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THERMAL RADIATION EFFECT ON MIXED CONVECTION FROM VERTICAL SURFACES IN SATURATED POROUS MEDIA

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An analysis is presented to investigate the effect of radiation on mixed convection from a vertical flat plate in a saturated porous medium. Both a hot surface facing upward and a cold surface facing downward are considered in the analysis. The conservation equations that govern the problem are reduced to a system of nonlinear ordinary differential equations. The important parameters of this problem are the radiation parameters R, the buoyancy parameter B, and the freestream to wall temperature ratio T_∞/T_w for the case of a hot surface or the wall to freestream to wall temperature T_w/T_∞ for the case of cold surface.

Key Words : Thermal Radiation; Mixed Convection; Vertical Surfaces

1. NOMENCLATURE

c-Specific heat;

f-dimensionless stream function defined by eq. (9);

g-acceleration due to gravity;

k-permeability of the porous medium;

k_t-thermal conductivity of the saturated porous medium;

p-pressure;

p_{ex}-Peclet number;

q-local heat flux;

q^r-radiative flux $\left(\dfrac{4\sigma}{3\beta^*}\right)\dfrac{\partial T^4}{\partial y}$;

Re-local Reynolds number $U_\infty x/\nu$;

Gr-local Groshof number;

T-temperature;

R-radiation parameter;

B-Buoyancy parameter;

Nu-local Nusselt number;

U_∞-velocity in x-direction outside the boundary layer;

u-velocity in x-direction;

v-velocity in y-directionl

x-coordinate along the plate;

y-coordinate normal to the plate;

Greek Symbols

α-equivalent thermal diffusively;

β-coefficient of thermal expansion;

β^*-extinction coefficient;

η-dimensionless similarity variable defined in eq (9);

θ-dimensionless temperature defined in eq. (9);

σ-Stefan-Boltzmann constant;

μ-viscosity of convective fluid;

ν-kinematic viscosity of the convective fluid;

ψ-stream function;.

Subscripts

∞-condition at infinity;

and r-reference conditions.

2. Introduction

Heat transfer by mixed convection in laminar boundary-layer flow has been analyzed extensively for flat plate geometry in saturated porous media in vertical, horizontal, and inclined orientations. Typical studies can be found, for example, in [1-4]. On the other hand, heat transfer by simultaneous natural convection and thermal radiation in a participating fluid has not received as much attention. This is unfortunate because thermal radiation will play a significant role in the overall surface heat transfer in situations where convection heat transfer coefficients are small, as is the case of natural convection. Gorla[10 & 11] has investigated the effects of radiation on mixed convection flow over vertical cylinders.

In the present study, the focus is on the effect of mixed convection and radiation about a vertical surface in a porous medium. Similarly solutions were obtained for aiding external flow over a vertical flat plate. The governing equations for mixed convection and boundary conditions are first transformed with the help of similarity variables. The resulting nonlinear similarity equations are solved numerically and the results are reported in graphical and tabular form.

3. Analysis

Consider a vertical flat plate embedded in a saturated porous media. Having invoked the Boussinesq and boundary-layer approximations, the governing equations may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \text{... (1)}$$

$$u=-\frac{K}{\mu}\left(\frac{\partial p}{\partial x}\pm\rho g\right), \qquad \text{... (2)}$$

$$v=-\frac{K}{\mu}\frac{\partial p}{\partial y}, \qquad \text{... (3)}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{(\rho\infty c)}\frac{\partial q^r}{\partial y} \qquad \dots (4)$$

and $$\rho=\rho_\infty[1-\beta(T-T_\infty)]. \qquad \dots (5)$$

Here the (+) sign in eq. (2) refers to the case of a heated impermeable surface facing upward and the (–) sign refers to the case of a cooled impermeable surface facing downward. In eqs. (1 – 5) u and v are the Darcy velocities in the x and y directions; ρ, μ and β are the density, viscosity, and thermal expansion coefficient of the convection fluid; K is the permeability of the porous medium; $\alpha \equiv k_t/(\rho_\infty c)_f$ is the equivalent thermal diffusivity; c the specific heat of fluid; and k_t the thermal conductivity of the saturated porous medium. T, p and g are the temperature, pressure and gravitational acceleration, respectively. In the energy balance equation, the temperature of fluid and solid are assumed to be equal at the boundary. The refractive index of the medium is assumed to be constant and uniform within the porous medium.

