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CONNECTEDNESS IN FUZZY SUPRABITOPOLOGICAL SPACES

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The concept of supraconnectedness, strong supraconnectedness and super supraconnectedness have been introduced and the interrelations that exist between them are obtained in fuzzy suprabitopological spaces.

Key Words : Supratopology; Fuzzy Pairwise Supraconnected; Strong Supraconnected; Super Supraconnected

1. INTRODUCTION

Abd E1_Monsef and Ramadan[1] introduced the concept of fuzzy supratopological spaces as a natural generalization of fuzzy topological spaces and studied some of its properties. In this paper we introduce supraconnectedness, strong supraconnectedness and super supraconnectedness and study the mutual relationship that exist between them in fuzzy suprabitopological spaces.

2. PRELIMINARIES

We recall the following definitions of the fuzzy supratopological theory, which are to be used in the sequel.

Let X be a non-empty set. A fuzzy set in X is a function from X to I = [0, 1]. 0 and 1 denotes the constant fuzzy sets taking the values 0 and 1 respectively.

Definition 2.1[2] — A family $\tau \subset I^X$, where I^X denotes fuzzy sets of X, is called a fuzzy supratopology on X if 0, $1 \in \tau$ and τ is closed under arbitrary union.

The pair (X, τ) is called a fuzzy supratopological space. The members of τ are called τ-fuzzy supraopen sets and their complements are called τ-fuzzy supraclosed sets. A triple (X, τ_1, τ_2), where X is a non-empty set and τ_1 and τ_2 are two arbitrary fuzzy supratopologies on X is called a fuzzy suprabitopological space.

Definition 2.2[2] — A mapping $f: (X, \tau) \to (Y, \sigma)$ between two fuzzy supratopological spaces is called fuzzy supracontinuous iff the inverse image of each σ-fuzzy supraopen set is τ-fuzzy supraopen.

Definition 2.3 — A mapping $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ between two fuzzy suprabitopological spaces is called fuzzy pairwise supracontinuous iff the induced mappings $f: (X, \tau_k) \to (Y, \sigma_k)$, k = 1, 2 are fuzzy supracontinuous.

Definition 2.4[2] — For a fuzzy set λ in a fuzzy supratopological space (X, τ), the suprainterior of λ (denoted by sint (λ)) and supraclosure of λ (denoted by scl (λ)) are defined as follows :

$$\text{sint } (\lambda) = \text{Sup } \{\nu : \nu \leq \lambda, \nu \text{ is } \tau\text{-fuzzy supraopen}\}$$

$$\text{scl } (\lambda) = \text{Inf } \{\nu : \nu \geq \lambda, \nu \text{ is } \tau\text{-fuzzy supraclosed}\}.$$

3. Fuzzy Pairwise Supra Connectedness

Definition 3.1 — Let (X, τ_1, τ_2) be a fuzzy suprabitopological space. A fuzzy set λ of X is said to be fuzzy pairwise supraconnected (FP-supraconnected, in short) if there exist no non-zero proper τ_i-fuzzy supraopen and τ_j-fuzzy supraclosed set in λ for $i \neq j$ and $i, j = 1, 2$. (A fuzzy set ν is said to be proper if $\nu \neq 0$ and $\nu \neq 1$).

The following notations will be used throughout this paper.

$$\tau_{k\nu} = \{\nu \wedge \mu : \mu \in \tau_k\} \text{ and } \tau_{k\nu}^c = \{\nu \wedge \xi : \xi \in \tau_k^c\},$$

we call the members of $\tau_{k\nu}$ (resp. $\tau_{k\nu}^c$) as τ_k-fuzzy supraopen (resp. τ_k-fuzzy supraclosed) sets in the fuzzy set $\nu, k = 1, 2$.

Definition 3.2 — A fuzzy suprabitopological space (X, τ_1, τ_2) is said to be fuzzy pairwise strongly supraconnected (FP-strongly supraconnected) if it has no non-zero τ_i-fuzzy supraclosed set μ and a τ_j-fuzzy supraclosed set ν such that $\mu + \nu \leq 1, i \neq j,\ i, j = 1, 2$.

If (X, τ_1, τ_2) is not FP-strongly supraconnected then it will be called FP-strongly supradisconnected.

Theorem 3.3 — *A fuzzy suprabitopological space* (X, τ_1, τ_2) *is FP-strongly supraconnected iff it has no non-zero fuzzy sets* μ *and* ν, *where* μ *is* τ_i*-fuzzy supraopen,* ν *is* τ_j*-fuzzy supraopen, such that* $\mu \neq 1, \nu \neq 1$ *and* $\mu + \nu \geq 1$.

PROOF : Suppose that (X, τ_1, τ_2) is not FP-strongly supraconnected. Then it has non-zero fuzzy sets λ and ξ, where λ is τ_i-fuzzy supraclosed, ξ is τ_j-fuzzy supraclosed, such that $\lambda + \xi \leq 1$. By our hypothesis it does not have a τ_i-fuzzy supraopen set $\mu = 1 - \lambda$, τ_j-fuzzy supraopen set $\nu = 1 - \xi$ such that $\mu \neq 1, \nu \neq 1$ and $\mu + \nu \geq 1$.

