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HALL EFFECT ON THERMOSOLUTAL INSTABILITY OF RIVLIN-ERICKSEN FLUID WITH VARYING GRAVITY FIELD IN POROUS MEDIUM

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The thermosolutal convection in Rivlin-Ericksen elastico-viscous fluid is considered in the presence of uniform horizontal magnetic field to include Hall currents with varying gravity field. For the case of stationary convection, the Hall currents hasten the onset of convection, the magnetic field postpones the onset of convection, medium permeability also postpones the onset of convection in the presence of the Hall currents, whereas the kinematic viscoelasticity has no effect on the onset of convection. The Hall currents, kinematic viscoelasticity, varying gravity field, the magnetic field, medium permeability and the solute parameter introduce oscillatory modes in the system, which were non-existent in their absence. The case of overstability is also considered wherein the sufficient conditions for the non-existence of overstability are also obtained. The effect of solute gradient, magnetic field, medium permeability and Hall currents have also been shown graphically.

Key Words : Thermosolutal Convection; Rivlin-Ericksen Fluid; Horizontal Magnetic Field; Viscoelasticity; Varying Gravity Field

1. INTRODUCTION

A detailed account of the theoretical and experimental results of the onset of thermal instability (Bénard convection) in an incompressible, viscous (Newtonian) fluid layer, under varying assumption of hydrodynamics and hydromagnetics, has been given in the celebrated monograph by Chandrasekhar[1]. If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current of flow across an electric field in the presence of magnetic field is called Hall effect. The Hall effect is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasmas. Sherman and Sutton[2] have considered the effect of Hall currents on the efficiency of a magneto-fluid-dynamic generator. Gupta[3] studied the thermal instability of fluid in the presence of Hall currents. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis[4]. The physics is quite similar in the stellar case in that Helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of single component fluid and rigid boundaries and therefore, it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The thermosolutal convection problems arise in oceanography, limnology and engineering. In all the above studies the fluid is considered to be Newtonian and gravity field is assumed to be constant.

There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. Two classes of such fluids are Rivlin-Ericksen[5] and Walters[6] (model B') fluid. Walters[6] has proposed the constitutive equations for such elastico-viscous fluids. The mixture of polymethyl methacrylate and pyridine at 25^{o}C containing 30.5 gram of polymer per litre behaves very nearly as the Walters (model B') visco-elastic fluid and which is proposed by Walters[7]. Rivlin and Ericksen[5] have proposed a theoretical model of such another elastico-viscous fluid and other polymers are used in agriculture, communication appliances and in biomedical applications. Joshi[8] has discussed the viscoelastic Rivlin-Ericksen incompressible fluid under time dependent pressure gradient. Srivastava and Singh[9] have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channel of different cross-sections in the presence of time dependent pressure gradient. In another study, Garg *et al.*[10] have studied the rectilinear oscillations of a sphere along its diameter in a conducting dusty Rivlin-Ericksen fluid in the presence of uniform magnetic field. Recently, Sharma *et al.*[11] have studied the Hall effect on the thermal instability of Rivlin-Ericksen fluid. Sharma and Kango[12] have studied the thermal convection in Rivlin-Ericksen elastico-viscous fluid in porous medium in hydromagnetics. In all the above studies, the gravity field is assumed to be constant.

The idealization of uniform gravity field can be hardly justified in the presence of large scale convection phenomenon occurring in atmosphere, the ocean or the mantle of the earth. Pradhan, Samal and Tripathy[13] studied the thermal instability of the fluid layer under variable gravitational field.

When the fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous term in the equations of Rivlin-Ericksen elastico-viscous fluid motion is replaced by $[-(1/k_1)(\mu+\mu'\,\partial/\partial t)\,q]$, where μ and μ' are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context (Mc Donnel[14]). The effect of Hall currents on thermosolutal convection in porous medium is likely to be important in many astrophysical situations and atmospheric physics.

Keeping in mind the importance in geophysics, soil physics, astrophysics, ground-water hydrology and various applications mentioned above, Hall effect on the thermosolutal instability of a Rivlin-Ericksen fluid in porous medium in the presence of varying gravitational field has been considered in the present paper.

2. Formulation of the Problem and Perturbation Equations

Consider the infinite, horizontal, incompressible Rivlin-Ericksen fluid layer of thickness d, heated and soluted from below so that, the temperatures, densities and solute concentrations at the bottom surface $z = 0$ are T_0, ρ_0 and C_0 and at the upper surface $z = d$ are T_d, ρ_d and C_d, respectively, and that a uniform temperature gradient $\beta(=|dT/dz|)$ and uniform solute gradient $\beta'(=|dC/dz|)$ are maintained. A uniform horizontal magnetic field $\mathbf{H}(H,0,0)$ and gravity field $\mathbf{g}(0,0,-g)$, where $g=\lambda g_0$, $(g_0>0)$ is the value of g at $z = 0$ and λ can be positive or negative according to whether gravity increases or decreases upwards from its value g_0; pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

