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WAVES DUE TO INITIAL AXISYMMETRIC DISTURBANCES AT THE INERTIAL INTERFACE BETWEEN TWO SUPERPOSED FLUIDS

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The generation of waves due to initial axisymmetric disturbances at the interface between two superposed fluids, the lower fluid being of uniform finite depth and the upper fluid extending infinitely upwards, is considered in this paper. The interface is composed of a thin but uniform distribution of non-interacting material. Assuming linear theory, the problem is formulated as a coupled initial value problem in the velocity potentials describing the ensuing motion in the two fluids. In the mathematical analysis the Laplace and Hankel transform techniques have been utilised to obtain the depression of the inertial interface in the form of an infinite integral. The interface depression is evaluated asymptotically for large time and distance by using the method of stationary phase. This is depicted graphically for two types of initial disturbances and the effect of the presence of the upper fluid and the inertial surface at the interface on the wave motion are discussed.

Key Words : Axisymmetric Disturbances; Inertial Interface; Linear Theory; Stationary Phase

1. INTRODUCTION

For deep water, the problem of two dimensional unsteady motion produced by initial disturbances in the form of surface elevation or impulse concentrated at a point on the free surface is discussed in the treatise of Lamb[1] and Stoker[2] assuming linear theory. In the mathematical analysis, Fourier and Laplace transform technique were used and the free surface elevation is obtained in the form of infinite integral. Various extensions of the problem to take into account three dimensional unsteady motion, liquid of uniform finite depth, liquid bounded by an inertial surface, axially symmetric initial disturbances, etc., have been investigated in the literature (cf. Kranzer and Keller[3], Chaudhuri[4], Wen[5], Mandal[6] , Mandal and Mukherjee[7] and others). These problems are formulated in terms of a potential function describing the ensuing irrotational motion in the liquid and the form of the free surface has been obtained asymptotically for large time and distance. Lé Méhauté and Wang[10] described in their book the present state of the art concerning explosion generated waves and a somewhat exhaustive list of references in this area are available in this book.

Wehausen and Laitone[8] discussed the possible extensions to two superposed fluids. Recently, Dolai considered generation of interface waves due to initial disturbances in the form of an initial depression or impulse at the interface when the upper fluid extends infinitely upwards and the lower fluid extends infinitely downwards. Here we consider, generation of interface waves due to initial axially symmetric disturbances at the inertial interface between two superposed fluids wherein the lower fluid is of uniform finite depth while the upper fluid extends infinitely upwards. Assuming linear theory, the potential functions describing the motions in the two fluids satisfy a coupled initial value problem. This reduces to a coupled boundary value problem after using Laplace transform in time. Because of axial symmetry, Hankel transform is used in the radial co-ordinate and the BVP is decoupled into two independent BVPs, each involving the unknown interface depression. The solution to these BVPs are easily found. Using the interface condition in these solutions, and taking Hankel and Laplace inversions, the interface depression is obtained in terms of an infinite integral. This is then evaluated asymptotically for large time and distance by using the method of stationary phase for the two cases when the initial disturbances at the inertial interface are concentrated at the origin or prescribed over a circular region. To visualize the effect of the upper fluid and the effect of the inertial surface on the wave motion at the interface, the non-dimensional interface depression is plotted graphically for both the cases in a number of figures.

2. Formulation of the Problem

We consider two-dimensional unsteady motion at the interface between two incompressible, inviscid superposed fluids generated due to an axially symmetric initial disturbances in the form of an interface depression or an impulsive pressure at the interface. The interface is composed of a thin but uniform distribution of non-interacting floating material. We choose a cylindrical co-ordinate system in which y-axis is taken vertically downwards and $y = 0$ is the undisturbed position of the interface. The motion in the two fluids is irrotational as it is assumed to start from rest and the velocity potentials $\phi_1(r, y; t)$ and $\phi_2(r, y; t)$ describing the motion in the lower and upper fluids satisfy the following initial value problem described by

$$\left.\begin{aligned} &\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_1}{\partial r}\right)+\frac{\partial^2 \phi_1}{\partial y^2}=0,\ 0\le y\le h,\\ &\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_2}{\partial r}\right)+\frac{\partial^2 \phi_2}{\partial y^2}=0,\ y\le 0,\end{aligned}\right\}\ t\ge 0, \qquad \text{... (2.1)}$$

$$\left.\begin{aligned} &\phi_{1y}=\phi_{2y}=\eta_t,\\ &(\phi_{1tt}-s\phi_{2tt})-\varepsilon(\phi_{1y}-s\phi_{2y})=g(\phi_{1y}-s\phi_{2y}),\end{aligned}\right\}\ \text{on } y=0,\ t>0, \qquad \text{... (2.2)}$$

