

FIG. 3. Couple stress $M_{z\theta}(r, 1)$ $M_{z\theta} = (4\pi/F_0)\, m_{z\theta}$

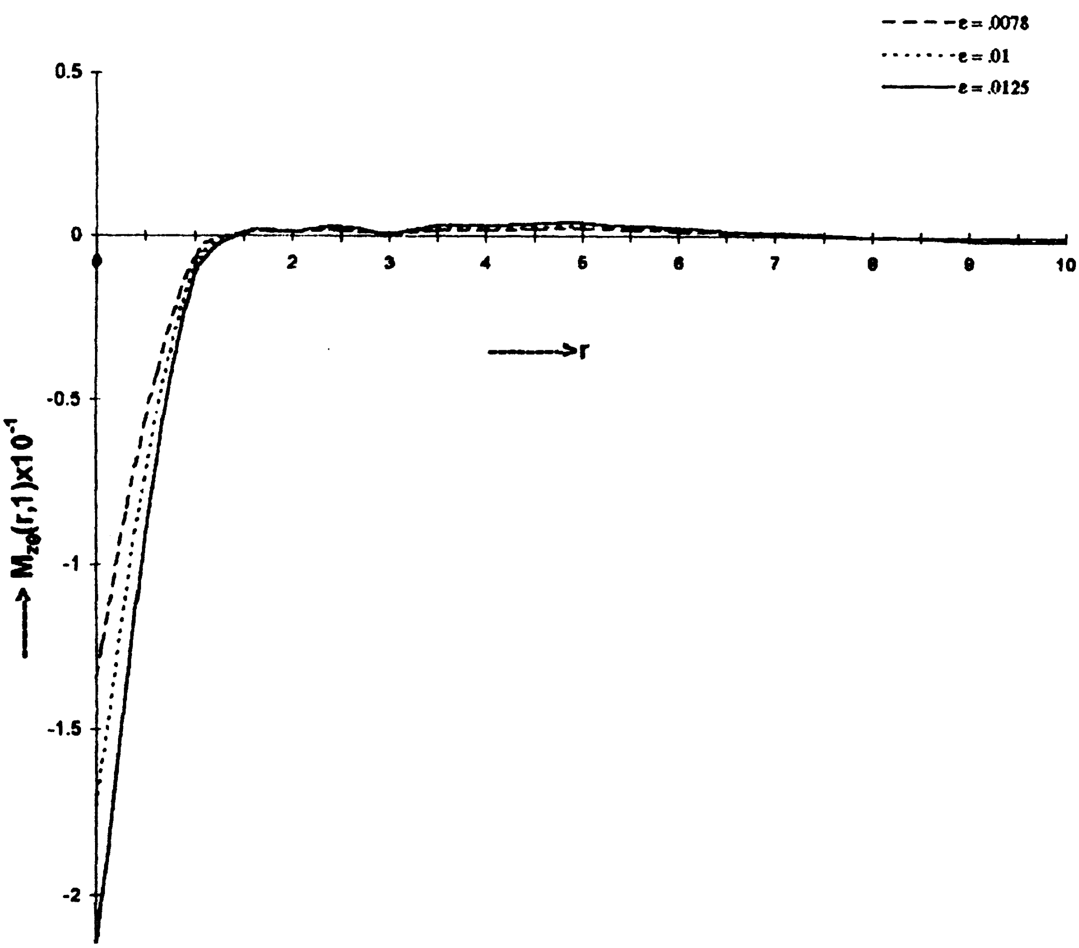

FIG. 4. Couple stress $M_{z\theta}(r, 1)$ $M_{z\theta} = (4\pi/F_0)\, m_{z\theta}$

