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FIRST ORDER REACTANT IN MHD TURBULENCE BEFORE THE FINAL PERIOD OF DECAY FOR THE CASE OF MULTI-POINT AND MULTI-TIME

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Following Deissler's approach the decay for the concentration fluctuation of a dilute contaminant undergoing a first order chemical reaction in Magneto Hydrodynamics (MHD) turbulence at times before the final period for the case of multi-point and multi-time correlation equations is studied. Two-point, two-time and three-point, three-time correlation equations have been obtained and to make the set of equations determinate, the terms containing quadruple correlation are neglected in comparison with second and third order correlation terms. The solution obtained gives the decay law for the concentration fluctuations before the final period.

Key Words : MHD Turbulence, First Order Reactant; Decay Before the Final Period

1. INTRODUCTION

Loeffer and Dissler[7] used the theory, developed by Deissler[3&4] to study temperature fluctuations in homogeneous turbulence before the final period. In their approach it is considered the two- and three-point correlation equations and solution were obtained of these equations after neglecting the fourth and higher order correlation terms. Using Deissmer's theory, Kumer and Patel[5] studied the first order reactant in homogeneous turbulence before the final period for the case of multi-point and multi-time consideration.

In our present study, the same approach of Deissler is applied to the study of magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period. In this problem, we considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally, we obtained the decay law for magnetic energy fluctuation of concentration before the final period.

2. FUNDAMENTAL EQUATIONS

The equations of motion for viscous, incompressible MHD turbulent flow are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k} \qquad \ldots (2.1)$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_j h_k) = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} \qquad \text{... (2.2)}$$

with
$$\frac{\partial u_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0, \qquad \text{... (2.3)}$$

where $u_i(\hat{x}, t)$, ith-component of turbulent velocity about the mean at a point $P(\hat{x})$ and time t; $h_i(\hat{x}, t)$, ith-component of magnetic field fluctuation of concentration about the mean at a point $P(\hat{x})$ and time t; $W(x, t) = p/\rho + \frac{1}{2}\langle h \rangle^2$, total MHD pressure; $p(\hat{x}, t)$, hydrodynamic pressure; ρ, fluid density; ν, kinematic viscosity; $\lambda = \nu/P_M$, magnetic diffusivity; P_M, magnetic prandtle number; x_k, space coordinate; the subsctipts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicates a summation.

Eqs. (2.1) through (2.3) are derived by Chandrasekhar[1], the basis of Batchelor's discussion by coupling Maxwell's equation for the electromagnetic field and the Navier-Stokes equations for the velocity field. The Maxwell equations are modified to include the induced electric field due to the fluid motion, and the Navier-Stokes equations are modified to include to the Lorentz force on fluid elements due to the magnetic field.

3. Two-Point, Two-Time Correlation and Spectral Equations

If the turbulence and the concentration magnetic field are homogeneous, chemical reaction and the local mass transfer have no effect on the velocity field, the reaction rate and the magnetic diffusivity are constant, then Induction equation of a magnetic field governing the concentration of a dilute contaminant undergoing a first order chemical reaction at the points p and p' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} - Rh_i \qquad \text{... (3.1)}$$

and
$$\frac{\partial h_i'}{\partial t'} + u_k' \frac{\partial h_j'}{\partial x_k'} - h_k' \frac{\partial u_j'}{\partial x_k'} = \lambda \frac{\partial^2 h_j'}{\partial x_k' \partial x_k'} - Rh_j' \qquad \text{... (3.2)}$$

where R is the constant reaction rate.

