THREE DIMENSIONAL MHD LAMINAR SOURCE FLOW BETWEEN POROUS DISKS

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The three dimensional magnetohydrodynamic laminar source flow between two infinite rotating porous insulating disks is investigated including the effect of induced radial electric field. The solution is obtained by perturbing the source free flow solution using reduced source Reynolds number R_e^* as the perturbation parameter. Asymptotic solutions are obtained for velocity perturbation when $R_w << M^2$, R_w and M being injection or suction Reynolds number and Hartmann number respectively. The interaction of source, injection, magnetic field and induced radial electric field with the flow are discussed in detail. It is observed that increase in source Reynolds number R_e^* decreases the magnitude of the angular velocity and increases the magnitude of radial velocity. Further it is noted that due to induced radial electric field there is a marked change in angular velocity profiles.

1. Introduction

The steady flow of an incompressible viscous fluid between two infinite disks offers one of the few situations where exact solutions of Navier-Stokes equations are possible and it has been the subject of investigation by many authors. Stewartson (1953) has discussed both theoretically and experimentally the flow between two co-axial disks. He finds experimentally that when the disks are rotating in the same directions the main body of the fluid rotates as well but if they rotate in opposite directions the main body of the fluid is almost at rest. The numerical solution for the above problem has been obtained by Lance and Rogers (1962) for various ratio of the angular velocities and they have shown that at high Reynolds numbers the main core of the fluid is in a state of solid body rotation for practically all ratios of the angular velocities. Subsequent numerical analysis of Pearson (1965) shows a marked asymmetry with an internal angular velocity of greater magnitude than that of either plate. Meller, Chappel and Stokes (1968) have obtained numerical solutions for the flow between a rotating and a stationary disk for arbitrary Reynolds number. They find that the main body of the fluid rotates with a constant angular velocity and the boundary layer develops on both disks as the Reynolds number increases. The above investigations are confined to source free flows. However, the laminar source flow between two non-porous disks has been investigated by Puebe (1963) and Savage (1964) for non-conducting flows and their theoretical results are in agreement with Moller's (1963) experimental results. The source flow between two parallel non-porous disks rotating at the same velocity has been examined by Breitner and Pohlhausen (1962), Kreith and Puebe (1965) and Khan (1968). The same problem with disks rotating at different speeds has been analysed by Kreith and Viviand (1967) and they have discussed the flow pattern in terms of the single parameter K which is the ratio of Taylor numbers.

Three dimensional MHD laminar source flow between non-porous disks has been discussed by Khan (1970). In his analysis, Khan has not considered the effect of induced radial electric field which completely modifies the angular velocity profiles. The aim of the present analysis is to discuss the steady laminar source flow of an incompressible conducting fluid between two porous non-conducting disks with uniform injection or suction in the presence of a uniform transverse magnetic field including the effect of radial electric field. The solution is obtained by perturbing the creeping flow solution using R_e^* the reduced Reynolds number as the perturbation parameter. The interaction of source, injection and the magnetic field with the flow is discussed in detail in terms of the parameters, the Hartman number (M), the injection Reynolds number (R_w), the reduced Reynolds number R_e^* and the ratio of Taylor numbers (K). The results of the present problem are compared with those of Khan (1970) to understand the part played by Radial electric field on the velocity profiles.

2. Formulation of the Problem

A homogeneous incompressible viscous electrically conducting fluid having density ρ, kinematic viscosity ν and electrical conductivity σ is bounded by two infinite non-conducting porous disks at $z = \pm h$ and rotate with angular velocities Ω_a and Ω_b. A source of strength Q is placed at the centre of the channel formed by the disks. The fluid is injected or extracted through the disks with uniform velocity V. A uniform magnetic field B is applied in the z direction.

