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REFLECTION AND REFRACTION OF MICROPOLAR THERMOELASTIC WAVES AT A LIQUID-SOLID INTERFACE

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The present investigation is concerned with the reflection and refraction of micropolar thermoelastic waves at an interface between a liquid half-space and a micropolar generalized thermoelastic solid half-space. The numerical results are calculated in terms of amplitude ratios for water/aluminium-epoxy composite model for L-S (Lord and Shulman) and G-L (Green and Lindsay) theories. The comparison between these theories reveals the effect of second thermal relaxation time taken by Green and Lindsay. The results are also compared with those without thermal effect.

Key Words : Reflection; Refraction; Micropolar Generalized Thermoelastic Solid; Amplitude Ratios

INTRODUCTION

Jeferrey's[1] and Gutenberg[2] considered the reflection of elastic plane waves at a solid half-space. Chadwick and Sneddon[3] and Lockett[4] studied the propagation of thermoelastic plane waves. Knot[5] derived the general equations for reflection and refraction at plane boundary.

In classical dynamical coupled theory of thermoelasticity, the thermal and mechanical waves propagate with an infinite velocity, which is not physically admissible. To overcome this contradiction, the coupled theory of thermoelasticity has been extended by including the thermal relaxation time in constitutive relations by Lord and Shulman[6] and Green and Lindsay[7]. Some problems on relfection in thermoelastic solid have been discussed by Deresiewicz[8], Sinha and Sinha[9] and Sharma[10].

A theory of micropolar continua was proposed by Eringen and Suhubi[11] and Eringen[12] to explain the continuum behaviour of materials possessing microstructure. The propagation of plane waves in an infinite micropolar elastic solid has been discussed by Parfitt and Eringen[13], Ariman[14] and Smith[15]. Parfitt and Eringen[13] have shown that four basic waves (a longitudinal displacement wave, two sets of coupled waves and a longitudinal microrotational wave) propagate in an infinite micropolar elastic solid.

The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effect by Eringen[16] and Nowacki[17] and is known as micropolar coupled thermoelasticity. Dost and Tabarrok[18] have presented the generalized micropolar thermoelasticity by using Green - Lindsay theory. Kumar and Singh[19] have also presented the generalized micropolar thermoelasticity with stretch by using Lord-Shulman and green-Lindsay

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theories. Wave propagation in a micropolar generalized thermoelastic body with stretch has been studied by Kumar and Singh[19]. Singh and Kumar[20, 21] have discussed a some problems on reflection of plane waves from flat boundary of a micropolar generalized thermoelastic half-space. Singh and Kumar[22] have also proposed a generalized thermo microstretch elastic solid and have discussed the reflection of plane waves from the free surface of a generalized thermo-microstretch elastic solid.

In the present paper, a problem of reflection and refraction of micropolar thermoelastic waves have been studied at an interface between a thermally conducting liquid and a micropolar generalized thermoelastic solid half-spaces.

FORMULATION OF THE PROBLEM

We consider a homogeneous micropolar generalized thermoelastic solid and thermally conducting liquid which occupy lower and upper half-spaces respectively. We assume that heat sources, external force loading and body forces are absent and consider a fixed rectangular cartesian coordinate system (x, y, z). We consider that the two semi-infinite media are in contact at a plane interface ($z = 0$) and suppose that the plane longitudinal displacement wave impinges on the interface from below which we take as first medium, the positive z-axis lying inside the solid half-space. We take the plane wave motion in the xz-plane (i.e. $\partial/\partial y = 0$). The complete geometry of the problem has been shown in Fig. 1. Following Eringen[16], Lord and Shulman[6] and Green and Lindsay[7], the constitutive and field equations of micropolar generalized thermoelastic solid without body forces and body couples are

$$\sigma_{kl} = \lambda u_{r,r}\,\delta_{kl} + \mu(u_{k,l} + u_{l,k}) + \kappa(u_{l,k} - \varepsilon_{klr}\,\phi_r) - \nu(\theta + t_1\dot{\theta})\,\delta_{kl} \quad \text{... (1)}$$