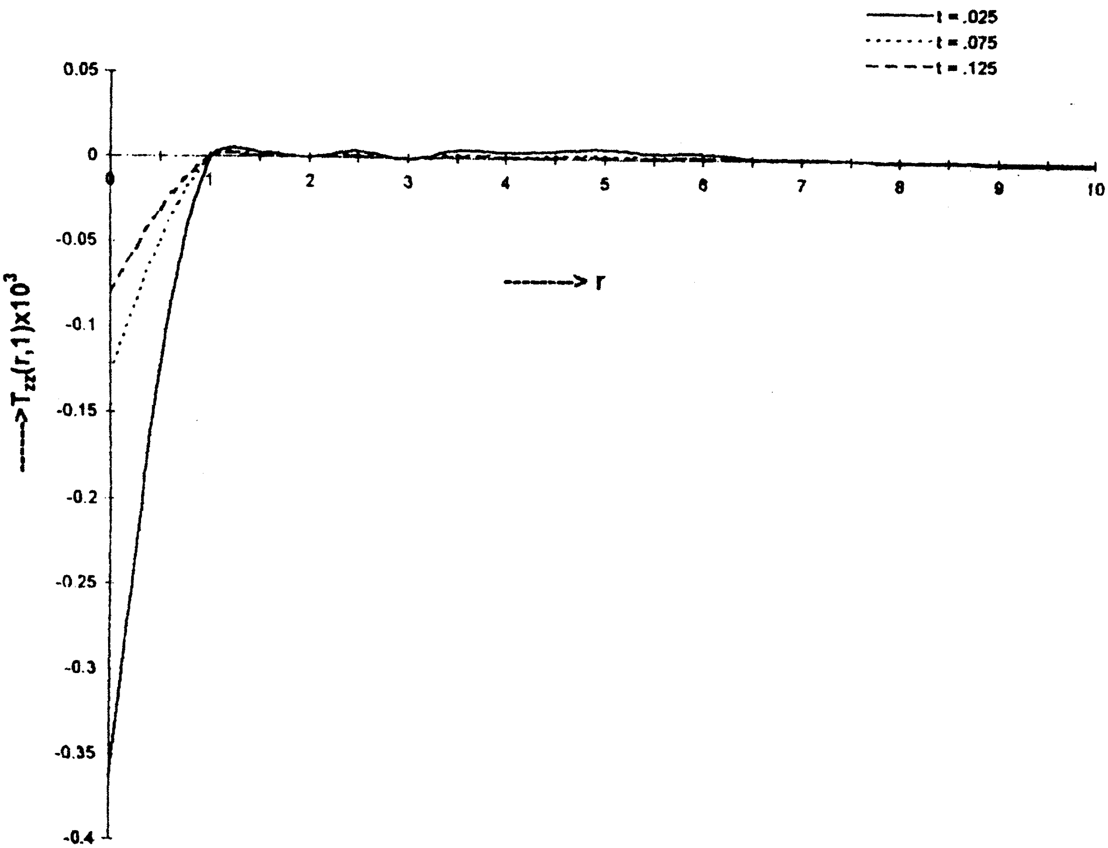

FIG. 5. Normal force stress $T_{zz}(r, 1)$ $T_{zz} = (4\pi/F_0)\, t_{zz}$

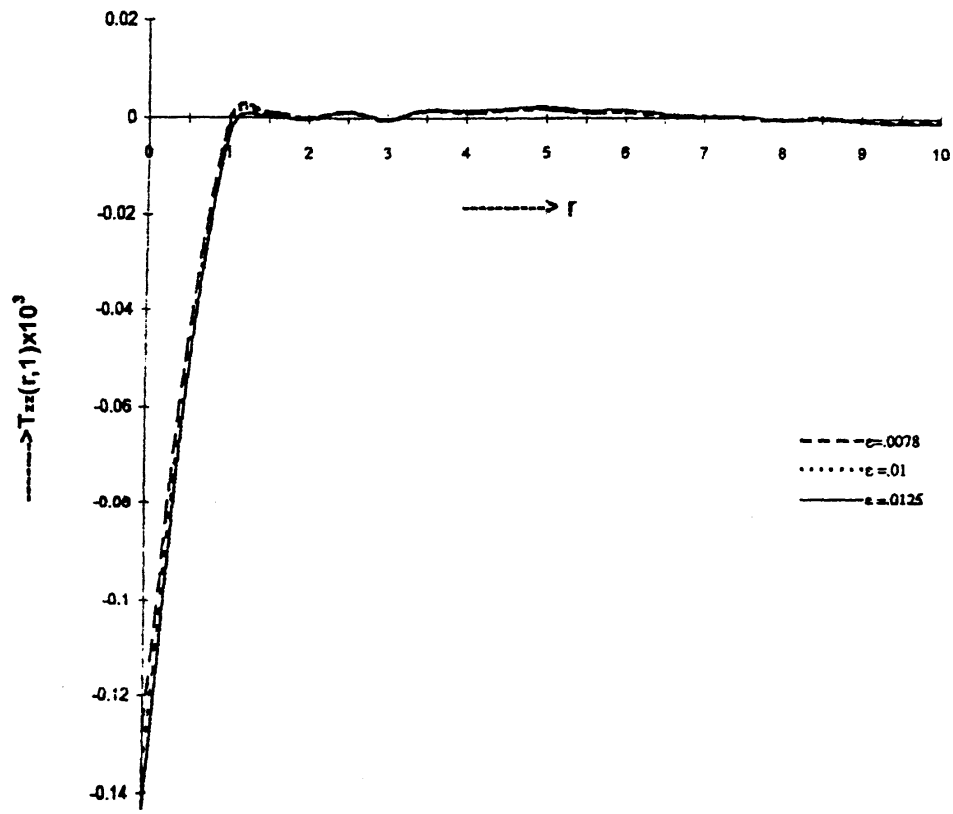

FIG. 6. Normal force stress $T_{zz}(r, 1)$ $T_{zz} = (4\pi/F_0)\, t_{zz}$

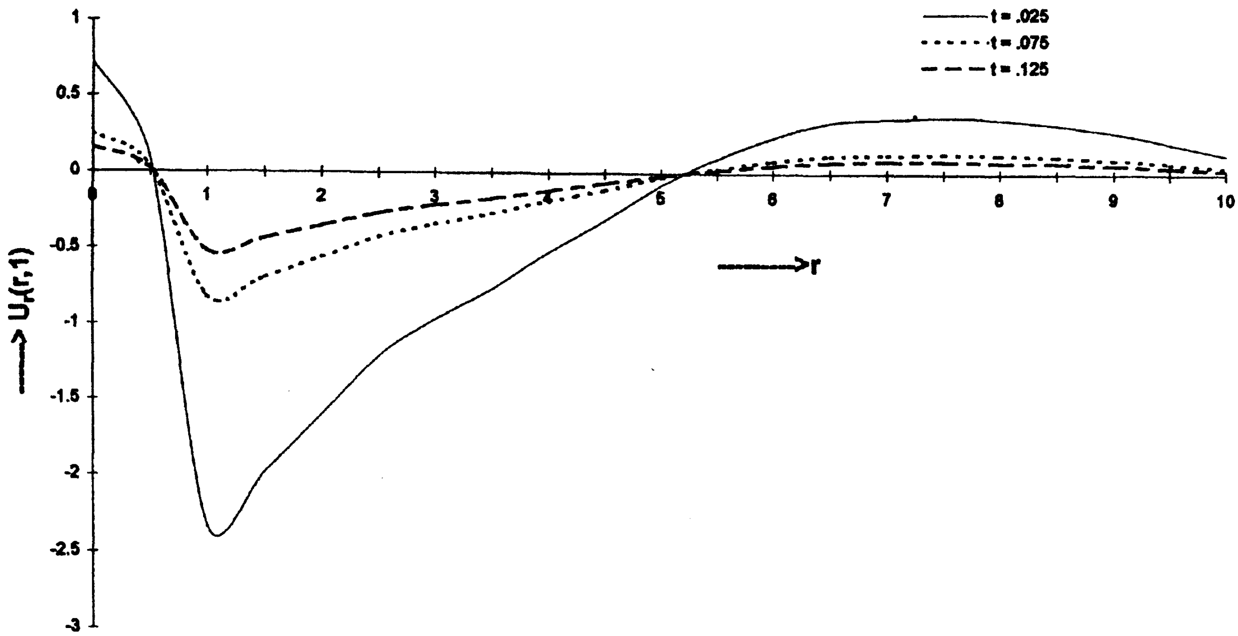

FIG. 7. Radial displacement $U_z(r, 1)$ $U_z = (4\pi/F_0)\, u_z$

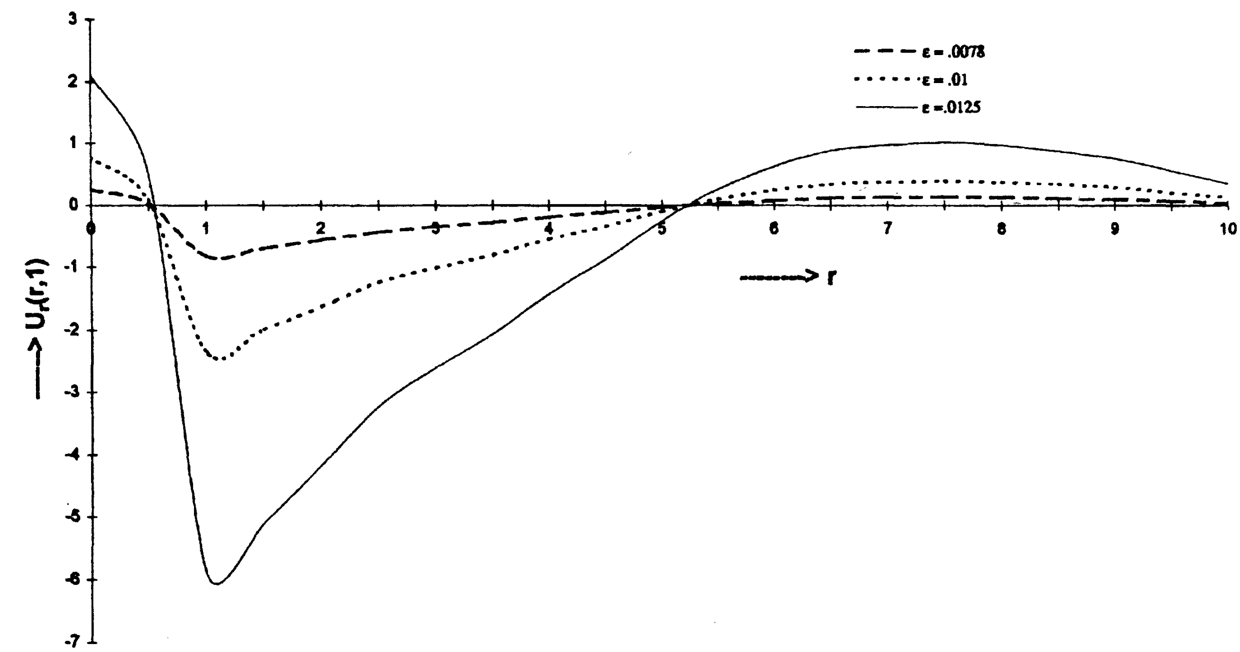

FIG. 8. Radial displacement $U_z(r, 1)$ $U_z = (4\pi/F_0)\, u_z$

Figure 7 shows the variation of radial displacement which decreases in the range $0 \le r \le 1, 8 \le r \le 10$ and increases in the range $1.5 \le r \le 7.5$ as time increases from 0.025 to .125 for fixed value of ε = .0078. Figure 9 shows the variation of micro rotation which increases in the range $0 \le r \le 1, r \le r \le 10$ and decreases in the range $1.5 \le r \le 10$ as find increases from .025 to .125 for fixed value of $\varepsilon = .0078$. Figure 11 shows the variation of shear stress which increases in the range $5 \le r \le 6.5$ and oscillates in the range $0 \le r \le 4.5, 7 \le r \le 10$ as time increases from .025 to .125 for fixed value of ε = .0078.