Let $p, \rho, T, C, \alpha, \alpha'$ and $\boldsymbol{q}$ denote, respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion and fluid velocity (initially zero). Then equations expressing the conservation of momentum, mass, temperature, solute mass concentration and equation of state of Rivlin-Ericksen fluid through porous medium are

$$\frac{1}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}\cdot\nabla)\boldsymbol{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\frac{\mu_e}{4\pi\rho_0}[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}]$$

$$+\,\boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\boldsymbol{q}, \quad \text{... (1)}$$

$$\nabla\cdot\boldsymbol{q}=0, \quad \text{... (2)}$$

$$E\frac{\partial T}{\partial t}+(\boldsymbol{q}\cdot\nabla)T=\kappa\nabla^2 T, \quad \text{... (3)}$$

$$E'\frac{\partial C}{\partial t}+(\boldsymbol{q}\cdot\nabla)C=\kappa'\nabla^2 C, \quad \text{... (4)}$$

and

$$\rho=\rho_0[1-\alpha(T-T_0)+\alpha'(C-C_0)], \quad \text{... (5)}$$

where the suffix zero refers to values at the reference level $z = 0$ and in writing eq. (1), use has been made of the Boussinesq approximation. $E=\varepsilon+(1-\varepsilon)(\rho_s c_s/\rho_0 c_i)$ is a constant and E' is a constant analogous to E but corresponding to solute rather than heat. $\rho_s, c_s; \rho_0$ and c_i denote the density and heat capacity of solid (porous matrix) material and fluid, respectively.

Maxwell's equations and the modified Ohm's law yields

$$\varepsilon\frac{d\boldsymbol{H}}{dt}=\nabla+(\boldsymbol{q}\times\boldsymbol{H})+\eta\,\varepsilon\nabla^2\boldsymbol{H}-\frac{\varepsilon c}{4\pi Ne}\nabla\times[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}], \quad \text{... (6)}$$

$$\nabla\cdot\boldsymbol{H}=0, \quad \text{... (7)}$$

where $d/dt=\partial/\partial t+\boldsymbol{q}\cdot\nabla$ stands for the covective derivative and c stands for speed of light.

Here $\mu_e, \eta, \mu, \nu\,(=\mu/\rho_0), \nu', \kappa, \kappa', N$ and e stands for the magnetic permeability, the electrical resistivity, the viscosity, the kinematic viscosity, ν, kinematic viscoelasticity ν', the thermal diffusivity, solute diffusivity, electron number density and charge of an electron, respectively.

The steady state solution is

$$\boldsymbol{q}=(0,0,0),\, T=-\beta z+T_0,\;\; C=-\beta' z+C_0,\;\; \rho=\rho_0(1+\alpha\beta z-\alpha'\beta' z). \quad \text{... (8)}$$

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution and let $\delta p, \delta\rho, \theta, \gamma, \boldsymbol{q}\,(u, v, w)$ and $\boldsymbol{h}\,(h_x, h_y, h_z)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C, velocity $\boldsymbol{q}\,(0,0,0)$ and magnetic field $\boldsymbol{H}\,(H,0,0)$. The change in density $\delta\rho$ caused mainly by the perturbations θ and γ in temperature and solute concentration, is given by

$$\delta\rho=-\rho_0(\alpha\theta-\alpha'\gamma). \quad \text{... (9)}$$

Then the linearized perturbation equations become

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{q}}{\partial t} = -\frac{1}{\rho_0}(\nabla \delta p) + \frac{\mu_e}{4\pi\rho_0}(\nabla \times \boldsymbol{h}) \times \boldsymbol{H} - g_0 \lambda (\alpha \theta - \alpha' \gamma)$$

$$-\frac{1}{k_1}\left(\nu + \nu' \frac{\partial}{\partial t}\right)\boldsymbol{q}, \qquad \dots (10)$$

$$\nabla \cdot \boldsymbol{q} = 0, \qquad \dots (11)$$

$$E\frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta, \qquad \dots (12)$$

$$E'\frac{\partial \gamma}{\partial t} = \beta' w + \kappa' \nabla^2 \gamma, \qquad \dots (13)$$

$$\nabla \cdot \boldsymbol{h} = 0, \qquad \dots (14)$$

and

$$\varepsilon \frac{\partial \boldsymbol{h}}{\partial t} = \nabla \times (\boldsymbol{q} \times \boldsymbol{H}) + \eta \varepsilon \nabla^2 \boldsymbol{h} - \frac{\varepsilon c}{4\pi N e} \nabla \times [(\nabla \times \boldsymbol{h}) \times \boldsymbol{H}]. \qquad \dots (15)$$

3. THE DISPERSION RELATION

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, \theta, \gamma, h_z, \zeta, \xi] = [W(z), \Theta(z), \Gamma(z), K(z), Z(z), X(z)] \exp(ik_x x + ik_y y + nt), \qquad \dots (16)$$

where k_x and k_y are the wave numbers along the x– and y– directions. $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and n is growth rate in general, a complex constant. $\zeta = (\partial v/\partial x - \partial u/\partial y)$ and $\xi = (\partial h_y/\partial x - \partial h_x/\partial y)$ are the z-components of the vorticity and current density respectively.