$$m_{kl} = \alpha\,\phi_{r,r}\,\delta_{kl} + \beta\,\phi_{k,l} + \gamma\phi_{l,k} \quad \text{... (2)}$$

$$(c_1^2 + c_3^2)\,\nabla(\nabla\cdot\boldsymbol{u}) - (c_2^2 + c_3^2)\,\nabla\times(\nabla\times\boldsymbol{u}) + c_3^2\,\nabla\times\boldsymbol{\phi} - \bar{\nu}\,\nabla(\theta + t_1\dot{\theta}) = \ddot{\boldsymbol{u}} \quad \text{... (3)}$$

$$(c_4^2 + c_5^2)\,\nabla(\nabla\cdot\boldsymbol{\phi}) - c_4^2\,\nabla\times(\nabla\times\boldsymbol{\phi}) + \omega_0\nabla\times\boldsymbol{u} - 2\,\omega_0^2\,\boldsymbol{\phi} = \ddot{\boldsymbol{\phi}}, \quad \text{... (4)}$$

$$\rho C^*(\theta + t_0\dot{\theta}) + \nu\,\theta_0[\dot{u}_{i,i} + \Delta\, t_0\,\ddot{u}_{i,i}] = K^*\,\nabla^2\theta, \quad \text{... (5)}$$

where $c_1^2 = (\lambda + 2\mu)/\rho,\ c_2^2 = \mu/\rho,\ c_3^2 = \kappa/\rho,$

$$c_4^2 = \gamma/\rho j,\ c_5^2 = (\alpha + \beta)/\rho j,\ \omega_0^2 = c_3^2/j = \kappa/\rho j,$$

$$\nu = (3\lambda + 2\mu + \kappa)\,\alpha_t,\ \bar{\nu} = \nu/\rho \quad \text{... (6)}$$

where symbols $\lambda, \mu, \kappa, \alpha, \beta, \gamma$ are material constants, ρ the density, j the rotational inertia, K^* the coefficient of thermal conductivity, α_t the coefficient of linear expansion. $\boldsymbol{u}$ and $\boldsymbol{\phi}$ are displacement vector and microrotation vector respectively. Superposed dots stand for derivatives with respect to time. δ_{kl} is the kronecker delta.

For the L-S (Lord-Shulman) theory $t_1 = 0, \Delta = 1$ and for G-L (Green-Lindsay) theory $t_1 > 0$ and $\Delta = 0$. The thermal relaxations t_0 and t_1 satisfy the inequality $t_1 \geq t_0 \geq 0$ for the G-L theory only.

We define the angle of incidence (I) as the angle between the propagation of plane longitudinal displacement wave and normal to the boundary of the micropolar generalized thermoelastic medium.

SOLUTION OF THE PROBLEM

To solve the problem in micropolar generalized thermoelastic medium, we decompose the displacement and microrotation vectors as

$$u = \nabla \phi + \nabla_X U, \;\; \nabla \cdot U = \mathbf{0}, \qquad \text{... (7)}$$

$$\phi = \nabla \xi + \nabla_X \boldsymbol{\Phi}, \;\; \nabla \cdot \boldsymbol{\Phi} = 0, \qquad \text{... (8)}$$

Using eqs. (7) and (8), eqs. (3) to (5) reduce as

$$(c_1^2 + c_3^2)\, \nabla^2 \phi = \ddot{\phi} + \bar{v}\,(\theta + t_1\, \dot{\theta}), \qquad \text{... (9)}$$

$$(c_2^2 + c_3^2)\, \nabla^2 U + c_3^2\, \nabla \times \boldsymbol{\Phi} = \ddot{U}, \qquad \text{... (10)}$$

$$c_4^2\, \nabla^2 \boldsymbol{\Phi} - 2\, \omega_0^2\, \boldsymbol{\Phi} + \omega_0^2\, \nabla \times U = \ddot{\boldsymbol{\Phi}} \qquad \text{... (11)}$$

$$(c_4^2 + c_5^2)\, \nabla^2 \xi - 2\, \omega_0^2\, \xi = \ddot{\xi} \qquad \text{... (12)}$$

From eqs. (9) to (12), we see that the longitudinal displacement wave (LD wave) is affected due to the thermal wave, the coupled transverse and microrotational waves (CD I and CD II waves) and longitudinal microrotational wave (LM wave) remain unaffected.