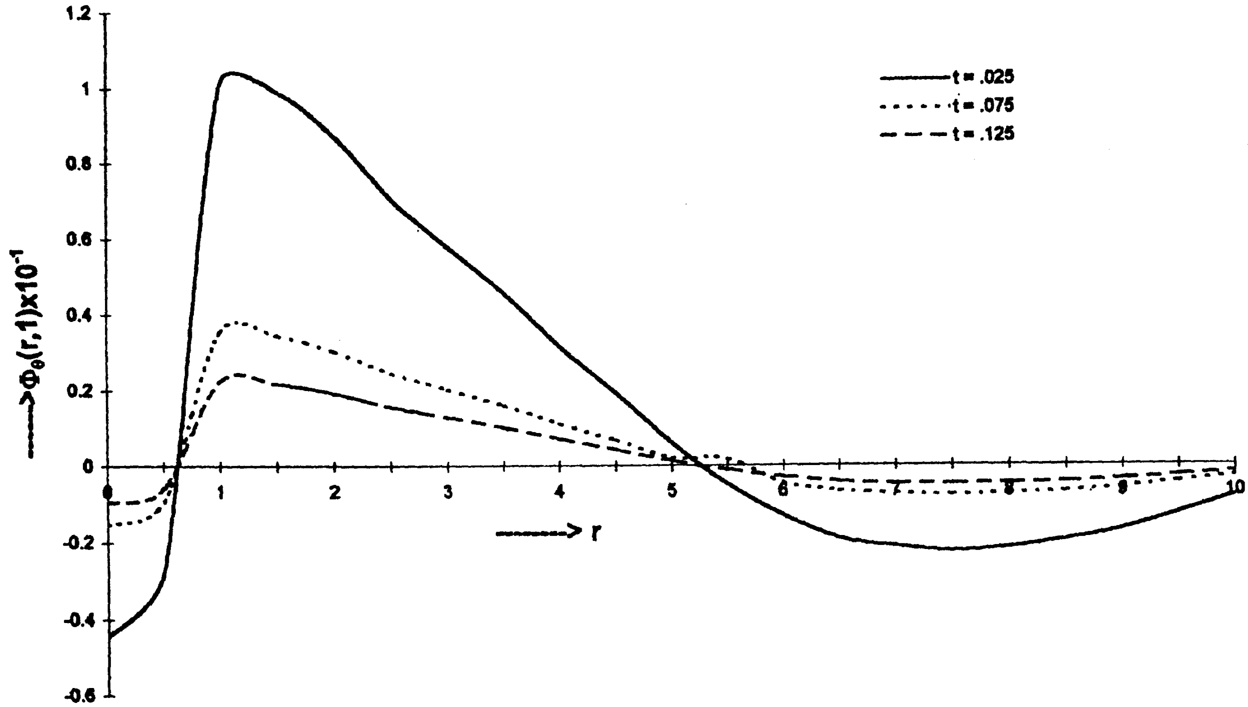

FIG. 9. Microrotation $\Phi_\theta(r,1)$ $\Phi_\theta = (4\pi/F_0)\,\phi_\theta$