Multiplying eq. (3.1) by h_j' and eq. (3.2) by h_i and taking ensemble average, we get

$$\frac{\partial \langle h_j h_j' \rangle}{\partial t} + \frac{\partial}{\partial x_k}[\langle u_k h_i h_j' \rangle - \langle u_i h_k h_j' \rangle] = \lambda \frac{\partial^2 \langle h_i h_j' \rangle}{\partial x_k \partial x_k} - R\langle h_i h_j' \rangle \qquad \text{... (3.3)}$$

and
$$\frac{\partial \langle h_i h_j' \rangle}{\partial t'} + \frac{\partial}{\partial x_k'}[\langle u_k' h_i h_j' \rangle - \langle u_j' h_i h_k' \rangle] = \lambda \frac{\partial^2 \langle h_i h_j' \rangle}{\partial x_k' \partial x_k'} - R\langle h_i h_j' \rangle. \qquad \text{... (3.4)}$$

Angular bracket $\langle \text{........} \rangle$ which is used to denote an ensemble average.

Using the transformation

$$\frac{\partial}{\partial x_k} = -\frac{\partial}{\partial r_k}, \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r_k}$$

$$\left(\frac{\partial}{\partial t}\right)t' = \left(\frac{\partial}{\partial t}\right)\Delta t - \frac{\partial}{\partial \Delta t}, \frac{\partial}{\partial t'} = \frac{\partial}{\partial \Delta t} \quad \text{... (3.5)}$$

into eqs. (3.3) and (3.4), one obtains

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial r_k}[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle](\hat{r}, \Delta t, t)$$

$$-\frac{\partial}{\partial r_k}[\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle](\hat{r}, \Delta t, t) = 2\lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} - 2R\langle h_i h'_j \rangle \quad \text{... (3.6)}$$

and $$\frac{\partial \langle h_i h'_j \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}[\langle u'_k h_i h'_j \rangle - \langle u'_j h_i h'_k \rangle](\hat{r}, \Delta t, t) = \lambda \frac{\partial^2 \langle h'_i h'_j \rangle}{\partial r_k \partial r_k} - R\langle h_i h'_j \rangle. \quad \text{... (3.7)}$$

Using the relations (cf. Chandrasekhar[1])

$$\langle u_k h_i h'_j \rangle = -\langle u'_k h_i h'_j \rangle, \ \langle u'_j h_i h'_k \rangle = -\langle u_i h_k h'_j \rangle.$$

Eqs. (3.6) and (3.7) become

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + 2\frac{\partial}{\partial r_k}[\langle u'_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] = 2\lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} - 2R\langle h_i h'_j \rangle \quad \text{... (3.8)}$$

and $$\frac{\partial \langle h_i h'_j \rangle}{\partial \Delta t} + \frac{\partial}{\partial r_k}[\langle u'_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle] = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial r_k \partial r_k} - R\langle h_i h'_j \rangle. \quad \text{... (3.9)}$$

Now, we write eqs. (3.8) and (3.9) in spectral form by use of the three dimensional Fourier transforms

$$\langle h_i h'_j \rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \psi_i \psi'_j \rangle(\hat{K}, \Delta t, t) \exp[\hat{i}(\hat{K} \cdot \hat{r})] \, d\hat{K} \quad \text{... (3.10)}$$

and $$\langle u_i h_k h'_j \rangle(\hat{r}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \alpha_i \psi_k \psi'_j \rangle(\hat{K}, \Delta t, t) \exp[\hat{i}(\hat{K} \cdot \hat{r})] \, d\hat{K}. \quad \text{... (3.11)}$$

Interchanging the subscripts i and j and then interchanging the points p and p' gives $\langle u_{i_k} h_i h'_j \rangle(\hat{r}, \Delta t, t) = \langle u_k h_i h'_j \rangle(-\hat{r}, -\Delta t, t + \Delta t)$

$$= \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi_j' \rangle (-\hat{K}, -\Delta t, t+\Delta t) \exp [\hat{i}(\hat{K} \cdot \hat{r}) \, d\hat{K}, \qquad \text{... (3.12)}$$

where $\hat{K}$ is known as a wave-number vector and $d\hat{K} = dK_1 \, dK_2 \, dK_3$. The magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size.