From eq. (9), we have

$$\theta = (V_1^2\, \nabla^2 \phi - \ddot{\phi})/\bar{\gamma}, \qquad \text{... (13)}$$

where $V_1^2 = c_1^2 + c_3^2,\;\; \bar{\gamma} = \bar{v}\,[1 + t_1\,(\partial/\partial t)].$ ... (14)

Eliminating θ from eqs. (5) and (13), we get

$$\nabla^4 \phi - \left[\frac{C^*}{\bar{K}^*} \left\{ \left(1 + t_0 \frac{\partial}{\partial t} \right) + \varepsilon \left(1 + t_1 \frac{\partial}{\partial t} \right) \left(1 + \Delta\, t_0 \frac{\partial}{\partial t} \right) \right\} + \frac{1}{V_1^2} \frac{\partial}{\partial t} \right] \frac{\partial}{\partial t} \nabla^2 \phi + \frac{C^*}{\bar{K}^*} \frac{1}{V_1^2} \left(1 + t_0 \frac{\partial}{\partial t} \right) (\partial^3 \phi / \partial t^3) = 0, \qquad \text{... (15)}$$

where $\bar{K}^*\, K^*/\rho,\;\; \varepsilon = \bar{v}^2\, \theta_0 / V_1^2\, C^*.$ ... (16)

We assume the solution of eq. (15) in the form

$$\phi = f(z) \exp [ik(ct - x)],\ (c > V_1) \qquad \text{... (17)}$$

With the help of eq. (17), eq. (15) reduces to

$$\frac{d^4 f(z)}{dz^4} + A \frac{d^2 f(z)}{dz^2} + B f(z) = 0, \qquad \text{... (18)}$$

where

$$A = k^2 \left(\frac{c^2}{V_1^2} - 2 \right) - kc\,(C^*/\overline{K}^*)\,[(i - t_0 kc) + \varepsilon (i - t_1 kc)(1 + ikc\, t_0 \Delta)], \qquad \text{.... (19)}$$

$$B = k^4 \left(1 - \frac{c^2}{V_1^2} \right) + ck^3 (C^*/\overline{K}^*)\,[(i - t_0 kc) + \varepsilon (i - t_1 kc)(1 + i\,kc\, t_0 \Delta)$$

$$- \frac{c^2}{V_1^2} (i - t_0 kc) \Bigg]. \qquad \text{... (20)}$$

The solution of eq. (18) is of the form

$$f(z) = A_1 \exp (m_1 z) + A_2 \exp (-m_1 z) + A_3 \exp (m_2 z) + A_4 \exp (-m_2 z), \qquad \text{... (21)}$$

where

$$m_1 = [\{(A^2 - 4B)^{1/2} - A\}/2]^{1/2}, \qquad \text{... (22)}$$

$$m_2 = [- \{(A^2 - 4B)^{1/2} + A,\}/2]^{1/2}$$

correspond to the thermal and modified LD waves respectively and A_1, A_2, A_3, A_4 are arbitrary constants.