$$\left.\begin{aligned} &\phi_{1y}=0,\ \text{on } y=h,\\ &\nabla\phi_2\to 0\ \text{as } y\to-\infty,\end{aligned}\right\}, \qquad \text{... (2.3)}$$

where $\eta(r, t)$ is the interface depression, g is the gravity, $s=\frac{\rho_2}{\rho_1}$ $(0\le s\le 1)$, ρ_1 and ρ_2 are the densities of the lower and upper fluids respectively. The initial conditions when the initial disturbance is in the form of a prescribed depression of the interface, are

$$\left.\begin{aligned} &(\phi_1-s\phi_2)-\varepsilon(\phi_{1y}-s\phi_{2y})=0,\ \text{on } y=0,\ t=0,\\ &\frac{\partial}{\partial t}(\phi_1-s\phi_2)-\varepsilon(\phi_{1y}-s\phi_{2y})=g(1-s)\,F(r),\ \text{on } y=0,\end{aligned}\right\} \qquad \text{... (2.4)}$$

where $F(r)$ is the prescribed initial depression of the interface at a distance r from the origin.

When the initial disturbance is due to axially symmetric impulse of strength $G(r)$ per unit area at a distance r from the origin, then the initial conditions (2.4) are replaced by

$$\left.\begin{aligned}&(\phi_1 - s\phi_2) - \varepsilon(\phi_{1y} - s\phi_{2y}) = -\frac{G(r)}{\rho_1}, \text{ on } y = 0, t = 0,\\ &\frac{\partial}{\partial t}(\phi_1 - s\phi_2) - \varepsilon(\phi_{1y} - s\phi_{2y}) = 0, \text{ on } y = 0.\end{aligned}\right\} \quad \dots (2.5)$$

3. Method of Solution

Let $\bar{\phi}_j(r, y; p)$ $(j = 1, 2)$ denotes the Laplace transform of $\phi_j(r, y; t)$ in time defined as

$$\bar{\phi}_j(r, y; p) = \int_0^\infty \phi_j(r, y; t)e^{-pt}\,dt, \text{ Re } p > 0. \ (j = 1, 2).$$

Using Laplace transform, the initial value problem described by (2.1)-(2.4) reduces to the following coupled boundary value problem.

$$\frac{1}{r}\frac{\partial}{rr}\left(r\frac{\partial\bar{\phi}_1}{\partial r}\right) + \frac{\partial^2\bar{\phi}_1}{\partial y^2} = 0, 0 \le y \le h,$$

$$\left.\begin{aligned}&\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\bar{\phi}_2}{\partial r}\right) + \frac{\partial^2\bar{\phi}_2}{\partial y^2} = 0, y \le 0,\\ &\bar{\phi}_{1y} = \bar{\phi}_{2y} = p\bar{\eta}(r, p) - F(r) \text{ on } y = 0,\\ &p(\bar{\phi}_1 - s\bar{\phi}_2) - \varepsilon p(1 - s) - \phi_{1y} = g(1 - s)\,\bar{\eta}(r, p), \text{ on } y = 0,\\ &\bar{\phi}_{1y} = 0 \text{ on } y = h,\\ &\nabla\bar{\phi}_2 \to 0 \text{ as } y \to \infty,\end{aligned}\right\} \quad \dots (3.1)$$

where the unknown function $\bar{\eta}(r, p)$ denotes the Laplace transform of $\eta(r, t)$.

Let $\bar{\bar{\phi}}_1(k, y; p), \bar{\bar{\phi}}_2(k, y; p)$ and $\bar{F}(k)$ denote respectively the Henkel transform of $\bar{\phi}_1(r, y; p), \bar{\phi}_2(r, y; p)$ and $F(r)$ with respect to r. Then the coupled BVP (3.1) reduces to the following two uncoupled boundary value problems

$$\left.\begin{aligned}&\bar{\bar{\phi}}_{1yy} - k^2\bar{\bar{\phi}}_1 = 0, 0 \le y \le h,\\ &\bar{\bar{\phi}}_{1y} = M(k, p) \text{ on } y = 0,\\ &\bar{\bar{\phi}}_{1y} = 0 \text{ on } y = h,\end{aligned}\right\} \quad \dots (3.3)$$

$$\left.\begin{aligned} &\bar{\bar{\phi}}_{2yy} - k^2 \bar{\bar{\phi}}_2 = 0, y \le 0, \\ &\bar{\bar{\phi}}_{2y} = M(k, p) \text{ on } y = 0 \\ &\bar{\bar{\phi}}_{2y} \to 0 \text{ as } y \to -\infty, \end{aligned}\right\} \quad \ldots (3.4)$$