The governing equations of steady axisymmetric flow, including the contribution of the induced radial electric field $-\sigma B\chi(\Omega_a+\Omega_b)\bar{r}$ (see Stephenson 1969) and using the dimensionless variables

$$r=\frac{\bar{r}}{h},\quad z=\frac{\bar{z}}{h},\quad u=\frac{\bar{u}h}{\nu},\quad v=\frac{\bar{v}h}{\nu},\quad w=\frac{\bar{w}h}{\nu}\quad \text{and}\quad P=\frac{\bar{P}h^2}{\rho\nu^2}$$

are

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{\partial P}{\partial r}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}-M^2u \quad \ldots (1)$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}-M^2\{v-(R_1+R_2)r\chi\} \quad \ldots (2)$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial P}{\partial z}+\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2} \quad \ldots (3)$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \quad \ldots (4)$$

Equations (1) to (4) are solved using the boundary conditions

$$\left.\begin{aligned} u(r,\pm 1)&=0\\ v(r,+1)&=R_2 r\\ v(r,-1)&=R_1 r\\ w(r,\pm 1)&=\pm R_w=\pm\frac{M^2}{N}\end{aligned}\right\} \quad \ldots (5)$$

together with the mass conservation equation

$$\int_{-1}^{+1} u\,dz=\frac{M^2}{N}\,r+\frac{2R_e}{r} \quad \ldots (6)$$

where

(u, v, w) are the velocity components in the radial, azimuthal and axial directions,

P is the pressure,

r and z are the radial and axial co-ordinates,

$M=Bh\sqrt{\sigma/\nu\rho}$ is the Hartman number,

$R_1=h^2\Omega_a/\nu$ is the Taylor number for lower disk,

$R_2=h^2\Omega_b/\nu$ is the Taylor number for upper disk,

$\chi = \dfrac{\omega_{av}}{\Omega_a+\Omega_b}$ is a dimensionless parameter,

ω_{av} is the average angular velocity of the fluid,

$N = B^2\sigma h/\rho\nu$ is the interaction parameter,

$R_w = Vh/\nu$ is the injection or suction Reynolds number,

$R_e = Q/4\pi\nu h$ is the source Reynolds number.

We define the stream function ψ such that

$$u = \frac{1}{r}\frac{\partial\psi}{\partial z} \quad \ldots \quad (7)$$

$$w = -\frac{1}{r}\frac{\partial\psi}{\partial r} \quad \ldots \quad (8)$$

The following expressions which are valid for small value of the reduced Reynolds number R_e^* $(= R_e/r^2)$ and away from the source at $r = 0$ are assumed for ψ, P and v :

$$\psi = \tfrac{1}{2}r^2\,\frac{M^2}{N}\,f_{-1}(z)+R_e\left[f_0(z)+\frac{R_e}{r^2}f_1(z)+0\left(\frac{R_e}{r^2}\right)^2+\ldots\right]. \quad \ldots \quad (9)$$

$$P = \tfrac{1}{4}\,r^2\,\frac{M^2}{N}\,h_{-1}(z)+h(z)+R_e\left[h_0(z)\log r+\frac{R_e}{r^2}\,h_1(z)+0\left(\frac{R_e}{r^2}\right)^2+\ldots\right] \quad \ldots \quad (10)$$

$$v = \tfrac{1}{2}r\,\frac{M^2}{N}\,g_{-1}(z)+\frac{R_e}{r}\left[g_0(z)+\frac{R_e}{r^2}\,g_1(z)+0\left(\frac{R_e}{r^2}\right)^2+\ldots\right]. \quad \ldots \quad (11)$$

Since flow has been assumed fully developed, the zeroth order function will be independent of r.

The expressions for radial and axial velocities, from equation (9), are

$$u = \frac{1}{2}r\,\frac{M^2}{N}\,f'_{-1}(z)+\frac{R_e}{r}\left[f'_0(z)+\frac{R_e}{r^2}f'_1(z)+0\left(\frac{R_e}{r^2}\right)^2+\ \ldots\right] \quad \ldots \quad (12)$$

$$w = -\frac{M^2}{N}\,f_{-1}(z)+\left[2\left(\frac{R_e}{r^2}\right)^2 f_1(z)+0\left(\frac{R_e}{r^2}\right)^3+\ \ldots\right] \quad \ldots \quad (13)$$

where the primes denote the differentiation with respect to z.