Making use of eq. (8) in eq. (10), we get

$$\phi_2 = \frac{1}{c_3^2} \frac{\partial^2 \psi}{\partial t^2} - \left(\frac{c_2^2}{c_3^2} + 1 \right) \nabla^2 \psi, \qquad \text{... (24)}$$

where

$$\psi = (-U)_y,\ \phi_2 = (-\phi)_y = \frac{\partial \Phi_x}{\partial z} - \frac{\partial \Phi_x}{\partial x}. \qquad \text{... (25)}$$

Using eq. (24) in eq. (11) and then the final solution is in the form

$$\psi = [A_5 \exp (m_3 z) + A_6 \exp (-m_3 z) + A_7 \exp (m_4 z)$$

$$+ A_8 \exp (-m_4 z] [\exp i\, \omega t - kx)], \qquad \text{... (26)}$$

where

$$m_3 = \{(1/2)\,[(C^2 - 4D)^{1/2} - C]\}^{1/2}$$

and

$$m_4 = \{(-1/2)\ [(C^2 - 4D)^{1/2} + C]\}^{1/2}, \qquad \text{... (27)}$$

correspond to coupled transverse microrotational waves and coupled transverse displacement waves (CDI- and CDII-waves) and

$$C = k^2\left(\frac{c^2}{c_2^2+c_3^2}+\frac{c^2}{c_4^2}-2\right)+\frac{\omega_0^2}{c_4^2}\left(\frac{c_3^2}{c_2^2+c_3^2}-2\right), \quad \text{... (28)}$$

$$D = k^4\left(1-\frac{c^2}{c_2^2+c_3^2}-\frac{c^2}{c_4^2}+\frac{c^4}{c_4^2(c_2^2+c_3^2)}\right)-k^2\frac{\omega_0^2}{c_4^2}\left(\frac{c_3^2}{c_2^2+c_3^2}+\frac{2c^2}{c_2^2+c_3^2}-2\right), \quad \text{... (29)}$$

and A_5, A_6, A_7, A_8 are arbitrary constants.

If we assume $\mu=\kappa=\alpha=\beta=\gamma=0$, we see that the longitudinal wave in a thermally conducting liquid medium is affected due to the presence of a thermal wave. In this case, there is no existence for other waves. We consider the variables with dashes in the liquid medium.

The appropriate potentials for two media will now be {dropping the exponential term $ik(ct - x)$}

$$\phi = B_0 \exp(m_2 z) + B_1 \exp(-m_1 z) + B_2 \exp(-m_2 z), \quad \text{... (30)}$$

$$\theta = (1/\bar{\gamma}_0)\,[b_2 B_0 \exp(m_2 z) + b_1 B_1 \exp(-m_1 z) + b_2 B_2 \exp(-m_2 z)] \quad \text{... (31)}$$

$$\psi = B_3 \exp(-m_3 z) + B_4 \exp(-m_4 z), \quad \text{... (32)}$$

$$\phi_2 = b_3 B_3 \exp(-m_3 z) + b_4 B_4 \exp(-m_4 z), \quad \text{... (33)}$$

$$\phi' = B_5 \exp(m_1' z) + B_6 \exp(m_2' z) \quad \text{... (34)}$$

and

$$\theta' = (1/\bar{\gamma}_0')\,[b_1' B_5 \exp(m_1' z) + b_2' B_6 \exp(m_2' z)], \quad \text{... (35)}$$

where B_i (i = 0, 1, 2, ..., 6) are arbitrary constants, and

$$b_{1,2} = k^2(c^2 - V_1^2) + m_{1,2}^2 V_1^2, \quad \text{... (36)}$$

$$b_{1,2}' = k^2(c^2 - \alpha_1^2) + m_{1,2}'^2 \alpha_1^2, \quad \text{... (37)}$$

$$b_{3,4} = k^2\left\{(1 + (c_2^2/c_3^2) - (c^2/c_3^2)\right\} - m_{3,4}^2\{1 + (c_2^2/c_3^2)\}, \quad \text{... (38)}$$

and

$$\bar{\gamma}_0 = \bar{v}(1 + i\omega t_1),\ \bar{\gamma}_0' = \bar{V}'(1 + i\omega t_1'), \quad \text{... (39)}$$

where α_1 is velocity of sound wave and m_1' and m_2' correspond to thermal wave and modified longitudinal wave in liquid medium and are obtained from equations (22) and (23), if we let $\mu=\kappa=\alpha=\beta=\gamma=0$.