Substituting eqs. (3.10) to (3.12) into eqs. (3.8) and (3.9), one obtains

$$\frac{\partial \langle \psi_i \psi_j' \rangle}{\partial t} + 2\,[\lambda k^2 + R] \langle \psi_i \psi_j' \rangle = 2ik_k \,[\langle \alpha_i \psi_k \psi_j' \rangle (\hat{K} \Delta t, t)$$

$$- \langle \alpha_k \psi_i \psi_j' (-\hat{K}, -\Delta t, t+\Delta t)] \qquad \text{... (3.13)}$$

and

$$\frac{\partial \langle \psi_i \psi_j' \rangle}{\partial \Delta t} + [\lambda k^2 + R] \langle \psi_i \psi_j' \rangle = ik_k \,[\langle \alpha_i \psi_k \psi_j' \rangle (\hat{K} \Delta t, t)$$

$$- \langle \alpha_k \psi_i \psi_j' (-\hat{K}, -\Delta t, t+\Delta t)]. \qquad \text{... (3.14)}$$

The tensors eqs. (3.13) and (3.14) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial \langle \psi_i \psi_i' \rangle}{\partial t} + 2\,[\lambda k^2 + R] \langle \psi_i \psi_i' \rangle = 2ik_k \,[\langle \alpha_i \psi_k \psi_i' \rangle (\hat{K}, \Delta t, t)$$

$$- \langle \alpha_k \psi_i \psi' \rangle_i (-\hat{K}, -\Delta t, t+\Delta t)]. \qquad \text{... (3.16)}$$

4. Three-Point, Three-Time Correlation Equations and Solution For Times Before the Final Period

In the present investigation, under the same assumption as before, we take the momentum equation of MHD turbulence at the point p and induction equations of magnetic field fluctuation of concentration at p' and p'' separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial W}{\partial x_l} + \nu \frac{\partial^2 u_l}{\partial x_k \partial x_k}, \qquad \text{... (4.1)}$$

$$\frac{\partial h_i'}{\partial t'} + u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial u_i'}{\partial x_k'} = \lambda \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} - Rh_i' \qquad \text{... (4.2)}$$

and

$$\frac{\partial h_i''}{\partial t''} + u_k'' \frac{\partial h_j''}{\partial x_k''} - h_k'' \frac{\partial u_i''}{\partial x_k''} = \lambda \frac{\partial^2 h_j''}{\partial x_k'' \partial x_k''} - Rh_i''. \qquad \text{... (4.3)}$$

Multiplying eqs. (4.1) to (4.3) by $h_i' h_j''$, $u_l h_j''$ and $u_l h_i'$ respectively and taking ensemble average, one obtains

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t}+\frac{\partial}{\partial x_k}[\langle u_k u_l h_i' h_j''\rangle-\langle h_k h_l h_i' h_j''\rangle]$$

$$=-\frac{\partial\langle W h_i' h_j''\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k \partial x_k} \quad \ldots (4.4)$$

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t'}+\frac{\partial}{\partial x_k'}[\langle u_l u_k' h_i' h_j''\rangle-\langle u_l u_i' h_k' h_j''\rangle]$$

$$=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k \partial x_k}-R\langle u_l h_i' h_j''\rangle \quad \ldots (4.5)$$

and

$$\frac{\partial\langle u_l h_i' h_j''\rangle}{\partial t''}+\frac{\partial}{\partial x_k''}[\langle u_l u_k'' h_i' h_j''\rangle-\langle u_l u_i' h_i' h_k''\rangle]$$

$$=\lambda\frac{\partial^2\langle u_l h_i' h_j''\rangle}{\partial x_k'' \partial x_k''}-R\langle u_l h_i' h_j''\rangle. \quad \ldots (4.6)$$