Remark 3.4 : FP-supraconnectedness of fuzzy set does not imply FP-strong supraconnectedness as the following example shows.

Example 3.5 — Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2$ and μ_3 be fuzzy sets of X defined as follows :-

$$\lambda_1 (a) = 0.5, \lambda_2 (a) = 0.6, \lambda_3 (a) = 0.6$$

$$\lambda_1 (b) = 0.7, \lambda_2 (b) = 0.5, \lambda_3 (b) = 0.7$$

$$\mu_1 (a) = 0.4, \mu_2 (a) = 0.7, \mu_3 (a) = 0.7$$

$\mu_1(b) = 0.6, \mu_2(b) = 0.4, \mu_3(b) = 0.6.$

Let $\tau_1 = \{0, \lambda_1, \lambda_2, \lambda_3, 1\}$ and $\tau_2 = \{0, \mu_1, \mu_2, \mu_3, 1\}$ be fuzzy supratopologies on X. Then (X, τ_1, τ_2) is FP-supraconnected but it is not FP-strongly supraconnected.

It is well known in general topology that connectedness is preserved under continuous surjections. We have the following result of FP-strong supraconnected spaces.

Theorem 3.6 — *Let $(X\ \tau_1, \tau_2)$ and (Y, σ_1, σ_2) be two fuzzy suprabitopological spaces and $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ a fuzzy pairwise supracontinuous surjection. If (X, τ_1, τ_2) is FP-strongly supraconnected, then (Y, σ_1, σ_2) is also FP-srongly supraconnected.*

PROOF : Suppose (Y, σ_1, σ_2) is not FP-strongly supraconnected. Then there exists a σ_i-fuzzy supraclosed set μ and a σ_i-fuzzy supraclosed set ν in (Y, σ_1, σ_2) such that $\mu + \nu \leq 1$. Since f is fuzzy pairwise supracontinuous, $f^{-1}(\mu)$ is a τ_i-fuzzy supraclosed set, $f^{-1}(\nu)$ is a τ_j-fuzzy supraclosed set in (X, τ_1, τ_2) and we also have $f^{-1}(\mu)(x) + f^{-1}(\nu)(x) \leq 1$ for each $x \in X$, since $(\mu\, o f)(x) + (\nu\, o f)(x) \leq 1$ follows from $\mu(y) + \nu(y) \leq 1$ which is a contradiction. Hence, (Y, σ_1, σ_2) is FP-strongly supraconnected.

Definition 3.7 — A fuzzy set λ in a fuzzy suprabitopological space (X, τ_1, τ_2) is said to be a (τ_1, τ_2)-fuzzy supraregular open set if $\lambda = \tau_1$-sint (τ_2-scl (λ)). The complement of a (τ_1, τ_2)-fuzzy supraregular open set is called a (τ_1, τ_2)-fuzzy supraregular closed set. A fuzzy set λ is called a fuzzy pairwise supraregular open (supra regular closed) set if it is (τ_i, τ_j)-fuzzy supraregular open (supraregular closed) set for $i \neq j$ and $i, j = 1, 2$.

Definition 3.8 — A fuzzy suprabitopological space (X, τ_1, τ_2) is called a fuzzy pairwise super supraconnected (FP-super supraconnected, in short) iff it has no proper (τ_i, τ_j)-fuzzy supraregular open set for $i \neq j$ and $i, j = 1, 2$.

Remark 3.9 : Since a τ_i-fuzzy supraopen, τ_j-fuzzy supraclosed set is (τ_i, τ_j)-fuzzy supraregular open for $i \neq j$ and $i, j = 1, 2$, FP-super supraconnectedness implies FP-supraconnectedness. But the converse is not true as the following example shows.

Example 3.10 — Let $X = \{a, b\}$ and let $\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2$ and μ_3 be fuzzy sets of X defined as follows :

$\lambda_1(a) = 0.3, \lambda_2(a) = 0.6, \lambda_3(a) = 0.6$

$\lambda_1(b) = 0.5, \lambda_2(b) = 0.2, \lambda_3(b) = 0.5$

$\mu_1(a) = 0.3, \mu_2(a) = 0.5, \mu_3(a) = 0.5$

$\mu_1(b) = 0.5, \mu_2(b) = 0.3, \mu_3(b) = 0.5.$

Let $\tau_1 = \{0, \lambda_1, \lambda_2, \lambda_3, 1\}$ and $\tau_2 = \{0, \mu_1, \mu_2, \mu_3, 1\}$ be fuzzy supratopologies on X. Then the fuzzy suprabitopological space (X, τ_1, τ_2) is FP-supraconnected. But if we define a fuzzy set ν by $\nu(a) = 0.3, \nu(b) = 0.5$, then ν is both (τ_1, τ_2)-fuzzy supraregular open and (τ_2, τ_1)-fuzzy supraregular open. Hence (X, τ_1, τ_2) is not FP-super supraconnected.

Remark 3.11 : In a fuzzy suprabitopological space (X, τ_1, τ_2), the classes of FP-supraconnected sets can be described by the following diagram :

FP-super supraconnectedness

$\downarrow$

FP-supraconnectedness

$\uparrow$

FP-strong supraconnectedness.

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