where

$$M(k, p) = p\, \bar{\bar{\eta}}\,(r, p) - \bar{F}\,(k), \quad \ldots (3.4)$$

where $\bar{\bar{\eta}}\,(k, p)$ being the Hankel transform of $\bar{\eta}\,(r, p)$. The solution of these BVPs (3.2) and (3.3) are given by

$$\bar{\bar{\phi}}_1\,(k, y; p) = -\frac{M(k, p) \cosh k(h-y)}{k \sinh kh}, 0 \le y \le h \quad \ldots (3.5)$$

and

$$\bar{\bar{\phi}}_2\,(k, y; p) = -\frac{M(k, p)\, e^{|k| y}}{|k|}, y < 0. \quad \ldots (3.6)$$

These solutions involve the unknown function $\bar{\bar{\eta}}\,(k, p)$. This is determined by using the relation

$$p(\bar{\bar{\phi}}_1 - s\bar{\bar{\phi}}_2) - \varepsilon p\,(1-s)\,\bar{\bar{\phi}}_{1y} = g(1-s)\,\bar{\bar{\eta}}\,(k, p), \quad \ldots (3.7)$$

obtained from the second interface condition.

Thus

$$\bar{\bar{\eta}}\,(k, p) = \frac{pA(k)\,\bar{F}(k)}{p^2 A(k) + gk\,(1-s) \sinh kh},$$

where

$$A(k) = \cosh kh + s(sgn\; k) \sinh kh + \varepsilon(1-s)\, k \sinh kh$$

Hence,

$$\bar{\eta}\,(r, p) = \int_0^\infty \frac{p\bar{F}\,(k)\, kJ_0(kr)}{p^2 + \omega^2}\, dk, \quad \ldots (3.8)$$

where

$$\omega^2 = \frac{gk(1-s)\sinh kh}{A(k)}.$$

Laplace inversion gives

$$\eta(r, t) = \int_0^\infty k\bar{F}\,(k)\, J_0(kr) \cos \omega t\, dk \quad \ldots (3.9)$$

By a similar analysis, for the case of initial axially symmetric impulse we obtain

$$\eta(r,t) = \frac{1}{\rho_1\, g(1-s)} \int_0^\infty k\,\overline{G}(k)\, J_0(kr)\, \omega(k) \sin \omega t\, dk \qquad \text{... (3.10)}$$

We note that when the upper fluid is absent and the lower fluid is of infinite depth, equations (3.9) and (3.10) coincides with the results obtained by Mandal and Mukherjee[7] earlier.

We introduce non-dimensional quantities $\tilde{r}, \tilde{t}$ and $\tilde{\eta}(\tilde{r},\tilde{t})$ defined by $\tilde{r} = r/l$, $\tilde{t} = t(g/l)^{1/2}$, $\tilde{\eta}(\tilde{r},\tilde{t}) = \eta(r,t)/l$, where l is a characteristic length scale. Then

$$\tilde{\eta}(\tilde{r},\tilde{t}) = \frac{1}{l} \int_0^\infty k\,\overline{F}(k)\, J_0(kl\tilde{r}) \cos\,\{\omega(l/g)^{1/2}\tilde{t}\}\; dk \qquad \text{... (3.11)}$$

when the initial disturbance is prescribed in the form of interface depression, and

$$\tilde{\eta}(\tilde{r},\tilde{t}) = \frac{1}{g\rho_1\,(1-s)l} \int_0^\infty k\,\overline{G}(k)\, J_0(kl\tilde{r})\, \omega(k) \sin\,\{\omega(l/g)^{1/2}\tilde{t}\}\; dk \qquad \text{... (3.12)}$$

when the initial disturbance is prescribed in the form of an impulse at the interface.

4. Asymptotic Expansion

To obtain the asymptotic form of $\tilde{\eta}(\tilde{r},\tilde{t})$ for large $\tilde{r}$ and $\tilde{t}$, we first use the result

$$J_0(kr) = \frac{2}{\pi} \int_0^\infty \cos\,(kr\cos\beta)\, d\beta$$

in (3.11) to obtain

$$\tilde{\eta}(\tilde{r},\tilde{t}) = \frac{1}{\pi l} \int_0^\infty \int_0^{\pi/2} k\,\overline{F}(k) \cos\,(kl\,\tilde{r}\cos\beta) \cos\,\{\omega(l/g)^{1/2}\tilde{t}\}\; d\beta\, dk.$$

$$= \frac{1}{\pi\, l}\,\mathrm{Re} \int_0^\infty \int_0^{\pi/2} k\,\overline{F}(k) \left[\exp\left\{ i\,\tilde{t}\left((l/g)^{1/2}\,\omega(k) + \frac{kl\,\tilde{r}}{\tilde{t}}\cos\beta \right)\right\} \right.$$

$$\left. + \exp\left\{ i\,\tilde{t}\left((l/g)^{1/2}\,\omega(k) - \frac{kl\,\tilde{r}}{\tilde{t}}\cos\beta \right)\right\} \right] d\beta\, dk \qquad \text{... (4.1)}$$