If we use the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\ \frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\ \frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'}$$

$$\left(\frac{\partial}{\partial t}\right)t',t''=\left(\frac{\partial}{\partial t}\right)\Delta t,\Delta t'-\frac{\partial}{\partial\Delta t}-\frac{\partial}{\partial\Delta t'},\ \frac{\partial}{\partial t'}=\frac{\partial}{\partial\Delta t},\ \frac{\partial}{\partial t''}=\frac{\partial}{\partial\Delta t'}$$

and the six dimensional Fourier transforms of the type

$$\langle u_l h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\phi_l\beta_i'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t')$$

$$\exp[\hat{i}(\hat{K}\cdot\hat{r}+\hat{k}\cdot\hat{r})]\,d\hat{K}\,d\hat{K}', \quad \ldots (4.7)$$

$$\langle u_l u_k' h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\phi_l\phi_k'\beta_i'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t')$$

$$\exp[\hat{i}(\hat{K}\cdot\hat{r}+\hat{k}\cdot\hat{r})]\,d\hat{K}\,d\hat{K}' \quad \ldots (4.8)$$

and

$$\langle W h_i' h_j''\rangle(\hat{r},\hat{r}',\Delta t,\Delta t',t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\langle\gamma\beta_i'\beta_j''\rangle(\hat{K},\hat{K}',\Delta t,\Delta t')$$

$$\exp[\hat{i}(\hat{K}\cdot\hat{r}+\hat{K}'\cdot\hat{r}')]\,d\hat{K}\,d\hat{K}' \quad \ldots (4.9)$$

into eqs. (4.4) to (4.6), we have

$$\frac{\partial}{\partial t}\langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda\left[(1+P_M)(k^2+k'^2)\, 2P_M kk' + \frac{2R}{\lambda}\right]$$

$$\times \langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0, \qquad \text{... (4.10)}$$

$$\frac{\partial}{\partial \Delta t}\langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda\left[k^2 + \frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_j'' \rangle$$

$$(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \qquad \text{... (4.11)}$$

and

$$\frac{\partial}{\partial \Delta t'}\langle \phi_l \beta_i' \beta_j'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda\left[k'^2 + \frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_j'' \rangle$$

$$(\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0, \qquad \text{... (4.12)}$$

where, three-point correlation equation are considered and the quadruple correlation are neglected (as they decay faster than the lower-order correlation terms). The term $\langle \gamma \beta_i' \beta_j'' \rangle$ associated with the pressure correlation are also neglected because it is related to the quadruple correlation's of Sarker and Kishore[9].

The tensor eqs. (4.10) to (4.12) can be converted to scalar equations by contraction of the indices i and j and inner multiplication by k_l

$$\frac{\partial}{\partial t} k_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda\,[(1+P_M)(k^2+k'^2) + 2P_M kk' + 2R/\lambda]$$

$$\times \langle \phi_l \beta_i' \phi_{''} \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0, \qquad \text{... (4.10}a\text{)}$$

$$\frac{\partial}{\partial \Delta t} k_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda\,[k^2 + R/\lambda]\langle \phi_l \beta_i' \beta_i'' \rangle$$

$$(\hat{K}, \hat{K}', \Delta t, \Delta t'\, t) = 0 \qquad \text{... (4.11}a\text{)}$$

and

$$\frac{\partial}{\partial \Delta t'} k_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda\left[k'^2 + \frac{R}{\lambda}\right]$$

$$\langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0. \qquad \text{... (4.12}a\text{)}$$

Integrating eqs. (4.10a) to (4.12a) between t_0 to t, we obtain

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = f_1 \exp\left\{ -\lambda\left[(1+P_M)(k^2+k'^2) + 2P_M kk' \cos\theta + \frac{2R}{\lambda}\right](t-t_0)\right\},$$

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = g_l \exp\left[-\lambda\left(k^2 + \frac{R}{\lambda}\right)\Delta t\right]$$

and

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = q_1 \exp\left[-\lambda\left(k'^2 + \frac{R}{\lambda}\right)\Delta t'\right].$$