Here we assume that the boundary conditions at the interface z = 0 are independent of x and t, so the values of the phae velocity and wave number in ϕ, ψ, θ, ϕ_2 must be same as those in

ϕ' and θ'. We consider the continuity of stresses and displacements at the interface $z = 0$ as

$$\sigma_{zz} = \sigma'_{zz},\ \sigma_{zz} = 0,\ u_3 = u'_3,$$

$$m_{zy} = 0,\ \theta = \theta',\ K^* (\partial \theta/\partial z) = K^{*'} (\partial \theta'/\partial z). \quad \ldots (40)$$

Making use of the potentials given by eqs. (30) to (35) in boundary conditions (40), after using the eqs. (1), (2), (7) and (8), we get a system of six nonhomogeneous equations which can be written as

$$\sum_{j}^{6} a_{ij} Z_j = b_i \ (i = 1, 2, \ldots, 6), \quad \ldots (41)$$

where
$$a_{11} = (\lambda + 2\mu + \kappa) m_1^2 - \lambda k^2 - \rho b_1,\ a_{12} = (\lambda + 2\mu + \kappa) m_2^2 - \lambda k^2 - \rho b_2,$$

$$a_{13} = -i (2\mu + \kappa) m_3 k,\ a_{14} = -i (2\mu + \kappa) m_4 k,$$

$$a_{15} = -\lambda' (m_1'^2 - k^2) + \rho' b'_1,\ a_{16} = -\lambda' (m_2'^2 - k^2) + \rho' b'_2,$$

$$a_{21} = i (2\mu + \kappa) m_1 k,\ a_{22} = i (2\mu + \kappa) m_2 k,\ a_{25} = 0 = a_{26},$$

$$a_{23} = \mu k^2 + (\mu + \kappa) m_3^2 - \kappa b_3,\ a_{24} = \mu k^2 + (\mu + \kappa) m_4^2 - \kappa b_4,$$

$$a_{31} = -m_1,\ a_{32} = -m_2,\ a_{33} = ik = a_{34},\ a_{35} = -m'_1,\ a_{36} = -m'_2,$$

$$a_{41} = a_{42} = a_{45} = a_{46} = 0,\ a_{43} = m_3 b_3,\ a_{44} = m_4 b_4,$$

$$a_{51} = b_1,\ a_{52} = b_2,\ a_{53} = a_{54} = 0,$$

$$a_{55} = -(\bar{\gamma}'_0/\bar{\gamma}_0)\, b'_1,\ a_{56} = -(\bar{\gamma}'_0/\bar{\gamma}_0)\, b'_2,$$

$$a_{61} = m_1 b_1,\ a_{62} = m_2 b_2,\ a_{63} = a_{64} = 0,$$

$$a_{65} = (K^{*'} \gamma_0/K^* \gamma'_0)\, m'_1 b'_1,\ a_{66} = (K^{*'} \gamma_0/K^* \gamma'_0)\, m'_2 b'_2$$

and
$$b_1 = -a_{12},\ b_2 = a_{22},\ b_3 = a_{32},\ b_4 = a_{42},\ b_5 = -a_{52},\ b_6 = a_{62}, \quad \ldots (42)$$

and (Z_j) are the amplitude ratios for various reflected and refracted waves.

NUMERICAL ANALYSIS

To explain the analytical procedure presented earlier, we now consider a numerical example. The results depict the variation of the angle of incidence with the modulus of the amplitude ratios in the context of water-aluminium epoxy composite.