Application of the stationary phase method to the β-integral transforms (4.1) into

$$\tilde{\eta}(\tilde{r},\tilde{t}) \approx \frac{1}{\pi l}\left\{\frac{2\pi}{l\tilde{r}}\right\}^{1/2} Re \int_0^\infty k^{1/2}\,\overline{F}(k) \left[\exp\left\{ i\,\tilde{t}\,\psi_1(k) \right\} + \exp\left\{ i\,\tilde{t}\,\psi_2(k) \right\} \right] dk \qquad \text{.. (4.2)}$$

where

$$\psi_1(k) = (l/g)^{1/2}\,\omega(k) + \frac{lk\tilde{r}}{\tilde{t}} - \frac{\pi}{4\tilde{t}}$$

and

$$\psi_2(k) = (l/g)^{1/2}\,\omega(k) - \frac{lk\tilde{r}}{\tilde{t}} + \frac{\pi}{4\tilde{t}}\,.$$

The integral in (4.2) is further approximated by using again the method of stationary phase. We note that the first integral of (4.2) does not contribute because it has no stationary point in the range of integration. The second integral has stationary points given by the solution of the equation

$$\psi_2(k) = 0.$$

Now

$$\psi_2(0) = \{hl(1-s)\}1/2 - \frac{l\tilde{r}}{\tilde{t}},\ \psi_2'(\infty) = -\frac{\tilde{r}}{\tilde{t}} \text{ as } k \to \infty.$$

Also, $\psi''_{2k} < 0$ for $0 < k < \infty$, so that $\psi_2'(k)$ is monotonic decreasing for $0 < k < \infty$. These show that in the range of integration there is only one stationary point at $k = \alpha_0$ (say) if $\frac{\tilde{r}}{\tilde{t}} < \left\{\frac{h}{l}(1-s)\right\}^{1/2}$ while there is no stationary point if $\frac{\tilde{r}}{\tilde{t}} > \left\{\frac{h}{l}(1-s)\right\}^{1/2}$ When $\frac{\tilde{r}}{\tilde{t}} = \left\{\frac{h}{l}(1-s)\right\}^{1/2}$, the stationary point is at $k = 0$. This gives a smaller contribution than the case when $\frac{\tilde{r}}{\tilde{t}} < \left\{\frac{h}{l}(1-s)\right\}^{1/2}$ so that this contribution may be neglected. Thus applying the method of stationary phase to the second integral of (4.2), $\tilde{\eta}(\tilde{r},\tilde{t})$ is approximated finally as

$$\tilde{\eta}(\tilde{r},\tilde{t}) \approx \frac{2}{\tilde{r}l^2}\left\{\frac{\alpha_0 l\tilde{r}}{(l/g)^{1/2}\,\tilde{t}\,|\,\psi_2''(\alpha_0)\,|}\right\}^{1/2} \bar{F}(\alpha_0)\cos\left\{\tilde{t}(l/g)^{1/2}\,\omega(\alpha_0) - \alpha_0 l\tilde{r}\right\} \quad \ldots (4.3)$$

By a similar technique, equation (3.12) produces

$$\tilde{\eta}(\tilde{r},\tilde{t}) \approx \frac{2}{\rho g(1-s)\tilde{r}l^2}\left\{\frac{\alpha_0 l\tilde{r}}{(l/g)^{1/2}\,\tilde{t}\,|\,\psi_2''(\alpha_0)\,|}\right\}^{1/2}$$

$$\times\,\omega(\alpha_0)\,\bar{G}(\alpha_0)'\sin\left\{\tilde{t}(l/g)^{1/2}\,\omega(\alpha_0) - \alpha_0 l\tilde{r}\right\} \quad \ldots (4.4)$$

If we consider that the initial axisymmetric depression of the interface is concentrated at the origin, then $\bar{F}(\alpha_0) = \frac{l^3}{2\pi}$, so that (4.3) produces

$$\tilde{\eta}(\tilde{r},\tilde{t}) \approx \frac{1}{\pi\sqrt{r}}\left\{\frac{\alpha_0 l\,\tilde{r}}{(l/g)^{1/2}\,\tilde{t}\,|\,\psi_2''(\alpha_0)\,|}\right\}^{1/2} \cos\left\{\tilde{t}(l/g)^{1/2}\,\omega(\alpha_0)-\alpha_0 l\,\tilde{r}\right\} \quad \text{... (4.5)}$$

Again when the initial depression is prescribed over a circle of radius a with centre at the origin, then

$$\overline{F}(\alpha_0)=\frac{W}{\pi a\alpha_0}J_1(a\alpha_0),$$

where W represents total volume of depressed fluid.