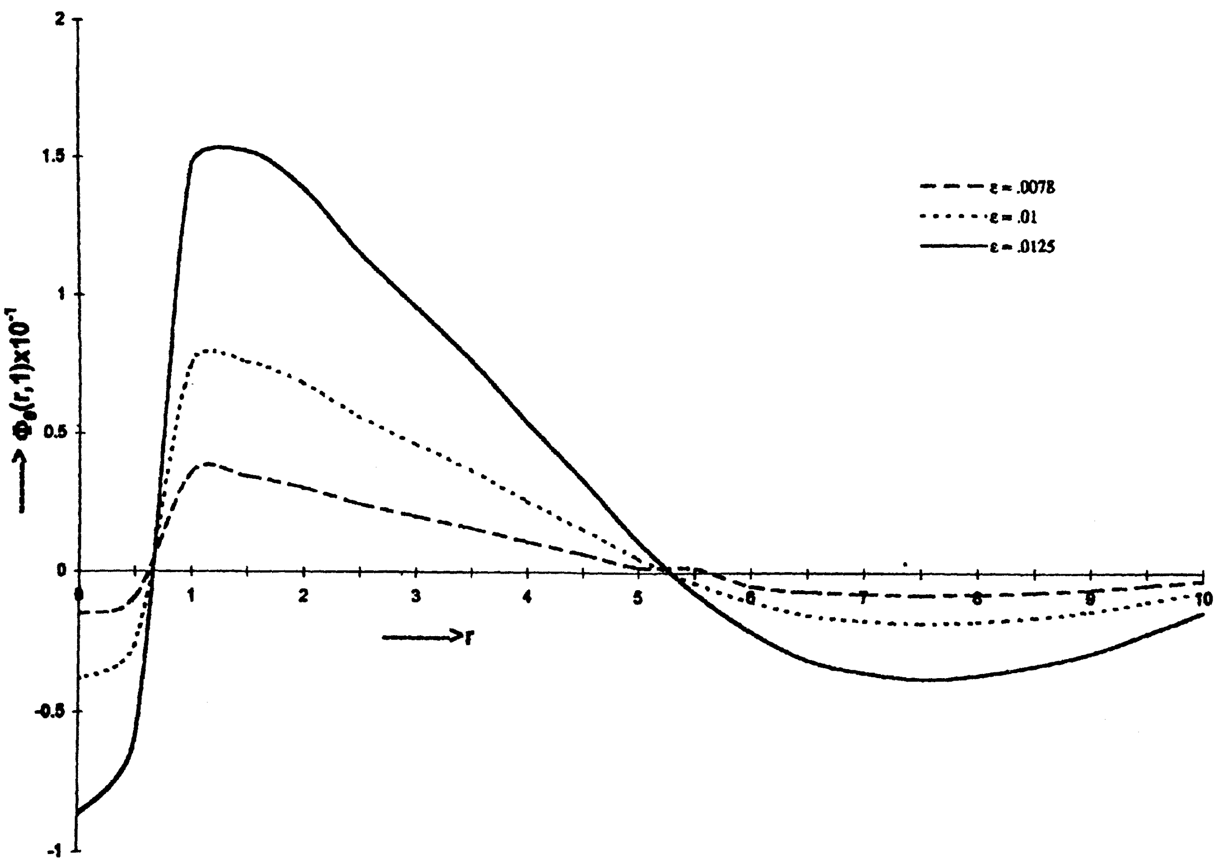

FIG. 10. Microrotation $\Phi_\theta(r,1)$ $\Phi_\theta = (4\pi/F_0)\,\phi_\theta$