The boundary conditions on f_n and g_n are

$$f'_n(\pm 1) = 0 \text{ for } n \geqslant -1$$
$$f_n(\pm 1) = 0 \text{ for } n \geqslant 1$$
$$f_{-1}(\pm 1) = \pm 1$$

and

$$f_0(1)-f_0(-1) = 2 \qquad \dots (14)$$

choosing $f_0(-1) = -1$, we have $f_0(1) = 1$

$$g_{-1}(-1) = 2R_1/R_w$$

$$g_{-1}(+1) = 2R_2/R_w$$

$$g_n(\pm 1) = 0 \text{ for } n \geqslant 0.$$

These expressions (12) and (13) are inserted into the governing equations and after the co-efficients of successive powers of r are equated to zero we get an infinite set of system of simultaneous equations. The first three systems are considered here.

System I

$$f'''_{-1}+\frac{M^2}{N}\left(f_{-1}f''_{-1}+\tfrac{1}{2}g^2_{-1}-\tfrac{1}{2}f'^2_{-1}\right) = h_{-1}+\frac{M^2}{N}f'_{-1} \qquad \dots (15)$$

$$g''_{-1}+\frac{M^2}{N}\left(f_{-1}g'_{-1}-f'_{-1}g_{-1}\right) = M^2g_{-1}-2M^2(R_1+R_2)\chi \qquad \dots (16)$$

$$h'_{-1} = 0 \quad \text{or} \quad h_{-1} = \text{constant} \qquad \dots (17)$$

$$h = -\frac{M^2}{N}\left(f'_{-1}+\tfrac{1}{2}\frac{M^2}{N}f^2_{-1}\right) + \text{constant}. \qquad \dots (18)$$

where the constant in equation (18) is determined from a known pressure at a point.

System II

$$f'''_0+\frac{M^2}{N}\left(f_{-1}f''_0+g_{-1}g_0\right) = h_0+M^2f'_0 \qquad \dots (19)$$

$$g''_0+\frac{M^2}{N}\left(f_{-1}g'_0-f'_0g_{-1}\right) = M^2g_0 \qquad \dots (20)$$

$$h'_0 = 0, \qquad h_0 = \text{constant}. \qquad \dots (21)$$

System III

$$f'''_1+\frac{M^2}{N}\left(f_{-1}f'_1+f_{-1}f''_1-f''_{-1}f_1\right)-M^2f'_1 = -2h_1-f'^2_0-g^2_0-g_{-1}g_1 \qquad \dots (22)$$

$$g''_1+\frac{M^2}{N}\left(f'_{-1}g_1-g_{-1}f'_1+f_{-1}g'_1-f_1g'_{-1}\right) = M^2g_1 \qquad \dots (23)$$

$$h'_1 = 0, \qquad h_1 = \text{constant}. \qquad \dots (24)$$

3. Asymptotic Analysis for $R_w \ll M^2$

The unknown functions f_n, g_n, h_n and χ are expanded in powers of $\frac{1}{N}$ (we note that in the limit $\frac{1}{N} \to 0$, $\frac{M^2}{N}$ remains finite and is equal to R_w) as follows

$$f_n = \sum_{\beta=0}^{\infty} \frac{1}{N^\beta} f_{n,\beta}$$

$$g_n = \sum_{\beta=0}^{\infty} \frac{1}{N^\beta} g_{n,\beta} \qquad \text{for } n = -1, 0, 1, 2, \ldots$$

$$h_n = \sum_{\beta=0}^{\infty} \frac{1}{N^\beta} h_{n,\beta} \qquad \ldots (25)$$

and

$$\chi = \sum_{\beta=0}^{\infty} \frac{1}{N^\beta} \chi_\beta$$

where $f_{n,\beta}$, $g_{n,\beta}$, $h_{n,\beta}$ and χ_β are independent of N.