In this case, (4.3) produces

$$\tilde{\eta}(\tilde{r},\tilde{t}) \approx \frac{2W}{\pi\tilde{r}\,l^2 a\alpha_0}\left\{\frac{\alpha_0 l\,\tilde{r}}{(l/g)^{1/2}\,\tilde{t}\,|\,\psi_2''(\alpha_0)\,|}\right\}^{1/2}$$

$$\times J_1(a\alpha_0)\cos\left\{\tilde{t}(l/g)^{1/2}\,\omega(\alpha_0)-\alpha_0 l\,\tilde{r}\right\} \quad \text{... (4.6)}$$

When the initial axisymmetric impulse of the interface is concentrated at the origin, $\overline{G}(\alpha_0)=\frac{P}{2\pi}$, where P is the total impulse, then equation (4.4) produces

$$\tilde{\eta}(\tilde{r},\tilde{t}) \approx \frac{P}{\rho g\pi(1-s)\,l^2\,\tilde{r}}\left\{\frac{\alpha_0 l\,\tilde{r}}{(l/g)^{1/2}\,\tilde{t}\,|\,\psi_2''(\alpha_0)\,|}\right\}^{1/2}$$

$$\times\,\omega(\alpha_0)\sin\left\{\tilde{t}(l/g)^{1/2}\,\omega(\alpha_0)-\alpha_0 l\,\tilde{r}\right\} \quad \text{... (4.7)}$$

Also when the initial impulse is prescribed over a circle of radius a with centre at the origin, then

$$\overline{G}(\alpha_0)=\frac{P}{\pi a\alpha_0}J_1(a\alpha_0)\cdot$$

In this case, (4.4) produces

$$\tilde{\eta}(\tilde{r},\tilde{t}) \approx \frac{2P}{\rho g\pi(1-s)\,l^2\,\tilde{r}\,a\alpha_0}\left\{\frac{\alpha_0 l\,\tilde{r}}{(l/g)^{1/2}\,\tilde{t}\,|\,\psi_2''(\alpha_0)\,|}\right\}^{1/2}$$

$$\times\,\omega(\alpha_0)\,J_1(a\,\alpha_0)\sin\left\{\tilde{t}(l/g)^{1/2}\,\omega(\alpha_0)-\alpha_0 l\,\tilde{r}\right\} \quad \text{... (4.8)}$$

5. Discussion

To visualize the effect of the upper fluid and the presence of inertial surface at the interface on the wave motion generated by axially symmetric initial disturbance applied at an inertial interface between

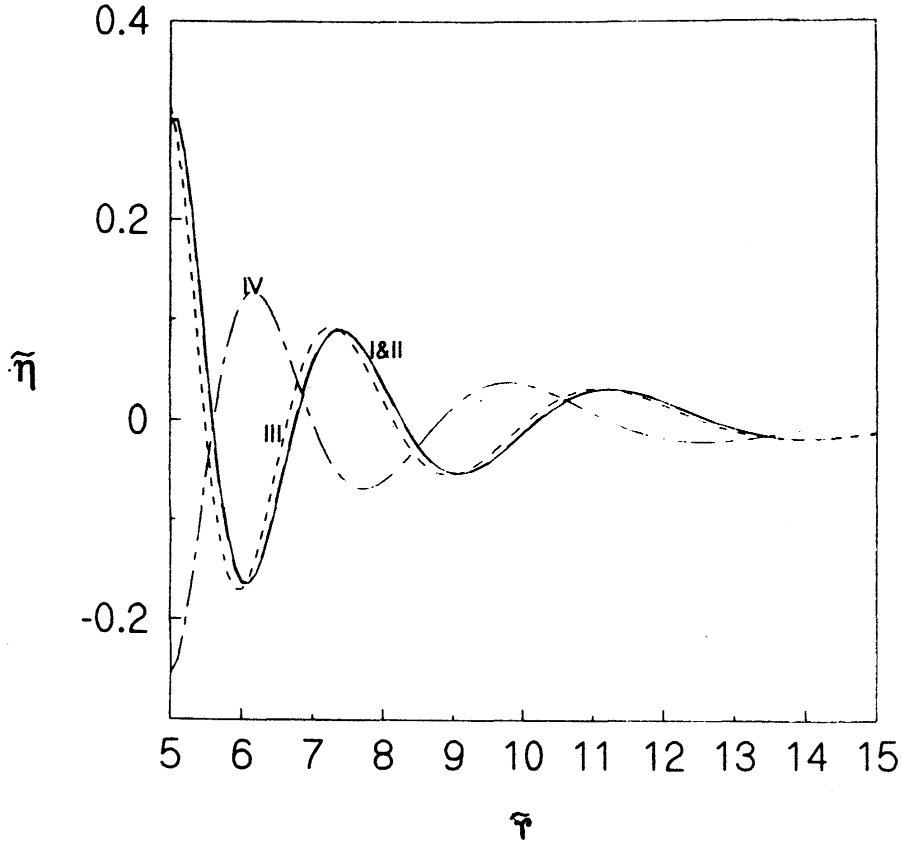
0.4
0.2
0
-0.2
IV
I&II
III
$\tilde{\eta}$
5 6 7 8 9 10 11 12 13 14 15
$\tilde{r}$