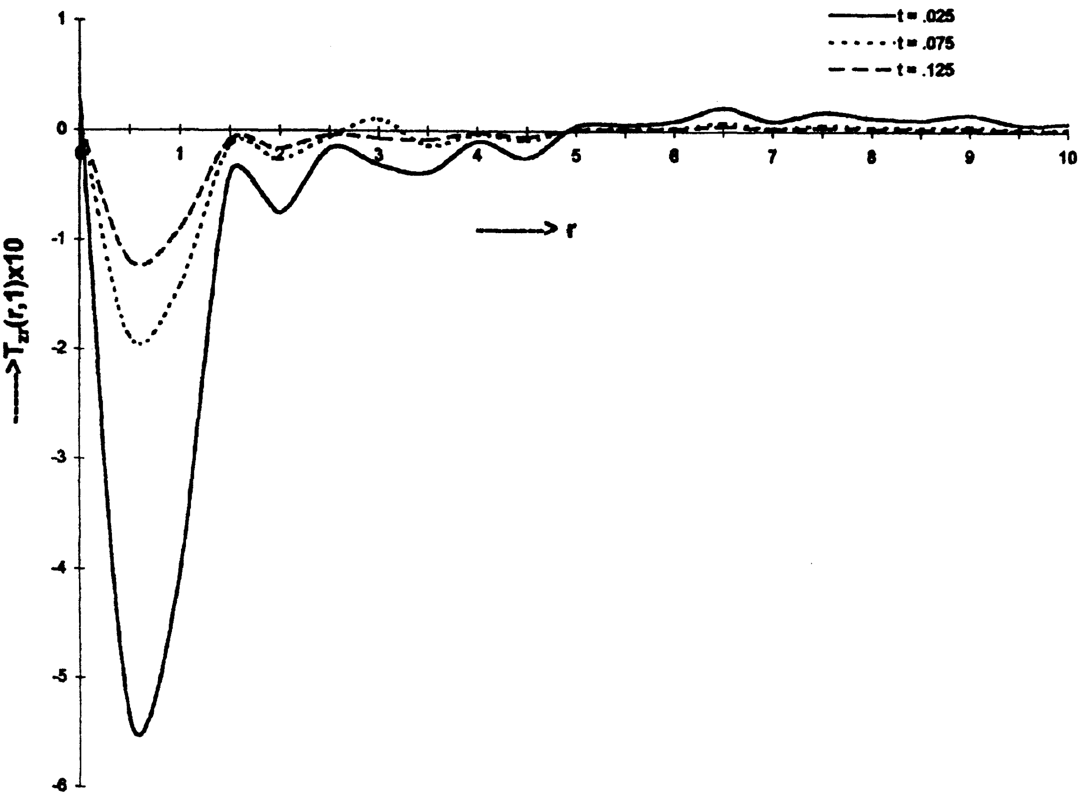

FIG. 11. Shear stress $T_{zr}(r, 1)$ $T_{zr} = (4\pi/F_0)\, t_{zr}$

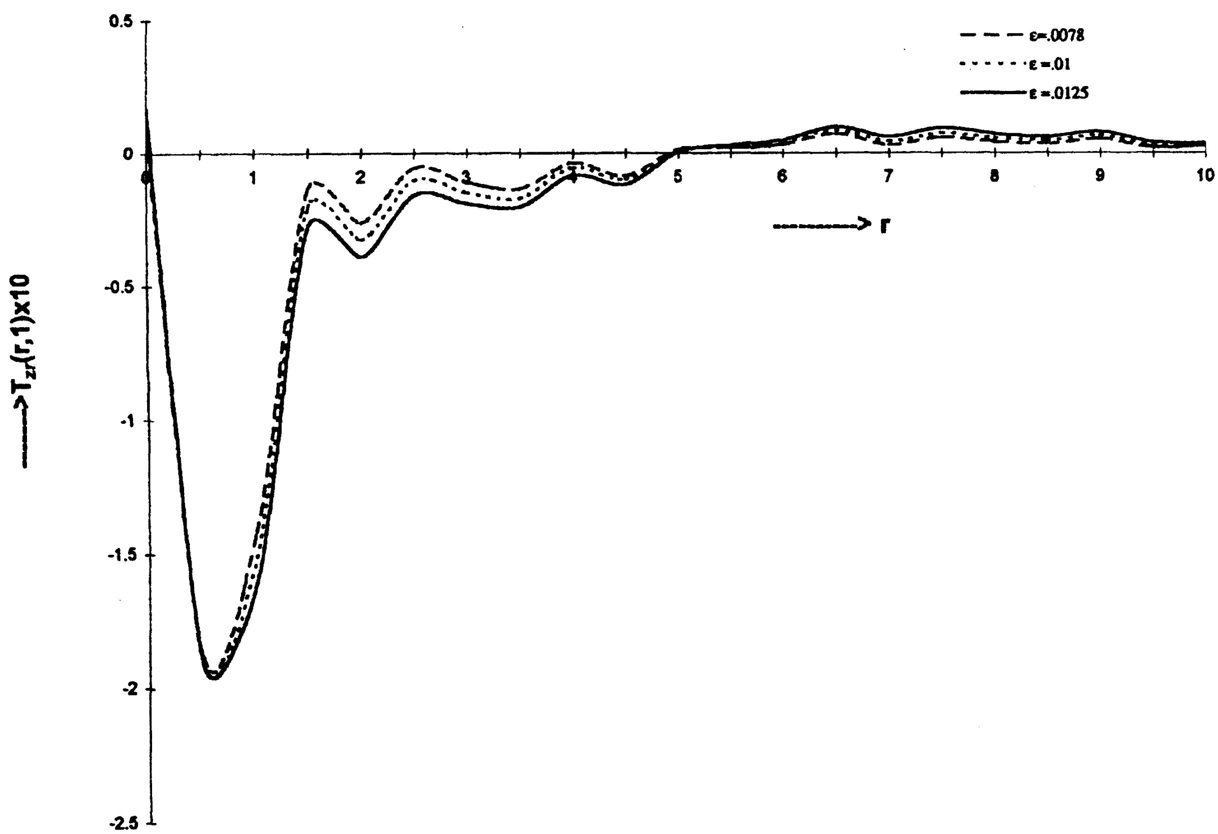

FIG. 12. Shear stress $T_{zr}(r, 1)$ $T_{zr} = (4\pi/F_0)\, t_{zr}$

Figure 2 shows the variation of normal displacement which decreases in range $0 \leq r \leq 1$, increases in $1.5 \leq r \leq 7.5$ and decreases in $8 \leq r \leq 10$ as ε increases from .0078 to .0125 fixed value of time .075. Figure 4 shows the variation of couple stress which decreases in the range $0 \leq r \leq 1, 8 \leq r \leq 10$ and increases in the range $1.5 \leq r \leq 7.5$ as ε increases from.0078 to .0125 for fixed value of time .075. Figure 6 shows the variation of normal force stress which decreases in the range $0 \leq r \leq 2.5, 8 \leq r \leq 10$ and increases in the range $3 \leq r \leq 7.5$ as ε increases from .0078 to .0125 for fixed value of time .075. Figure 8 shows the variation of radial displacement which decreases in the range $0 \leq r \leq 1$, $8 \leq r \leq 10$ and increases in the range $1.5 \leq r \leq 7.5$ as ε increases from .0078 to .0125 for fixed value of time .075. Figure 10 shows the variation of the microrotation which increases in the range $0 \leq r \leq 1, 8 \leq r \leq 10$ and decreases in the range $1.5 \leq r \leq 7.5$ as ε increases from .0078 to .0125 for fixed value of time .075. Figure 12 shoes the variation of shear stress which increases in the range $5 \leq r \leq 6.5$ and oscillates in the range $0 \leq r \leq 4.5, 7 \leq r \leq 10$ as ε increases from .0078 to .0125 for fixed value of time .075.