For these relations to be consistent, we have

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = k_1 \langle \phi_l \beta_i' \beta_i'' \rangle_0 \exp \{-\lambda [(1+P_M)(k^2+k'^2)(t-t_0)$$

$$+ k^2 \Delta t + k'^2 \Delta t' + 2P_M kk' \cos\xi (t-t_0) + \frac{2R}{\lambda}\left(t-t_0+\frac{\Delta t+\Delta t'}{2}\right)\Bigg] \quad \text{... (4.13)}$$

where $\langle \phi_l \beta_i' \beta_i'' \rangle_0$ is the value of $\langle \phi \beta_i' \beta_i'' \rangle$ at $t=t_0, \Delta t = \Delta t' = 0$ and ξ is the angle between $\hat{K}, \hat{K}'$. By letting $\hat{r}' = 0, \Delta t' = 0$ in the eq. (4.10) and comparing with eqs. (3.11) and (3.12), we obtain the relations

$$\langle \alpha_i \psi_k \psi_i' \rangle (\hat{K}, \Delta t, t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, 0, t)\, d\hat{K}' \quad \text{... (4.14)}$$

and

$$\langle \alpha_k \psi_i \psi_i' \rangle (-\hat{K}, -\Delta t, t+\Delta t) = \int_{-\infty}^{\infty} \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}', -\Delta t, 0, t)\, d\hat{K}'. \quad \text{... (4.15)}$$

Substituting eqs. (4.13) to (4.15) into eq. (3.15), one obtains

$$\frac{\partial}{\partial t} \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda \left[k^2 + \frac{R}{\lambda} \right] \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t)$$

$$= \int_{-\infty}^{\infty} 2ik_l [\langle \phi_l \beta_i' \beta_i'' \rangle - \langle \phi_l \beta_i' \beta_i'' \rangle]_0 \exp [-\lambda \{(1+P_M)(k^2+k'^2)(t-t_0)$$

$$+ k^2 \Delta t + 2P_M (t-t_0) kk' \cos\xi\, d(\cos\xi) + \frac{2R}{\lambda}(t-t_0+\Delta t\}]\, dk' \quad \text{... (4.16)}$$

and

$$\frac{\partial}{\partial t} \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t) + 2\lambda [k^2 + R/\lambda] \langle \psi_i \psi_i' \rangle (\hat{K}, \Delta t, t)$$

$$= \int_{-\infty}^{\infty} 2ik_l [\langle \phi_; \beta_i' \beta_i'' \rangle - \langle \phi_l \beta_i' \beta_i'' \rangle]_0$$

$$\times \left[\int_{-1}^{1} \exp [-\lambda \{(1+P_M)(k^2+k'^2)(t-t_0) + k^2 \Delta t + 2P_M (t-t_0) kk' \cos\xi \right.$$

$$+ 2R/\lambda (t-t_0+\Delta t/2]\}\, d(\cos\xi)]\, dk' \quad \text{... (4.17)}$$

where $d\hat{K}'$ is written in terms of k' and ξ as $-2\pi k'^2 d(\cos\xi)\, dk'$ (cf. Deissler[4]) and the quantity $[\langle \phi_l \beta_i', \beta_i'' \rangle (\hat{K}, \hat{K}') - \langle \phi_l \beta_i' \beta_i'' \rangle (-\hat{K}, -\hat{K}')]_0$ depends on the initial condition of the turbulence.

In order to make further calculation it is necessary to assume a relation which gives $ik_l\,[\langle \phi_l\,\beta_i^{'}\,\beta_i^{''}\rangle(\hat{K},\hat{K}') - \langle \phi_l\,\beta_i^{'}\,\beta_i^{''}\rangle(-\hat{K},-\hat{K}')]_0$ as a function of k and k' (cf. Loeffer and Deissler[7]). The relation assumed here is

$$(2\,\pi)^2\, ik_l\,[\langle \phi_l\,\beta_i^{'}\,\beta_i^{''}\rangle(\hat{K},\hat{K}') - \langle \phi_l\,\beta_i^{'}\,\beta_i^{''}\rangle(-\hat{K},-\hat{K}')]_0 = -\,\delta_0^{\cdot}\,[k^2\,k'^4 - k^4\,k'^2], \quad \dots (4.18)$$

where δ_0 is a constant depending on the initial conditions.