Eqs. (10)-(15), with the help of eq. (9)and eq. (16), in non-dimensional form become

$$\left[\sigma + \frac{\varepsilon}{P_l}(1 + F\sigma)\right](D^2 - a^2) W + \frac{g_0 \lambda \varepsilon \alpha a^2 d^2}{\nu} \Theta - \frac{g_0 \lambda \varepsilon \alpha' a^2 d^2}{\nu} \Gamma -$$

$$-\frac{ik_x \varepsilon \mu_e H}{4\pi\rho_0 \nu} d^2 (D^2 - a^2) K = 0, \qquad \dots (17)$$

$$\left[\sigma + \frac{\varepsilon}{P_l}(1 + F\sigma)\right] Z = \frac{ik_x \mu_e H \varepsilon d^2}{4\pi\rho_0 \nu} X, \qquad \dots (18)$$

$$(D^2 - a^2 - p_2 \sigma) X = -\frac{ik_x H d^2}{\eta \varepsilon} Z - \frac{ik_x H}{4\pi N e \eta}(D^2 - a^2) K, \qquad \dots (19)$$

$$(D^2 - a^2 - p_2\,\sigma)\,K = -\frac{ik_x Hd^2}{\eta\,\varepsilon}\,W + \frac{ik_x Hd^2}{4\,\pi N e\,\eta}\,X, \qquad \text{... (20)}$$

$$(D^2 - a^2 - Ep_1\,\sigma)\,\Theta = -\frac{\beta d^2}{\kappa}\,W, \qquad \text{... (21)}$$

$$(D^2 - a^2 - E'\,q\,\sigma)\,\Gamma = -\frac{\beta' d^2}{\kappa'}\,W. \qquad \text{... (22)}$$

where we have put $a = kd$, $\sigma = nd^2/\nu$, $F = \nu'/d^2$, $x/d = x^*$, $y/d = y^*$, $z/d = z^*$, $D = d/dz^*$ and

$P_l = k_1/d^2$ is the dimensionless medium permeability,

$p_1 = \nu/\kappa$ is the thermal Prandtl number,

$p_2 = \nu/\eta$ is the magnetic Prandtl number,

$q = \nu/\kappa'$ is the Schmidt number and the superscript * is suppressed.

Eliminating Θ, Γ, Z, X and K between eqs. (17)-(22), we have

$$\left[(D^2 - a^2 - p_2\,\sigma)^2 \left\{\sigma + \frac{\varepsilon}{P_l}(1 + F\,\sigma)\right\} - (D^2 - a^2 - p_2\,\sigma)\right.$$

$$\left. k_x^2\,Qd^2 - k_x^2\,M\,d^2\,(D^2 - a^2)\left\{\sigma + \frac{\varepsilon}{P_l}(1 + F\,\sigma)\right\}\right]$$

$$\times\left[\left\{\sigma + \frac{\varepsilon}{P_l}(1 + F\,\sigma)\right\}(D^2 - a^2 - Ep_1\,\sigma)\,(D^2 - a^2 - E'\,q\,\sigma)\,(D^2 - a^2)\,W\right.$$

$$\left. -\, R\,\lambda\,\varepsilon\,a^2\,(D^2 - a^2 - E'\,q\,\sigma)\,W + S\,\lambda\,\varepsilon\,a^2\,(D^2 - a^2 - E\,p_1\,\sigma)\,W\right]$$

$$-\, k_x^2\,Qd^2\,(D^2 - a^2 - Ep_1\,\sigma)$$

$$(D^2 - a^2 - E'\,q\,\sigma)\left[(D^2 - a^2 - p_2\,\sigma)\left\{\sigma + \frac{\varepsilon}{P_l}(1 + F\,\sigma)\right\} - k_x^2\,Qd^2\right]$$

$$(D^2 - a^2)\,W = 0, \qquad \text{... (23)}$$

where $Q = \left(\dfrac{\mu_e H^2 d^2}{4\,\pi\,\rho_0\,\nu\,\eta}\right)$ is the Chandrasekhar number,

$R = \left(\dfrac{g_0\,\alpha\,\beta\,d^4}{\nu\,\kappa}\right)$ is the thermal Rayleigh number,

$S = \left(\dfrac{g_0\,\alpha'\,\beta'\,d^4}{\nu\,\kappa'}\right)$ is the analogous solute Rayleigh number, and $M=\left(\dfrac{Hc}{4\pi N e \eta}\right)^2$ is the non-dimensional number accounting for Hall currents.

Consider the case in which both the boundaries are free as well as maintained at constant temperatures while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which the eqs. (17)-(22) must be solved are (Chandrasekhar[1]).

$$W = D^2 W = \Theta = \Gamma = DZ = 0 \text{ at } z = 0 \text{ and } 1, \qquad \ldots (24)$$

and h_x, h_y, h_z are continuous.

On the perfectly conducting boundaries $DX = 0$ and $K = 0$. The case of two free boundaries, though little artificial, the most appropriate case for stellar atmospheres (Spiegel[15]). Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence proper solution of eq. (23) characterizing the lowest mode is

$$W = W_0 = \sin \pi z, \qquad \ldots (25)$$

where W_0 is constant. Substituting the proper solution $W = W_0 \sin \pi Z$, in eq. (23) and letting $x = a^2/\pi^2$, $R_1 = R/\pi^4$, $S_1 = S/\pi^4$, $Q_1 = Q/\pi^2$, $P = \pi^2 P_l$, $k_x = k\cos\theta$ and $i\sigma_1 = \sigma/\pi^2$, we obtain the dispersion relation

$$R_1 = \left[\frac{1+x}{x\lambda}\left\{ i\sigma_1 + \frac{\varepsilon}{P}(1+iF\sigma_1\pi^2)\right\}(1+x+iEp_1\sigma_1) + S_1\frac{(1+x+i\sigma_1 Ep_1)}{(1+x+i\sigma_1 E' q)} \right.$$