FIG. 1

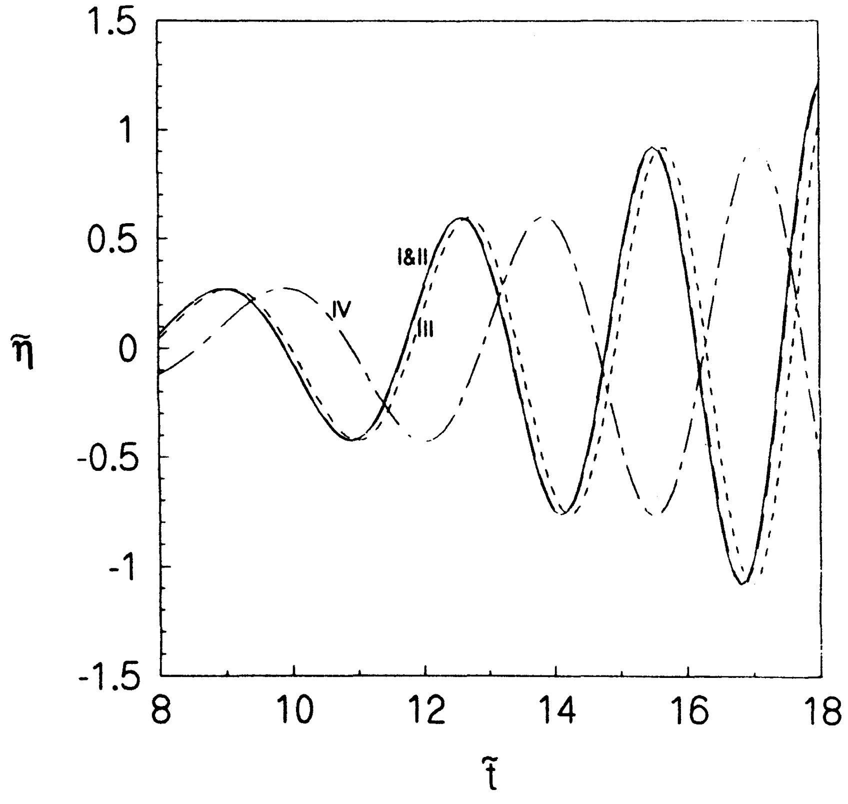
1.5
1
0.5
0
-0.5
-1
-1.5
I&II
IV
III
$\tilde{\eta}$
8 10 12 14 16 18
$\tilde{t}$