Physical constants for water

$$\rho' = 1.0\, gm/cm^3,\ \alpha_1 = 1.439 \times 10^5 \text{ cm/s},$$

$$K^{*\prime} = 0.144 \text{ cal/cm s}^{o}\text{C},\ C^{*\prime} = 1.0\text{cal/gm}^{o}\text{ C}.$$

Following Gauthier[23], the physical constants for aluminium-epoxy composite

$$\rho = 2.19 \text{ gm/cm}^3,\ \lambda = 7.59 \times 10^{11} \text{ dyne/cm}^2,$$

$$\mu = 1.89 \times 10^{11} \text{ dyne/cm}^2,\ \kappa = 0.0149 \times 10^{11} \text{ dyne/cm}^2,$$

$$\gamma = 0.0268 \times 10^{11} \text{ dyne},\ j = 0.0196 \text{ cm}^2,$$

$$K^* = 0.48 \text{ cal/cm s}^{o}\text{C},\ C^* = 0.206 \text{ cal/gm}^{o}\text{C},$$

$$\theta_0 = 20^{o}\text{C},\ \ \varepsilon = 0.073,\ \omega^2/\omega_0^2 = 200.$$

Nayfeh and Nasser[24] took $t_0 = 3K^*/\rho C^* \alpha^2$. We, therefore, take $t_0' = 3K^{*\prime}/\rho' C^* \alpha_1^2$ and $t_0 = 3K^*/\rho C^* V_1^2$. t_1' and t_1 are considered to be of same order as that of t_0' and t_0.

For the above values of relevant physical constants, the system of equatrions (41) in reduced form for L-S theory, G-L theory and in absence of thermal effect has been solved for amplitude ratios by using the Gauss elimination method for different angle of incidence varying from 0^o to 90^o. The variations of the amplitude ratios for various reflected and refracted waves with the angle of incidence have been shown graphically in figures 2 to 7.

The variations of the amplitude ratios for reflected thermal waves with the angle of incidence have been shown in Fig. 2 for L-S theory and G-L theory by solid line and solid line with centre symbols respectively. The amplitude ratios decrease with the increase in angle of incidence for both of L-S and G-L cases and they attain their minima near I = 72^o. The comparison between these two line curves shows the effect of second thermal relaxation time. Also, if we neglect the thermal effect, these thermal waves will disappear.

The amplitude ratios for reflected longitudinal displacement waves (LD waves) for L-S theory, G-L theory first decrease and then increase to their respective maxima. The variations for these amplitude ratios with the angle of incidence have been depicted in Fig. 3. The solid curve in Fig. 3. represents the variations for L-S theory whereas the solid curve with centre symbols represents the variations for G-L theory. Also, if thermal effect is neglected, then these variations reduce to those shown by dashed line in Fig. 3.

The variations of the amplitude ratios for two sets of reflected coupled waves with the angle of incidence have been shown in Figs 4 and 5. These two sets of coupled waves show the oscillatory variation with the angle of incidence for both L-S and G-L cases. The comparison between solid line and solid line with centre symbols shows the importance of second thermal relaxation time. Also, if thermal effect is neglected, then these variations reduce to those shown by dashed lines in Figs. 4 and 5.

The variations of the refracted thermal waves for L-S theory and G-L theory have been shown in Fig. 6. If we compare the solid line with the solid line with centre symbol, we observe the significance of second thermal relaxation time taken by Green and Lindsay[7].

The amplitude ratios for refracted longitudinal waves for both of the L-S and G-L cases first decrease to their minima and then increases sharply. The variations of these amplitude ratios for L-S case and G-L case have been shown in Fig. 7 by solid line and solid line with centre symbols respectively. The dashed line in Fig. 7. represent the variations of the refracted LD wave with the angle of incidence in absence of thermal effect.

CONCLUSIONS

Details numerical calculations have been presented for the case of micropolar thermoelastic waves incident at the interface of the model considered. The variations of the amplitude ratios for various reflected and refracted waves in G-L case are different from those L-S case. The comparison between the amplitude ratios for L-S case and G-L case reveals the effect of second thermal relaxation time. The results are also compared with the results obtained after neglecting the thermal effect.

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