$$+ \cos^2\theta \frac{Q_1}{\lambda\varepsilon}(1+x)(1+x+iEp_1\sigma_1)\left[(1+x+i\sigma p_2)\left\{ i\sigma_1 + \frac{\varepsilon}{P}(1+iF\sigma_1\pi^2)\right\}\right.$$

$$+ \cos^2\theta Q_1 x]]\left[(1+x+ip_2\sigma_1)^2\left\{ i\sigma_1 + \frac{\varepsilon}{P}(1+iF\sigma_1\pi^2)\right\}\right.$$

$$\left.\left. + (1+x+i\sigma_1 p_2)\cos^2\theta x Q_1 + \cos^2\theta\, Mx(1+x)\left\{ i\sigma_1 + \frac{\varepsilon}{P}(1+iF\sigma_1\pi^2\right\}\right]\right]^{-1} \qquad \ldots (26)$$

Eq. (26) is the required dispersion relation studying the effects of magnetic field, kinematic viscoelasticity, medium permeability, varying gravity field, stable solute gradient and Hall current on thermosolutal instability of Rivlin-Ericksen fluid in the presence of horizontal magnetic field in porous medium.

4. The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (26) reduces to

$$R_1 = \frac{1}{\varepsilon\lambda}\left[\cos^2\theta Q_1 (1+x) \left\{ \frac{\varepsilon(1+x)}{P} + \cos^2\theta Q_1 x \right\} \right.$$

$$\left. \left\{ \cos^2\theta x Q_1 + \frac{\varepsilon(1+x)}{P} + \cos^2\theta \frac{Mx\varepsilon}{P} \right\}^{-1} + \frac{(1+x)^2 \varepsilon}{xP} + S_1 \lambda \varepsilon \right], \quad \text{... (27)}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters S_1, Q_1, P and M. The parameter F accounting for viscoelasticity effect disappears for the stationary convection.

To study the effects of stable solute gradient, magnetic field, Hall currents and medium permeability, we examine the nature of dR_1/dS_1, dR_1/dQ_1, dR_1/dM and dR_1/dP analytically. Eq. (27) yields,

$$\frac{dR_1}{dS_1} = +1, \quad \text{... (28)}$$

which implies that the stable solute gradient has a stabilizing effect on the thermosolutal convection. The adverse solute gradient has destabilizing effect on the system since dR_1/dS_1 then becomes negative.

Eq. (27) also yields

$$\frac{dR_1}{dQ_1} = \frac{\cos^2\theta(1+x)}{\varepsilon\lambda}\left[\frac{(1+x)^2\varepsilon^2}{P^2} + \frac{2Q_1\cos^2\theta x(1+x)\varepsilon}{P} + Q_1^2\cos^4\theta x^2 \right.$$

$$\left. + \frac{M\cos^2\theta x(1+x)\varepsilon^2}{P^2} + \frac{2MQ_1\varepsilon}{P}\cos^4\theta x^2 \right] \times \left[\cos^2\theta x Q_1 + \frac{\varepsilon(1+x)}{P} \right.$$

$$\left. + \frac{\cos^2\theta\,\varepsilon Mx}{P} \right]^{-2}, \quad \text{... (29)}$$

which implies that magnetic field stabilizes the system when gravity is increasing upwards i.e., $(\lambda > 0)$ and destabilizes the system when gravity is decreasing upwards.

It is evident from eq. (27) that

$$\frac{dR_1}{dM} = -\frac{\cos^4\theta Q_1 x(1+x)}{\lambda P}\left[\left\{ \frac{(1+x)\varepsilon}{P} + Q_1 x\cos^2\theta \right\} \right.$$

$$\left. \left\{ \frac{(1+x)\varepsilon}{P} + Q_1 x\cos^2\theta + \frac{\varepsilon M x\cos^2\theta}{P} \right\}^{-2} \right], \quad \text{... (30)}$$

the Hall currents, therefore, has a stabilizing or destabilizing effects on the thermosolutal convection as gravity decreases or increases upwards. It is evident from eq. (27) that

$$\frac{dR_1}{dP}=\frac{1}{\lambda P^2}\left[-\frac{(1+x)^2}{x}+(Q_1^2 M\cos^6\theta x^2(1+x))\right.$$

$$\left.\left\{\cos^2\theta xQ_1+\frac{(1+x)\varepsilon}{P}+\frac{\cos^2\theta\varepsilon Mx}{P}\right\}^{-2}\right], \quad \text{... (31)}$$

In the absence of Hall currents and for constant varying field, dR_1/dP is given by

$$\frac{dR_1}{dP}=-\frac{(1+x)^2}{\lambda xP^2}, \quad \text{... (32)}$$

which is always negative. The medium permeability, therefore, has a destabilizing effect on the thermosolutal instability of a fluid in the absence of Hall currents and for constant varying gravity field. In the presence of Hall currents and varying gravity field, the system is stable if

$$\frac{(1+x)^2}{x}<\frac{Q_1^2 M\cos^6\theta x^2(1+x)}{\left\{\cos^2\theta xQ_1+\frac{(1+x)\varepsilon}{P}+\frac{\cos^2\theta\varepsilon Mx}{P}\right\}^2} \text{ and } \lambda>0. \quad \text{.... (33)}$$

