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EFFECT OF CHARGE ON THE SHIFT BETWEEN THE PERIODS OF θ-VIBRATIONS AND r (OR ϕ)-VIBRATIONS

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Considering circular trajectory (r = constant) in the plane $\theta = \pi/2$ in R-N field, a relation analogus to a relation between perihelic shifts $\delta\phi_{RN}$ and $\delta\phi_{\text{Schl'd}}$ is obtained by using Shirokov's technique. Thereby our result supports the conclusion that the charge on gravitating particle instead of helping the matter to curve the spacetime more, decurves it.

Key Words : Effect of Charge; Shift; θ-Vibration; ϕ-Vibrations; Theory of Relativity

In the general theory of relativity the equation of deviation from the geodesic is [1],

$$\frac{d^2 \xi^i}{ds^2} + 2\,\Gamma_{jk}^{\;\;i}\, u^j \frac{d\xi^k}{ds} + \frac{\partial \Gamma^i_{jk}}{\partial x^1} u^j \xi^1 = 0, \qquad \text{... (1)}$$

where ξ^i is the infinitesimal 4-vector giving the deviation from the basic geodesic, $u^i = \frac{dx^i}{ds}$ is the 4-vector velocity tangential to the basic geodesic and Γ^i_{jk} are Christoffel symbols defined as

$$\Gamma^i_{jk} = \frac{1}{2} g^{li} \left(\frac{\partial g_{li}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l} \right).$$

We suppose that the basic geodesic is a circular trajectory with radius r = constant in the plane $\theta = \pi/2$ in Reissner Nordstrom (R – N) field,

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\phi^2) + \left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right) dt^2, \qquad \text{... (2)}$$

where $\quad r = x^1, \theta = x^2, \phi = x^3, t = x^4,$

For the field (2), metric tensors are

$$g_{11} = -\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1}, g_{22} = -r^2,\ g_{33} = -r^2 \sin^2\theta,\ g_{44} = \left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right) \qquad \text{... (3)}$$

and the non-vanishing components of the Christoffel symbol are

$$\Gamma^1_{11} = -\left(\frac{m}{r^2} - \frac{e^2}{r^3}\right)\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1},$$

$$\Gamma^1_{22} = -r\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right), \Gamma^1_{33} = -r\sin^2\theta\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)$$

$$\Gamma^2_{21} = \frac{1}{r} = \Gamma^3_{13}, \Gamma^4_{41} = \left(\frac{m}{r^2} - \frac{e^2}{r^3}\right)\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1}, \Gamma^2_{33} = -\sin\theta\cos\theta$$

$$\Gamma^1_{44} = \left(\frac{m}{r^2} - \frac{e^2}{r^3}\right)\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right), \Gamma^3_{23} = \cot\theta \qquad \text{... (4)}$$

Following Howes[2], if the basic geodesics are circular in the axisymmetric stationary field, θ- disturbances are independent of r, ϕ and t-perturbations.

Therefore for $i = 2$, eqn (1) assumes the form

$$\frac{d^2\xi^2}{ds^2} + \frac{\partial \Gamma^2_{jk}}{\partial x^2} u^j u^k \xi^2 = 0, \; (j, k = 1, 2, 3, 4) \qquad \text{... (5)}$$

If we suppose that $\xi^2 = \xi^2_0 e^{i\Omega s}$ (ξ^2_0 is the amplitude of θ-vibrations), then from (5) we obtain

$$\Omega^2 = \Gamma^2_{jk,2} u^j u^k, \qquad \text{... (6)}$$

where comma denotes the partial differentiation and Ω is the frequency of θ-vibrations.

For $i = 1, 3, 4$ from (1) we get

$$\frac{d^2\xi^1}{ds^2} + 2\Gamma^1_{j3} u^j \frac{d\xi^3}{ds} + 2\Gamma^1_{j4} u^j \frac{d\xi^4}{ds} + \Gamma^1_{jk,1} u^j u^k \xi^1 = 0,$$

$$\frac{d^2\xi^3}{ds^2} + 2\Gamma^3_{j1} \frac{d\xi^1}{ds} u^j = 0 \text{ and}$$

$$\frac{d^2\xi^4}{ds^2} + 2\Gamma^4_{j1} \frac{d\xi^1}{ds} u^j = 0. \qquad \text{... (7)}$$

Further, if we suppose that $\xi^j = \xi^j_0 e^{i\omega s}$ $(j = 1, 3, 4)$

(ξ^j_0-the amplitude of r, ϕ and t-vibrations), then from (7) we get,

$$(\Gamma^1_{jk,1} u^j u^k - \omega^2)\xi^1_0 + 2i\omega\Gamma^1_{j3} u^j \xi^3_0 + 2i\omega\Gamma^1_{j4} u^j \xi^4_0 = 0,$$

$$2i\,\omega\,\Gamma^3_{j1}\,u^j\,\xi^1_0 - \omega^2\,\xi^3_0 = 0$$

and
$$2\,i\,\omega\,\Gamma^4_{j1}\,u^j\,\xi^1_0 - \omega^2\,\xi^4_0 = 0. \quad \text{... (8)}$$

where ω is the frequency of r, ϕ and t-vibrations, all the Chirstoffel symbols and their derivatives are evaluated at $\theta = \pi/2$.