Substituting eq. (4.18) in eq. (4.17), and multiplying both sides by k^2 terms of the magnetic energy spectrum function $E = 2\,\pi\,k^2\,\langle\,\psi_i\,\psi_j^{'}\rangle$, we get

$$\frac{\partial E}{\partial t} + 2\,\lambda\,k^2\,E = G, \quad \dots (4.19)$$

where G is the magnetic energy transfer term and is given by

$$G = -\,2\,\delta_0 \int_0^{\infty} (k^2\,k'^4 - k^4\,k'^2)\,k^2\,k'^2 \left[\int_{-1}^{1} \exp\;\{-\,\lambda\,[(1+P_M)\,(k^2+k'^2)\,(t-t_0) + k^2\,\Delta\,t + 2P_M\,(t-t_0)\,kk'\cos\xi\,\frac{2R}{\lambda}\,(t-t_0+\Delta\,t/2)]\}\;d\,(\cos\xi)\right] dk'. \quad \dots (4.20)$$

Integrating eq. (4.20) with respect to cos ξ and k', we have

$$G = -\frac{\delta_0\,P_M\,\sqrt{\pi}}{4\,\lambda^{3/2}\,(t-t_0^{3/2}\,(1+P_M)^{5/2}}\exp\left[-\,k^2\,\lambda\left(\frac{1+2\,P_M}{1+P_M}\right)\left(t-t_0+\frac{1+P_M}{1+2\,P_M}\,\Delta\,t\right) - 2R\,(t-t_0+\Delta\,t/2)\right]\left[\frac{15\,k^4}{4P_M^2\,(t-t_0)^2\,\lambda^2}\left(\frac{P_M}{1+P_M}\right) + \left\{5\left(\frac{P_M}{1+P_M}\right)^2 - \frac{3}{2}\right\}\frac{k^6}{P_M\,\lambda\,(t-t_0)} + \left\{\left(\frac{P_M}{1+P_M}\right)^3 - \frac{P_M}{1+P_M}\right\}k^8\right] - \frac{\delta_0\,P_M\,\sqrt{\pi}}{4\,\lambda^{3/2}\,(t-t_0+\Delta\,t)^{3/2}\,(1+P_M)^{5/2}}$$

$$\times\exp\left[-\,k^2\,\lambda\left(\frac{1+2\,P_M}{1+P_M}\right)\left(t-t_0+\frac{P_M}{1+P_M}\,\Delta\,t\right) - 2R\,(t-t_0+\Delta\,t/2)\right]\left[\frac{15\,k^4}{4\,\lambda^2\,P_M^2\,(t-t_0+\Delta\,t)}\left(\frac{P_M}{1+P_M}\right) + \left\{5\left(\frac{P_M}{1+P_M}\right)^2 - \frac{3}{2}\right\}\frac{k^6}{P_m\,\lambda\,(t-t_0+\Delta\,t)} + \left\{\left(\frac{P_M}{1+P_M}\right)^3 - \frac{P_M}{1+P_M}\right\}k^8\right]. \quad \dots (4.21)$$

The series of eq. (4.18) contains only even power of k and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times. If we put $\Delta t = 0$, $R = 0$, eq. (4.21) reduces to eq. (4.9) of Sarker and Kishore[9].

If we integrate eq. (4.21) for $\Delta t = 0$ over all wave number, we find that

$$\int_0^\infty G \cdot d\hat{k} = 0, \qquad \text{... (4.22)}$$

which indicating that the expression for G satisfying the conditions of continuity and homogeneity. Physically it was to be expected as G is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

For obtaining the magnetic energy spectrum function G eq. (4.19) can be written in integral form as

$$E = \exp[-2\lambda k^2 (t - t_0 + \Delta t/2)] \int G \exp[2\lambda (k^2 + R/\lambda)(t - t_{n0} + \Delta t/2]\, dt$$

$$+ J(k) \exp[-2\lambda (k^2 + R/\lambda)(t - t_0 + \Delta t/2)], \qquad \text{... (4.23)}$$

where $J(k) = \dfrac{N_0 k^2}{\pi}$ is a constant of integration and can be obtained as by Corrsin[2].