5. Stability of the System and Oscillatory Modes

Here we examine the possibility of oscillatory modes, if any, on stability problem due to the presence of kinematic viscosity, varying gravity field, medium permeability, stable solute gradient, magnetic field and Hall currents. Multiplying eq. (17) by W^*, the complex conjugate of W, and using eqs. (18)-(22) together with the boundary conditions (24), we obtain

$$\left[\sigma+\frac{\varepsilon}{P_l}(1+\sigma F)\right]I_1+\frac{\varepsilon\lambda g_0\alpha\kappa a^2}{\nu\beta}(I_2+p_1\sigma^* I_3)$$

$$-\frac{\varepsilon\lambda g_0\alpha'\kappa' a^2}{\nu\beta'}(I_4+E'q\sigma^* I_5)-\frac{\mu_e\eta d^2\varepsilon^2}{4\pi\rho_0\nu}$$

$$(I_6+p_2\sigma I_7)-\frac{\mu_e\eta\varepsilon^2}{4\pi\rho_0\nu}(I_8+p_2\sigma^* I_9)+d^2\left[\sigma^*+\frac{\varepsilon}{p_l}(1+\sigma^* F)\right]I_{10}=0, \quad \text{... (34)}$$

where
$$I_1=\int_0^1(|DW|^2+a^2|W|^2)\,dz,$$

$$I_2=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)\,dz,$$

$$I_3 = \int_0^1 |\Theta|^2 \, dz,$$

$$I_4 = \int_0^1 (|D\Gamma|^2 + a^2 |\Gamma|^2) \, dz,$$

$$I_5 = \int_0^1 |\Gamma|^2) \, dz,$$

$$I_6 = \int_0^1 (|DX|^2 + a^2 |X|^2) \, dz,$$

$$I_7 = \int_0^1 |X|^2 \, dz,$$

$$I_8 = \int_0^1 (|D^2K|^2 + a^4 |K|^2 + 2a^2 |DK|^2) \, dz,$$

$$I_9 = \int_0^1 (|DK|^2 + a^2 |K|^2) \, dz,$$

and

$$I_{10} = \int_0^1 |Z|^2 \, dz. \qquad \dots (35)$$

The integrals $I_1, I_2, \dots, I_{10}$ are all positive definite. Substituting $\sigma = \sigma_r + i\,\sigma_i$, in eq. (34) and equating the real and imaginary parts, we get

$$\sigma_r \left[\left(1 + \frac{F_\varepsilon}{P_l} \right) I_1 + \frac{\varepsilon \lambda g_0 \alpha \kappa a^2}{\nu \beta} E p_1 I_3 - \frac{\varepsilon \lambda g_0 \alpha' \kappa' a^2}{\nu \beta'} E' q I_5 - \frac{\mu_e \eta p_2 \varepsilon^2}{4 \pi \rho_0 \nu} (d^2 I_7 + I_9) + d^2 \left(1 + \frac{F \varepsilon}{P_l} \right) I_{10} \right]$$

$$= - \left[\frac{I_1 \varepsilon}{P_l} + \frac{\varepsilon \lambda g_0 \alpha \kappa a^2}{\nu \beta} I_2 - \frac{\varepsilon \lambda g_0 \alpha' \kappa' a^2}{\nu \beta'} I_4 - \frac{\mu_e \eta \varepsilon^2}{4 \pi \rho_0 \nu} (I_6 d^2 + I_8) + \frac{d^2 \varepsilon}{P_l} I_{10} \right], \quad \dots (36)$$

and

$$\sigma_i \left[\left(1 + \frac{F \varepsilon}{P_l} \right) - \frac{\varepsilon \lambda g_0 \alpha \kappa a^2}{\nu \beta} E p_1 I_3 + \frac{\varepsilon \lambda g_0 \alpha' \kappa' a^2}{\nu \beta'} \right.$$

$$E'ql_5 - \frac{\mu_e \eta p_2 \varepsilon^2}{4\pi\rho_0 \nu}(d^2 I_7 - I_9) - d^2\left(1 + \frac{F\varepsilon}{P_l}\right)I_{10}\Bigg] = 0. \qquad \text{... (37)}$$

It is evident from eq. (36) that σ_r is positive or negative. The system is, therefore, stable or unstable. It is clear from eq. (37) that σ_i may be zero or non-zero, meaning thereby that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of kinematic viscoelasticity, Hall currents, varying gravity field, magnetic field and stable solute gradient, which were non-existent in their absence.

6. The Case of Overstability

Here we discuss the possibility of whether instability may occur as overstability. Since we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which eq. (26) will admit solutions with σ_1 real.