For non-trivial solution of (8) we equate the determinant of coefficients to zero and obtain

$$\omega^2 = u^j\,u^k\,\Gamma^1_{jk,\,1} - 4u^j\,u^k\,\Gamma^3_{j1}\,\Gamma^1_{k3} - 4u^j\,u^k\,\Gamma^4_{j1}\,\Gamma^1_{k4}$$

$$= (\Gamma^1_{33,\,1} - 4\,\Gamma^3_{31}\,\Gamma^1_{33})\,(u^3)^2 + (\Gamma^1_{44,\,1} - 4\,\Gamma^4_{14}\,\Gamma^1_{44})\,(u^4)^2. \quad \text{... (9)}$$

To find u^3 : Consider geodesic equation

$$\frac{du^i}{ds} + \Gamma^i_{jk}\,u^j\,u^k = 0 \;\; (i,\, j,\, k \,=\, 1,\, 2,\, 3,\, 4), \quad \text{... (10)}$$

in the Einstein's theory of gravitation.

For circular orbits in the equatorial plane from (10) we find

$$\frac{dt}{d\,\phi} = \frac{u^4}{u^3} = (-\,\Gamma^1_{33}/\Gamma^1_{44})^{1/2}, \quad \text{... (11)}$$

which provides the angular velocity of the test particle as seen from the infinity.

Using (4), from (11) we get

$$(u^4)^2 = \frac{r^2}{\frac{m}{r} - \frac{e^2}{z^1}}\,(u^3)^2. \quad \text{... (12)}$$

For equatorial circular orbit in the field (2) using (12) we get

$$(u^3)^2 = \left(\frac{m}{r^3}\right)\left(1 - \frac{e^2}{mr}\right)\left(1 - \frac{3m}{r} + \frac{2e^2}{r^2}\right)^{-1} \quad \text{... (13)}$$

The corresponding frequencies of θ-vibrations and r (or ϕ-vibrations in (6) and (9) simplify to

$$\Omega^2 = (u^3)^2 \quad \text{... (14)}$$

and
$$\omega^2 = (u^3)^2 \left\{ 1 - \frac{6m}{r} + \frac{3e^2}{r^2} + \frac{e^2}{rm} + \frac{e^4}{r^2\,m^2} + O\,(\eta)^{2\,\frac{1}{2}} \right\}, \quad \text{... (15)}$$

respectively in which $\frac{m}{r} = \frac{e}{r} = O\,(\eta)$, η is small..

Therefore, the periods of θ-vibrations and r (or ϕ) vibrations are

$$T_\theta = \frac{2\pi}{\Omega} = T_0 \left\{ 1 - \frac{3m}{2r} - \frac{9m^2}{8r^2} + \frac{e^2}{2mr} + \frac{e^2}{3r^2} + \frac{3e^4}{8r^2 m^2} + O(\eta)^{2\frac{1}{2}} \right\} \quad \text{... (16)}$$

and

$$T_r \text{ (or } T_\phi) = \frac{2\pi}{\omega} = T_0 \left\{ 1 + \frac{3m}{2r} + \frac{63\, m^2}{8\, r^2} - \frac{7e^2}{2r^2} + O(\eta)^{2\frac{1}{2}} \right\}, \quad \text{... (17)}$$

where $T_0 = 2\pi \left(\frac{r^3}{m} \right)^{1/2}$ is the Newtonian period of test particle in the circular orbit of radius r. The difference ΔT_{RN} between the periods of θ-vibrations and r (or ϕ)-vibrations is

$$\Delta T_{RN} = \left(-\frac{3m}{r} + \frac{e^2}{2rm} \right) T_0 \quad \text{... (18)}$$

to the $1\frac{1}{2}$ order approximation

For $e = 0$, (18) reduces to

$$\Delta T_{\text{Schl'd}} = -\frac{3m}{r} T_0, \quad \text{... (19)}$$

the result obtained by Shirokov (1973) as a new effect of Einstein's theory of gravitation.

From (18) and (19) we find a relation between shift in periods of θ-vibrations and r (or ϕ) vibration in R-N field and Schwarzschild field as

$$\Delta T_{RN} = \Delta T_{Schl'd} \left(1 - \frac{e^2}{6m^2} \right), \quad \text{... (20)}$$

which is analogus to the relation,

$$\delta \phi_{RN} = \delta \phi_{Schl'd} \left(1 - \frac{e^2}{6m^2} \right), \quad \text{...(21)}$$

between perihelic shift in R-N field and Schwarzschild field obtained by H J Treder, H H V Borzeszkowski, A Van Der Merwe, W Y Yourgrau[3].

According to [4], $\delta \phi_{RN} < \delta \phi_{Schl'd}$ shows that charge on the gravitating particle instead of helping the matter to curve the space-time more, decurves the space-time, which means that the nature of the gravitational fields due to the matter and charged matter may be of different type.

In our case from equation (20), we find a similar relation $\Delta T_{RN} < \Delta T_{Schl'd}$ which supports the conclusion of [4].

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REFERENCES

1. M. F. Shirokov, *Gen. Rel. Grav.* **4** (1973) 131.
2. Robert J Howes, *Gen Rel. Grav.* **13** (1981) 830.
3. H J. Treder *et al.*, *Fundamental Principle of General Relativity Theory*, New York/London : Plenum Press, 1980.
4. G. D. Rathod and T. M. Karade (1989), *Analen der Physik*, **7**, Folge Band 46, Heft.