Moreover all three quantities observed more variation in their magnitude at all small times and small coupling coefficients and decreases with increase of time and ε.

REFERENCES

1. N. C. Das, S. N. Dass and B. Das, *J. therm. Stress.* **6** (1983), 35-43.
2. N. C. Das, A. Lahiri and R. R. Giri, *Indian J. pure appl. Math* **28** (1997), 1573-94.
3. A. C. Eringen and E. S. Suhubi, *Int. J. Engng. Sci.* **2** (1964), 189-203.
4. A. C. Eringen and E. S. Suhubi, *Int. J. Engng. Sci.* **2** (1964), 389-404.
5. A. C. Eringen, *J. Math. Mech.* **15** (1966), 909-24.
6. R. D. Gauthier, In *Experimental Investigations on Micropolar media, Mechanics of Micropolar Media*, ed. O. Brulin and R.K.T. Haseieh, World Scientific, Singapore 1982.
7. G. Honig and U. Hirdes, *J. Comp. Appl. Math.* **10** (1984), 113-132.
8. R. K. Mahalabanabis and J. Manna, *Ind. J. Pure Appl. Math* **20** (1989), 1237-1250.
9. J. Manna and R. K. Mahalabanabis, *J. Indian Acad. Math* **19** (1997), 69-86.
10. W. H. Press, S. A. Teukolsky, W. T. Vellerling and B. P. Flannery (Second Edition), *Numerical Recipes in FORTRAN*, Cambridge University Press, Cambridge, 1986.
11. H. S. Saxena and R. S. Dhaliwal, *J. Thermal Stresses* **13** (1990), 161-75.
12. J. N. Sharma and D. Chand, *Int. J. Engng. Sci.* **30** (1992), 223-30.
13. J. N. Sharma and V. Kumar, *J. therm. Stress.*, **19** (1996), 781-94.