Equating real and imaginary parts of eq. (26) and eliminating R_1 between them, we obtain

$$A_4 c_1^4 + A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0, \qquad \text{... (38)}$$

where we have put $c_1 = \sigma_1^2$, $b = 1 + x$ and

$$A_4 = bp_2^4 E'^2 q^2\left[\left(b + \frac{Ep_1\varepsilon}{P}\right) + \frac{F\pi^2\varepsilon}{P}\left(3b + \frac{Ep_1\varepsilon}{P}\right)\left(1 + \frac{F\pi^2\varepsilon}{P}\right)\right]$$

$$+\; b^2\frac{p_2^4 F^3 \pi^6 E'^2 q^2 \varepsilon^3}{P^3}, \qquad \text{... (39)}$$

$$A_3 = b^4\left[p_2^4\left(1 + \frac{F\pi^2\varepsilon}{P}\right)^3 + 2p_2^2 E'^2 q^2 (1 - M\cos^2\theta)\left(1 + \frac{F\pi^2\varepsilon}{P}\right)^3\right]$$

$$+\; b^3\left[\frac{p_2^4 Ep_1\varepsilon}{P}\left(1 + \frac{F\pi^2\varepsilon}{P}\right)^2 + \frac{2p_2^2 F\pi^2 Ep_1 E'^2 q^2\varepsilon^2}{P^2}(1 - 3M\cos^2\theta)\right.$$

$$+\; \frac{2p_2^2 Ep_1 E'^2 q^2\varepsilon}{P}\left\{\frac{F\pi^2\varepsilon}{P} + (1 - M\cos^2\theta)\right\} + \frac{2p_2 F^2\pi^4 Ep_1 E'^2 q^2\varepsilon^3}{P^3}(1 - M\cos^2\theta)$$

$$+\cos^2\theta\left[p_2^2 E'^2 q^2 (Ep_1 - p_2)\left(1 + \frac{F\pi^2\varepsilon}{P}\right)^2 Q_1 + 2p_2^2 E'^2 q^2 (M - p_2 Q_1)\left(1 + \frac{F\pi^2\varepsilon}{P}\right)^2\right.$$

$$+\frac{2p_2^2F\pi^2E'^2q^2\varepsilon}{P}\left(1+\frac{F\pi^2\varepsilon}{P}\right)^2M+\frac{p_2^2F\pi^2Ep_1E'^2q^2\varepsilon^2}{P^2}M\Bigg]\Bigg]$$

$$+b^2\left[\frac{p_2^4E'^2q^2\varepsilon^2}{P^2}\left(1+\frac{F\pi^2\varepsilon}{P}\right)+\cos^2\theta\left[p_2^2E'^2q^2Ep_1\left(\frac{M\varepsilon}{P}-Q_1\right)\left(1+\frac{F\pi^2\varepsilon}{P}\right)^2\right.\right.$$

$$+\frac{p_2^2Ep_1E'^2q^2\varepsilon}{P}(M-2p_2Q_1)\left(1+\frac{F\pi^2\varepsilon}{P}\right)+3p_2^3E'^2q^2\left(1+\frac{F\pi^2\varepsilon}{P}\right)^2Q_1$$

$$+\frac{p_2^2F^2\pi^4Ep_1E'^2q^2\varepsilon^3}{P^3}\Bigg]\Bigg]+b\left[\frac{p_2^4Ep_1E'^2q^2\varepsilon^3}{P^3}+\cos^2\theta\frac{2p_2^3Ep_1E'^2q^2\varepsilon}{P}\left(1+\frac{F\pi^2\varepsilon}{P}\right)Q_1\right.$$

$$\left.+\lambda S_1\varepsilon(b-1)p_2^4(Ep_1-E'q)\left(1+\frac{F\pi^2\varepsilon}{P}\right)^2\right] \qquad \ldots (40)$$

As σ_1 is real for overstability, the four values of $c_1\ (=\sigma_1^2)$ {italic mu }st be positive. The sum of the roots of eq. (38) is $-A_3/A_4$, which is impossible if

$$Ep_1>p_2,\ Ep_1,\ E'q,\ 1>M\cos^2\theta,\ M>p_2Q_1,\ \frac{M\varepsilon}{P}>Q_1 \text{ and } \lambda>0 \qquad \ldots (41)$$

which implies that

$$\kappa<\min\left\{E\eta,\frac{E'}{E}\kappa'\right\},\ \frac{Hc\cos\theta}{4\pi\eta e}<N<\frac{c}{2de}\left(\frac{\rho_0}{\pi\mu_e}\right)^{1/2},$$

$$\eta<\frac{\rho_0\nu c^2\varepsilon}{4\pi^3N^2e^2P_l\mu_e d^2} \text{ and } \lambda>0. \qquad \ldots (42)$$

Thus, eq. (42) are, therefore, the necessary conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

We have plotted the variation of the critical Rayleigh number R_1 with solute gradient S_1, magnetic field Q_1, Hall current M and medium permeability P for the wave numbers x = 0.5 and x = 1.0 by using eq. (27). In Fig. 1, R_1 is plotted against S_1 for fixed values of λ = 2, ε = 0.5, Q_1 = 30, M = 30, $\theta = 45^0$, P = 1 and wave number x = 0.5 and x = 1.0. For both the wave numbers, R_1 increases as S_1 increases implying the stabilizing effect of salinity on the system. In Fig. 2, R_1 is plotted against Q_1 for fixed values of λ = 2, ε = 0.5, S_1 = 6, M = 30, $\theta = 45^0$, P = 1 and wave number x = 0.5 and x = 1.0. For both the wave numbers, R_1 increases as Q_1 increases