Substituting the value of G as given by eq. (4.21) into eq. (4.23), gives the equation

$$E = \frac{N_0 k^2}{\pi} \exp[-2\lambda (k^2 + R/\lambda)(t - t_0 + \Delta t/2] + \frac{\delta_0 P_M \sqrt{\pi}}{4\lambda^{3/2}(1 + P_M)^{7/2}}$$

$$\times \exp\left[-k^2 \lambda \left(\frac{1 + 2P_M}{1 + P_M}\right)\left((t - t_0 + \frac{1 + P_M}{1 + 2P_M}\Delta t\right) - 2R(t - t_0 + \Delta t/2)\right]$$

$$\left[\frac{3k^4}{2P_M (t - t_0)^{5/2}\lambda^2} + \frac{(7P_M - 6)k^6}{3\lambda(1 + P_M)(t - t_0)^{3/2}} - \frac{4(3P_M^2 - 2P_M + 3)}{3)1 + P_M)^2 (t - t_0)^{1/2}}\right.$$

$$\left. + \frac{8\lambda^{1/2}(3P_M^2 - 2P_M + 3)}{3(1 + P_M)^{5/2}} k^9 F(\omega) + \frac{\delta_0 \sqrt{\pi} P_M}{4\lambda^{3/2}(1 + P_M)}\right]$$

$$\exp\left[-\lambda k^2 \left(\frac{1 + 2P_M}{1 + P_M}\right)\left(t - t_0 + \frac{P_M}{1 + P_M}\Delta t\right) - 2R(t - t_0 + \Delta t/2)\right]$$

$$+ \left[\frac{3k^4}{2\lambda^2 P_M (t - t_0 + \Delta t)^{5/2}} + \frac{(7P_M - 6)k^6}{3\lambda(1 + P_M)(t - t_0 + \Delta t)^{3/2}} + \frac{4(3P_M^2 - 2P_M + 3)}{3(1 + P_M)^2 (t - t_0 + \Delta t)^{1/2}}\right.$$

$$\left. + \frac{8\lambda^{1/2}(3P_M^2 - 2P_M + 3)}{(1 + P_M)^{5/2}} k^9 F(\omega)\right], \qquad \text{... (4.24)}$$

where $$F(\omega) = e^{-\omega^2} \int_0^{\omega} e^{x^2}\, dx, \quad \omega = k\sqrt{\frac{\lambda(t-t_0)}{1+P_M}} \text{ or } k\sqrt{\frac{\lambda(t-t_0+\Delta t)}{1+P_M}}.$$

The expression for the magnetic energy decay is obtained from eq. (3.10) by setting $\hat{r} = 0$, $j = i$

$$d\hat{K} = -2\pi k^2\, d(\cos\xi)\, dk \text{ and } E = 2\pi k^2 \langle \psi_i \psi_j' \rangle \text{ as}$$

$$\frac{\langle h_i h_j' \rangle}{2} = \int_0^{\infty} E dk. \qquad \text{... (4.25)}$$