FIG. 2

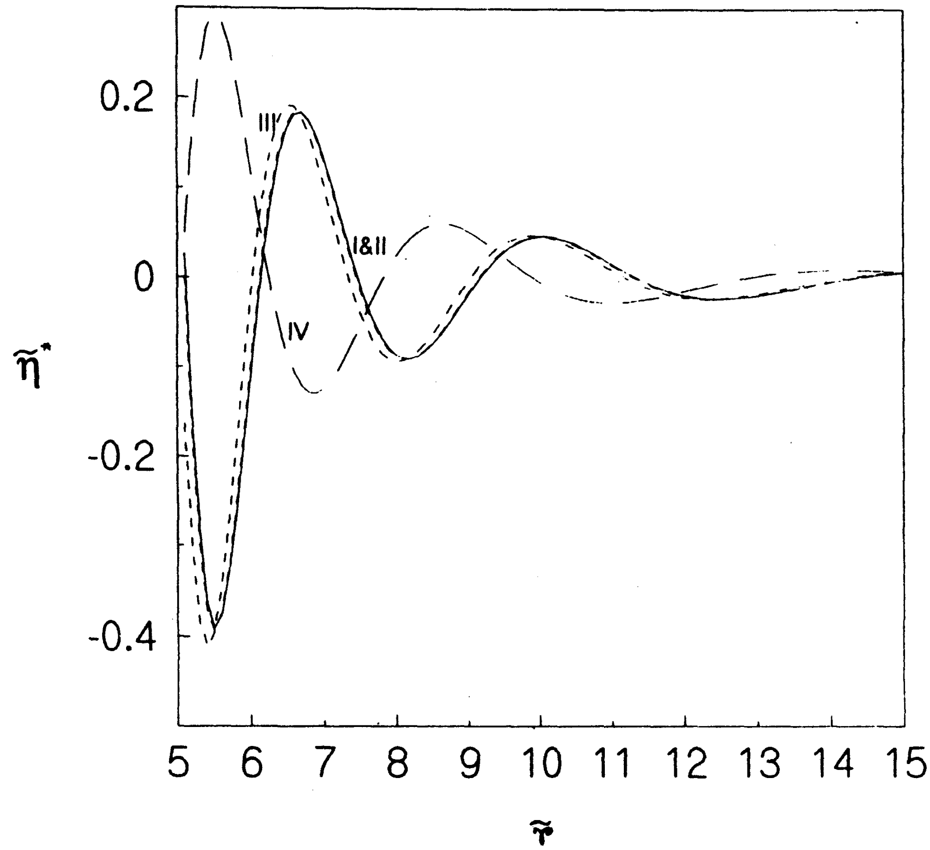
0.2
0
-0.2
-0.4
III
I&II
IV
η̃*
5 6 7 8 9 10 11 12 13 14 15
r̃

FIG. 3

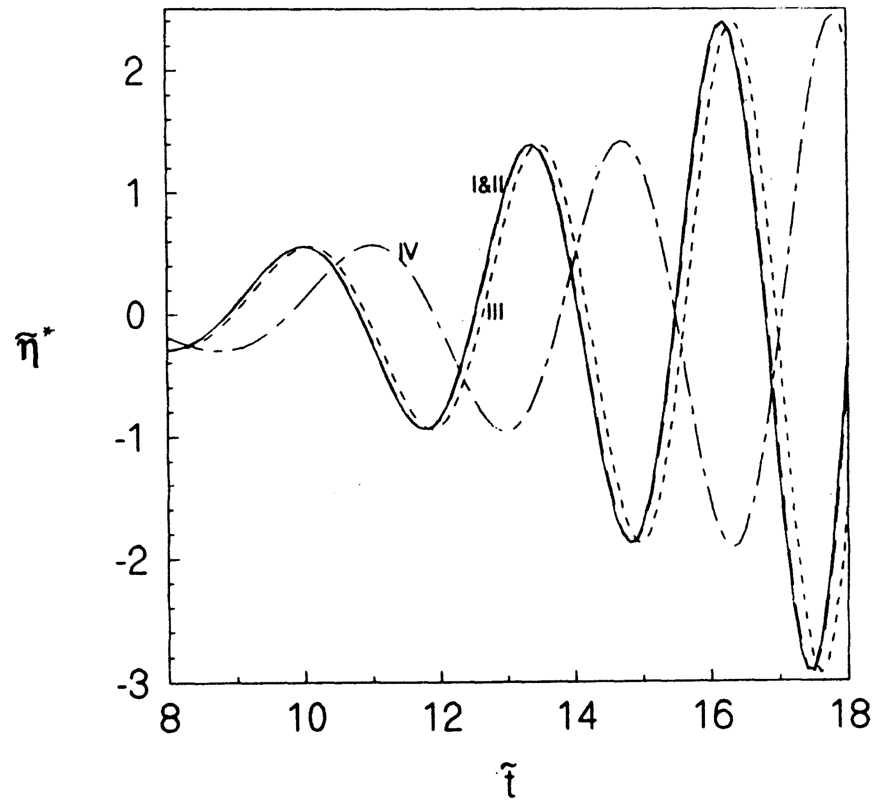
2
1
0
-1
-2
-3
I&II
IV
III
η̃*
8 10 12 14 16 18
t̃

FIG. 4

two superposed fluids, the interface profiles $\tilde{\eta}(\tilde{r},\tilde{t})$ and $\tilde{\eta}^*(\tilde{r},\tilde{t})\left(=\dfrac{\rho_0 g^{1/2} l^{7/2}\,\tilde{\eta}(\tilde{r},\tilde{t})}{P}\right)$ obtained from (4.5) and (4.7) respectively, are depicted graphically in a number of figures. For this purpose, the unique positive root $k=\alpha_0$ of the equation $\psi_2'(k)=0$ is obtained first. Obviously α_0 is a function of $\tilde{r},\tilde{t}$ and other parameters and is computed numerically for different values of $\tilde{r},\tilde{t}$ and other parameters.