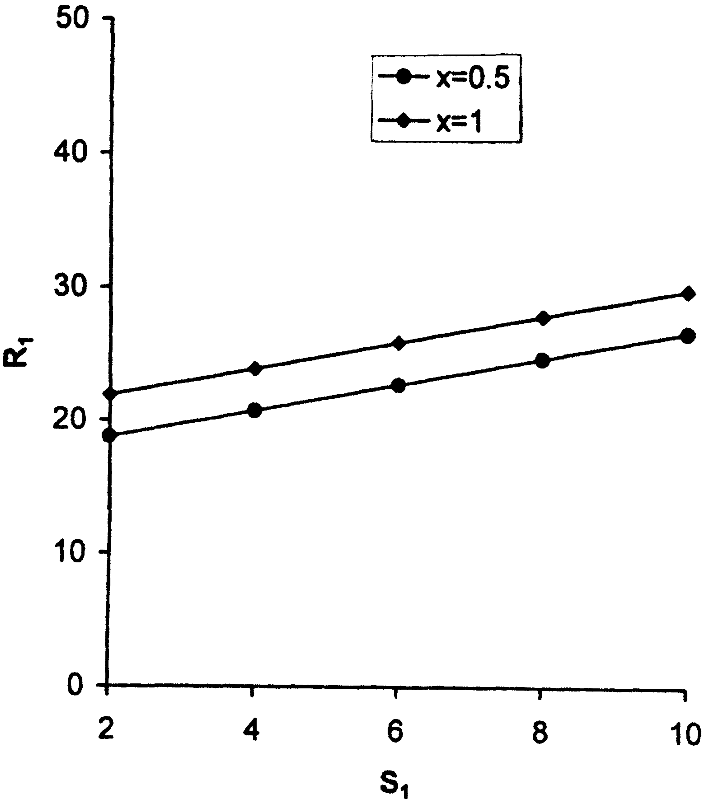

FIG. 1. Variation of critical Rayleigh number R_1 with S_1 for a fixed $\lambda = 2$, $\varepsilon = 0.5$, $P = 1, Q_1 = 30$, $M = 30$ and $\theta = 45^{\circ}$

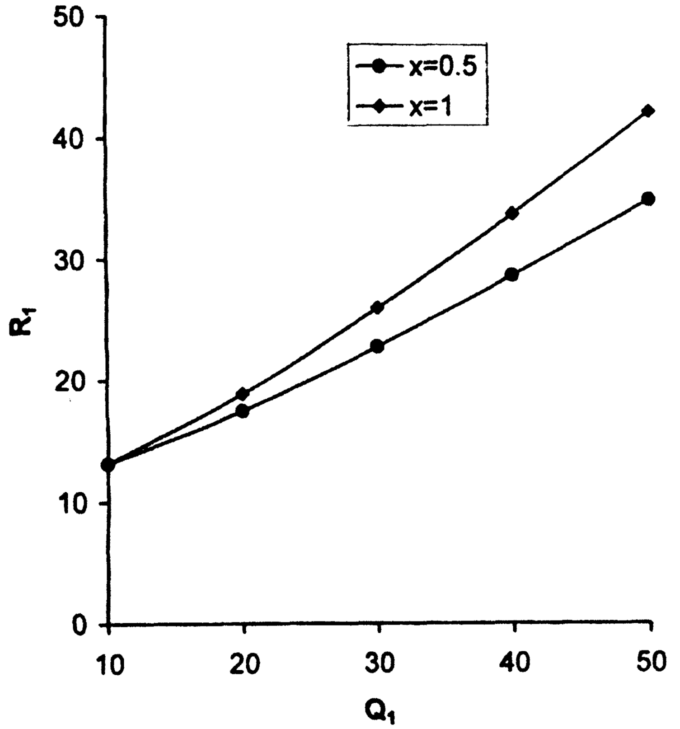

FIG. 2. Variation of critical Rayleigh number R_1 with Q_1 for a fixed $\lambda = 2$, $\varepsilon = 0.5$, $P = 1, S_1 = 6$, $M = 30$ and $\theta = 45^{\circ}$

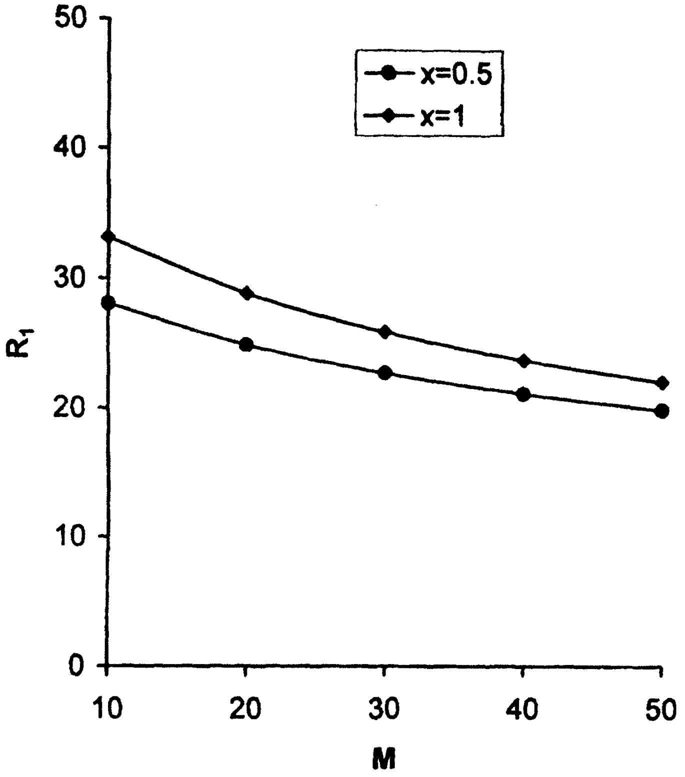

FIG. 3. Variation of critical Rayleigh number R_1 with M for a fixed $\lambda = 2$, $\varepsilon = 0.5$, $P = 1$, $Q_1 = 30$, $S_1 = 6$ and $\theta = 45^{\circ}$