Substituting eq. (4.24) into eq. (4.25) and after integration, one can obtain

$$\frac{\langle h_i h_j' \rangle}{2} = \frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}\,(T+\Delta T/2)^{3/2}} \exp[-2R(T+\Delta t/2)]$$

$$+ \frac{\pi \delta_0}{4\lambda^2 (1+P_M)(1+2P_M)^{5/2}}$$

$$\exp[-2R(T+\Delta t/2)] \left[\frac{9}{16 T^{5/2}\left(T + \frac{1+P_M}{1+2P_M}\Delta T\right)^{5/2}} \right.$$

$$+ \frac{9}{16(T+\Delta T)^{5/2}\left(T + \frac{P_M}{1+P_M}\Delta T\right)^{5/2}}$$

$$+ \frac{35 P_M (3P_M^2 - 2P_M + 3)}{8(1+2P_M)\, T^{1/2}\left(T + \frac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}} + \frac{35 P_M (3P_M^2 - 2P_M + 3)}{8(1+2P_M)(T+\Delta T)^{1/2}\left(T + \frac{P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$+ \frac{8 P_M (3P_M^2 - 2P_M + 3)(1+2P_M)^{5/2}}{3.2^{23/2}(1+P_M)^{11/2}} \Sigma \frac{1\cdot 3\cdot 5\cdot \ldots (2n+9)}{n!\,(2n+1)\, 2^{2n}(1+P_M)^n}$$

$$\left. \times \left\{ \frac{T^{(2n+1)/2}}{(T+\Delta T)^{(2n+1)/2}} + \frac{(T+\Delta T)^{(2n+1)/2}}{(T+\Delta T/2)^{(2n+1)/2}} \right\} \right], \qquad \text{... (4.26)}$$

where $T = t - t_0$.

Thus the decay law for magnetic energy fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction before the final period may be written as

$$\frac{\langle h^2 \rangle}{2} = \exp[-2RT_m]\Bigg[\frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}\,T_m^{3/2}} + \frac{\pi\,\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}} \times$$

$$\Bigg[\frac{9}{16(T_m-\Delta T/2)^{5/2}\left(T_m+\dfrac{\Delta T}{1+2P_M}\right)^{5/2}} + \frac{9}{16(T_m+\Delta T/2)^{5/2}\left(T_m-\dfrac{\Delta T}{2(1+P_M)}\right)^{5/2}}$$

$$\frac{5P_M(7P_M-6)}{16(1+2P_M)(T_m-\Delta T/2)^{3/2}\left(T_m+\dfrac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)(T_m+\Delta T/2)^{3/2}\left(T_m\dfrac{\Delta T}{2(1+2P_M)}\right)^{7/2}} + \cdots\Bigg]\Bigg], \quad \ldots (4.27)$$

where $T_m = T + \Delta T/2$,

which is analogous to the eq. (43) of Kumer and Patel[5&6].

If we put $\Delta T = 0$, $R = 0$, we can easily find-out that

$$\frac{\langle h^2 \rangle}{2} = \frac{N_0}{8\lambda^{3/2}\sqrt{2\pi}}T^{-3/2} + \frac{\pi\,\delta_0}{4\lambda^6(1+P_M)(1+2P_M)}T^{-5}\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+\cdots\right\}$$

$$= \frac{N_0}{8\sqrt{2\pi}\,\lambda^{3/2}}T^{-3/2} + \frac{\delta_0 S}{2\lambda^6}T^{-5} \quad \ldots (4.28)$$

where $S = \dfrac{\pi}{(1+P_M)(1+2P_M)}\left\{\dfrac{9}{16}+\dfrac{5}{16}\dfrac{P_M(7P_M-6)}{1+2P_M}+\cdots\right\}$,

which is obtained by Sarker and Kishore[9] eq. (4.17). In the absence of chemical reaction, i.e., if we put $R = 0$, the decay low for magnetic energy fluctuation before the final period is completely same as with the result obtained earlier by Sarker and Islam[10].

5. Concluding Remarks

The concentration fluctuation decays with time in a natural manner in the pure mixing case. This study shows that the chemical reaction ($R \neq 0$) in the MHD turbulence causes the concentration fluctuations to decay more they would for pure mixing and it is governed by $\exp[-R(t-t_0)]$.

A lot of experimental data exists for the case of pure mixing. Since any experimental data is not available concerning the chemical reactant in MHD turbulence, the authors have been avoided the numerical work for this study.

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