If we introduce the stream function ψ into the governing equations we get :

$$\frac{\partial^2 \psi}{\partial y^2}=\pm\frac{K\rho g\beta}{\mu}\frac{\partial T}{\partial y} \qquad \dots (6)$$

and $$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x}+\frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}-\frac{1}{k_t}\frac{\partial q^r}{\partial y}\right), \qquad \dots (7)$$

where $$q^r=-\left(\frac{4\sigma}{3\beta^*}\right)\frac{\partial T^4}{\partial y}.$$

The boundary conditions are given by —

$$y=0:\ b=0,\ T=T_w,$$

$$y\to\infty: u\to U_\infty, T\to T_\infty. \qquad \dots (8)$$

Proceeding with the analysis, we now introduce a similarity variable η with a reduced stream function $f(\eta)$ and a dimensionless temperature $\theta(\eta)$ as follows :

$$\eta=\frac{y}{x}(Pe_x)^{1/2},$$

$$\psi(x,y)=\alpha(Pe_x)^{1/2}f(\eta),$$

$$\theta(\eta)=T/T_r,$$

and $$Pe_x=\frac{U_\infty x}{\alpha}. \qquad \dots (9)$$

Here, $T_r=T_w$ for the hot wall $T_r=T_\infty$ for the cold wall.

The stream function $\psi(x, y)$ automatically satisfies the continuity eq. (1) with

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}. \quad \text{... (10)}$$

We assume an optimally thick approximation for the radiative heat flux q^r. Upon substituting the expressions in eq. (9) into the governing momentum and energy equations, we may write the transformed equations as

$$f'' = B\,\theta'$$

and

$$\theta''\left(1 + \frac{4}{3} R\,\theta^3\right) + 4R\,\theta^2\,(\theta')^2 + \frac{f\,\theta'}{2} = 0. \quad \text{... (12)}$$

The primes above denote differentiation with respect to η. In the previous equations, we have used the following :

$$B = \frac{Gr}{Re},$$

$$Gr = \frac{gk\,\beta\,T_r x}{\nu^2},$$

$$Re = U_\infty x/\nu$$

and

$$R = \frac{4\,T_r^3\,\sigma}{\beta^*\,k}. \quad \text{... (13)}$$

The transformed boundary conditions are given by

$f(0) = 0$, $\theta(0) = 1$ for the hot surface case,

$\theta(0) = T_w/T_\infty$ for the cold surface case,

$f'(\infty) = 1$, $\theta(\infty) = T_\infty/T_w$ for the hot surface case,

$= 1$ for the cold surface case. ... (14)

The heat transfer coefficient h is given by the following energy balance at the surface :

$$-k_t \cdot \frac{\partial T}{\partial y}\uparrow_{y=0} + q^r = h\,(T_w - T_\infty).$$

The local Nusselt number therefore becomes

$$Nu = \frac{hx}{k} = -\theta'(0)\,Pe_x^{1/2}\left[1 + \left(\frac{4}{3}\right) R\,\theta^3\right]\left(\frac{T_r}{T_w - T_\infty}\right). \quad \text{... (15)}$$

4. Discussion

Eqs. (11) and (12) were solved to satisfy the boundary defined by eq. (14) by means of the fourth-order Runge-Kutta method of numerical integration.

Shooting techniques were implemented in order to determine the missing wall conditions. Tables I-IV summarize the numerical results for the wall temperature gradient for the cases of a hot surface and a cold surface.

TABLE I

Summary of similarity solution for $-\theta'(0)$ with various R and θ for hot surface.

$R \setminus T_\infty/Tw$	0.0	0.2	0.4	0.6	0.8
0.0	.5306	.4304	.3272	.2211	.1120
0.4	.5570	.4562	.3447	.2237	.1042
0.6	.5707	.4694	.3532	.2248	.1011
0.8	.5846	.4828	.3617	.2255	.0972
1.0	.5988	.4964	.3700	.2247	.0089

TABLE II

Summary of similarity solution for $-\theta'(0)$ with various B and θ for hot surface.

$B \setminus T_\infty/Tw$	0.0	0.2	0.4	0.6	0.8
0.0	.4696	.3787	.2872	.1943	.0990
1.0	.6001	.4647	.3371	.2172	.1049
10.0	1.2687	.9306	.6271	.3633	.1480
50.0	2.6730	1.9350	1.2772	.7124	.2649
100.0	3.7502	2.7096	1.7829	.9881	.3612

TABLE III

Summary of similarity solution for $\theta'(0)$ with various R and θ for cold surface.

$R \ T_\infty/Tw$	0.0	0.2	0.4	0.6	0.8
0.0	.5306	.4304	.3272	.2211	.1120
0.2	.5436	.4432	.3360	.2224	.1077
0.4	.5570	.4562	.3447	.2237	.1042
0.6	.5707	.4694	.3532	.2248	.1011
0.8	.5846	.4828	.3617	.2255	.0973
1.0	.5988	.4964	.3700	.2246	.0089

TABLE IV
Summary of similarity solution for $\theta'(0)$ with various B and θ for cold surface.