The non-dimensional form of $\tilde{\eta}(\tilde{r},\tilde{t})$ obtained from (4.5) (due to an initial axially symmetric depression concentrated at the origin) is plotted graphically against $\tilde{r}$ between 5 to 15 for fixed $\tilde{t}$ = 20 in Fig. 1 and against $\tilde{t}$ between 8 to 18 for fixed $\tilde{r}=3$ in fig. 2 such that $\frac{\tilde{r}}{\tilde{t}}<\left|h/l(1-s)\right|^{1/2}$ for the following four cases:(I) s = 0.0 (ie, upper fluid is vaco); (II) s = 0.0013 (for air-water combination); (III) s = 0.01; (IV) s = 0.1. In all this cases we have taken ε/l = 0.01 and h/l = 1.

When the initial disturbances is in the form of initial depression over a circle whose centre is at the origin and radius a, then $\frac{l^3}{W}\tilde{\eta}(\tilde{r},\tilde{t})$ obtained from (4.6) is plotted graphically against $\tilde{r}$ between 5 to 15 for fixed $\tilde{t}$ in Fig. 3 and in Fig. 4, against $\tilde{t}$ between 8 to 18 for fixed $\tilde{r}$ = 3, for the same case as in Figs. 1 and 2 where a/l = 1 has been taken.

From Figs. 1 and 3, it is observed that for fixed $\tilde{t}$ when $\tilde{r}$ increases amplitude of the interface profile decreases so that at large distances they die out. Also the variation of $\tilde{\eta}(\tilde{r},\tilde{t})$ at a particular place with time is shown in Figs. 2 and 4. From Fig. 2, it is observed that for fixed $\tilde{r}$, when $\tilde{t}$ increases, amplitude of the wave motion increases. This is somewhat unrealistic and is consequence of the presence of a strong singularity at the origin as has also been remarked upon by Stoker[2] [p. 167] for the case of a single fluid. However, if the initial disturbance is spread over a finite region, the amplitude of the wave motion against $\tilde{t}$ shows an oscillatory behaviour which dies out ultimately. Fig. 4 depicts this to some extent. In fact, if we depict $\tilde{\eta}$ or $\tilde{\eta}^*$ for fixed $\tilde{r}$ against large $\tilde{t}$, this behaviour will be apparent. This is not shown here.

Again, if the density of the upper fluid is small (eg. s = 0.0013 for air-water combination) then the effect of the upper fluid is of not much significance (cf. I and II in figs. 1, 2, 3 and 4). As the density of the upper fluid increases to 0.1, say, there is not much change in the amplitude of the interface profile but the phase of the wave motion changes significantly. This implies that the presence of the upper fluid affects mainly the phase of the wave motion at the interface.

Again $\tilde{\eta}^*(\tilde{r},\tilde{t})$ obtained from (4.7) (due to an initial axially symmetric impulse concentrated at the origin) and $\tilde{\eta}^*(\tilde{r},\tilde{t})$ obtained from (4.9) (due to an initial axially symmetric impulse prescribed over a circle) when plotted for the same range and the same parameter values used in figs 1 and 2, where a/l = 1, shows similar behaviour as in Figs. 1 and 2.

TABLE I

	$\tilde{\eta}$		$\tilde{\eta}^*$	
$\tilde{r}$	ε = 0.0	ε = 0.01	ε = 0.0	ε = 0.01
5	0.162300	0.300757	0.617257	0.212410
8	0.051409	0.034592	−0.077781	−0.087504
11	0.028656	0.030214	0.019710	0.015212
14	−0.017880	−0.017984	−0.004710	−0.003762

For the case of air-water model (ie, s = 0.0013), a representative set of values of $\tilde{\eta}(\tilde{r}, \tilde{t})$ and $\tilde{\eta}^* (\tilde{r}, \tilde{t})$ (due to initial axi-symmetric depression concentrated at the origin) against $\tilde{r}$ between 5 to 15 for fixed $\tilde{t} = 20$ for the two cases, ε/l = 0.01 and ε/l = 0.0 respectively is given in Table I.

The entries on the first and second columns of Table I correspond to an ordinary interface and inertial interface respectively for s = 0.0013 due to an axially symmetric depression concentrated at the origin while third and fourth columns correspond to the same for an initial axially symmetric impulse concentrated at the origin. This table shows that the ordinary interface profile changes significantly when it is replaced by an inertial interface.

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