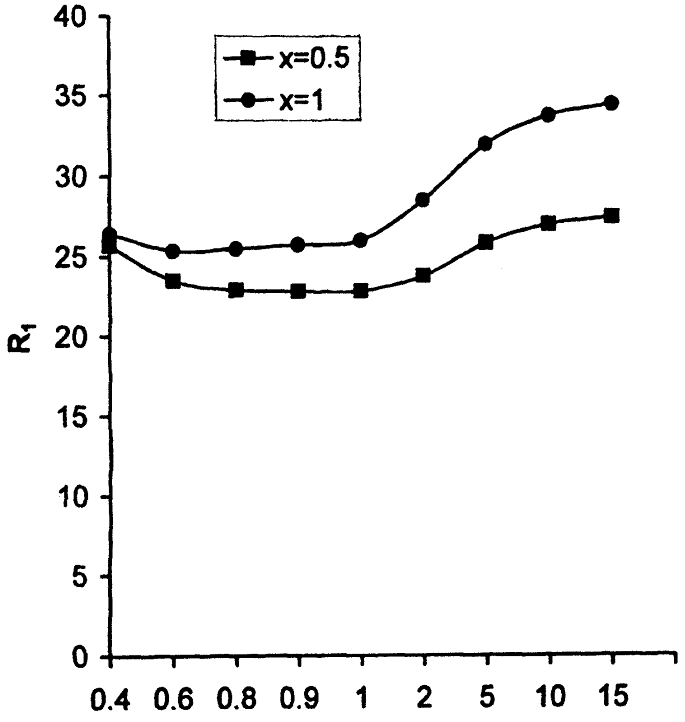

FIG. 4. Variation of critical Rayleigh number R_1 with P for a fixed $\lambda = 2$, $\varepsilon = 0.5$, $Q_1 = 30$, $S_1 = 6$, $M = 30$ and $\theta = 45^{\circ}$

implying the stabilizing effect of magnetic field. In Fig. 3, R_1 is plotted against M for fixed values of $\lambda = 2$, $\varepsilon = 0.5$, $Q_1 = 30$, $S_1 = 6$, $\theta \bullet = 45^0$, $P = 1$ and wave number $x = 0.5$ and $x = 1.0$. For both the wave numbers, R_1 decreases as M increases implying the destabilizing effect of Hall currents on the thermosolutal hydromagnetic instability of Rivlin-Ericksen fluid in porous media. In Fig. 4, R_1 is plotted against P for fixed values of $\lambda = 2$, $\varepsilon = 0.5$, $Q_1 = 30$, $S_1 = 6$, $M = 30$, $\theta = 45^0$ and wave number $x = 0.5$ and $x = 1.0$. For both the wave number Rayleigh number R_1 decreases from $P = 0.4$ to 0.8 showing thereby the destabilizing effect and increases for $P = 0.9$ to 15, implying the stabilizing effect.

7. CONCLUSION

The inclusion of Hall currents gives rise to a cross flow i.e. a flow at right angles to the primary flow in a channel in the presence of a transverse magnetic field, has been shown by Sato[16] and Tani[17] has found that Hall effect produces a cross-flow of double-swirl pattern in incompressible flow through a straight channel with arbitrary cross-section. This breakdown of the primary flow and information of a secondary flow may be attributed to the inherent instability of the primary flow in the presence of Hall currents. Sato[16] has pointed out that even if the distribution of the primary flow velocity be stable to external disturbances, the whole layer may become turbulent if the distribution of the cross-flow velocity is unstable. A similar situation occurs on the three dimensional boundary layer along a swept-back wing. Gupta[3] has found that the presence of Hall currents induces a vertical component of vorticity and this may well be the reason for the destabilizing influence.

The Hall currents, therefore, has a destabilizing influence for the stationary convection for $\lambda > 0$ and stabilizing for $\lambda < 0$. Also medium permeability has stabilizing as well as destabilizing effects on the thermosolutal instability of Rinlin-Ericksen elasticoviscous fluid in porous medium. The Hall currents, kinematic viscoelasticity, stable solute gradient, varying gravity field, medium permeability and magnetic field introduce the oscillatory modes in the system which were non-existent in their absence. The sufficient conditions for the non-existence of overstability for thermosolutal instability of Rivlin-Ericksen fluid with stable solute gradient, Hall currents (hence magnetic field), viscoelasticity, medium permeability and varying gravity field are

$$\kappa < \min\left\{ E\eta, \frac{E}{E'}\kappa' \right\}, \frac{Hc\cos\theta}{4\pi\eta e} < N < \frac{c}{3de}\left(\frac{\rho_0}{\pi\mu_e}\right)^{1/2}, \quad \eta < \frac{\rho_0 \nu c^2 \varepsilon}{4\pi^3 N^2 e^2 P_l \mu_e d^2}$$

and $\lambda > 0$.

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