$B \backslash T_\infty/Tw$	0.0	0.2	0.4	0.6	0.8
0.0	.5847	.4713	.3522	.2279	.1069
0.2	.5489	.4485	.3395	.2225	.1056
0.4	.4696	.3992	.3128	.2113	.1030
0.6	.3754	.3435	.2837	.1995	.1004
0.8	.0501	.1518	.1959	.1668	.0935
1.0	0.0	.0002	.0009	.0294	.0697

Fig. 1 shows the Nusselt number variation with the ratio of wall temperature to free stream temperature for a cold surface where the Nusselt number decreases as (T_w/T_∞) increases, while the radiation parameter R increases.

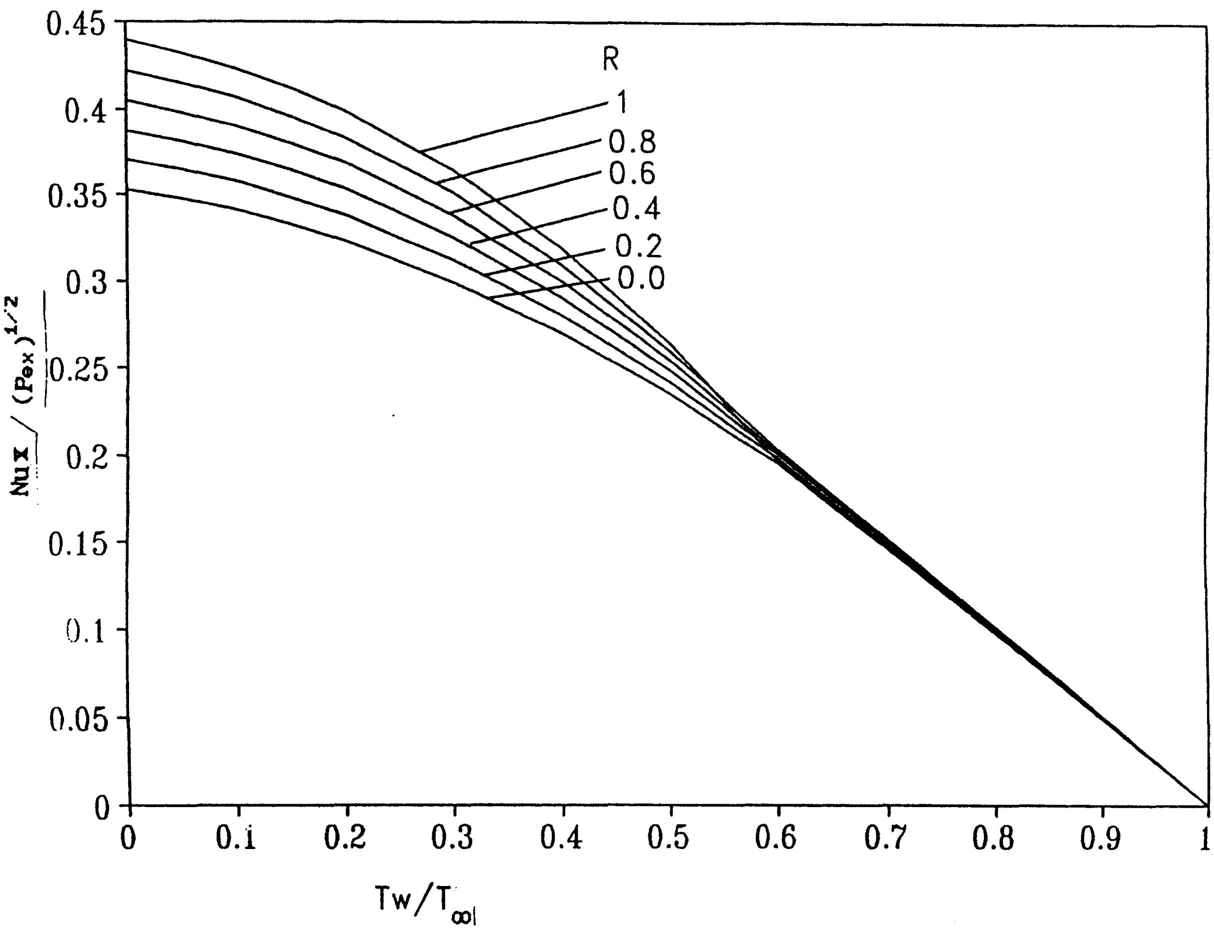

FIG. 1. Temperature gradient at the wall for a cold surface with $B = 1.0$

The radiation parameter has a considerable influence on the augmentation of surface heat transfer rate. The effect of increasing buoyancy is to increase the streamwise velocity at the surface and augment the heat transfer rates from the surface.

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