The boundary conditions to be satisfied by $f_{n,\beta}$ and $g_{n,\beta}$ are

$$f'_{n,\beta}(\pm 1) = 0 \qquad \text{for } n = -1, 0, 1, 2, \ldots \quad \text{and all } \beta$$

$$f_{n,\beta}(\pm 1) = 0 \qquad \text{for } n = 1, 2, \ldots \quad \text{and all } \beta$$

$$f_{n,0}(\pm 1) = \pm 1 \qquad \text{for } n = -1, 0.$$

$$f_{n,\beta}(\pm 1) = 0 \qquad \text{for } n = -1, 0 \quad \text{and } \beta \geqslant 1$$

$$g_{-1,0}(+1) = \frac{2R_2}{R_w} \qquad \ldots (26)$$

$$g_{-1,0}(-1) = \frac{2R_1}{R_w}$$

$$g_{-1,\beta}(\pm 1) = 0 \qquad \beta \geqslant 1$$

$$g_{n,\beta}(\pm 1) = 0 \qquad n = 0, 1, 2, \ldots \quad \text{and all } \beta.$$

Solution of system I

Equations (15) and (16) using equation (25) and equating the coefficients of like powers of $1/N$ give the following set of linear ordinary differential equations.

$$f''''_{-1,0} = h_{-1,0} + M^2 f''_{-1,0} \qquad \ldots (27a)$$

$$f''''_{-1,1} - M^2 f''_{-1,1} = h_{-1,1} - M^2 (f_{-1,0} f'_{-1,0} + \tfrac{1}{2} g^2_{-1,0} - \tfrac{1}{2} f^2_{-1,0} \qquad \ldots (27b)$$

$$g''_{-1,0}-M^2 g_{-1,0} = -2M^2(R_1+R_2)\chi_0 \quad \dots (27c)$$

$$g''_{-1,1}-M^2 g_{-1,1} = M^2(f'_{-1,0}g_{-1,0}-f_{-1,0}g'_{-1,0})-2M^2(R_1+R_2)\chi_1 \quad \dots (27d)$$

The analysis is confined to first and second orders since the higher order perturbations are algebraically complicated and also the effect of higher order terms are negligible compared to second order terms.

Solutions of eqn. (27) using the boundary conditions (26) are

$$f_{-1,0} = \frac{A}{M}(Mz \cosh M - \sinh Mz) \quad \dots (28)$$

where,

$$A = M/(M \cosh M - \sinh M)$$

also,

$$h_{-1,0} = M^3/(\tanh M - M) \quad \dots (29)$$

$$f_{-1,1} = B_1 \cosh Mz + D_1 \sinh Mz - \frac{h_{-1,1}}{M^2} z + a_3 z + a_4 \sinh 2Mz$$

$$+a_5 z \cosh Mz + a_6 z^2 \sinh Mz + a_7 z \sinh Mz + a_8 \cosh 2Mz + C_1 \quad \dots (30)$$

where,

$$a_3 = \frac{1}{4}(a_0^2 - a_1^2 + 2a_2^2) - \frac{A^2}{2}(3/2 + \cosh^2 M)$$

$$a_4 = -\frac{1}{24M}(a_0^2 + a_1^2 + A^2)$$

$$a_5 = -\frac{1}{4}(2a_0 a_2 + 5A^2 \cosh M); \qquad a_6 = \frac{A^2}{4} M \cosh M.$$

$$a_7 = -\frac{a_1 a_2}{2}, \qquad a_8 = \frac{-a_0 a_1}{12M}$$

$$B_1 = \frac{-a_7}{M \sinh M}(\sinh M + M \cosh M) - 4a_8 \cosh M$$

$$D_1 = \frac{1}{k}[a_4(2M \cosh 2M - \sinh 2M) + a_5 M \sinh M + a_6(\sinh M + M \cosh M)]$$

$$C_1 = \frac{a_7}{2M \sinh M}(2M + \sinh 2M) + a_8(2 + \cosh 2M)$$

$$\frac{h_{-1,1}}{M^2} = a_3 + (D_1 + a_6)\sinh M + a_4 \sinh 2M + a_5 \cosh M$$

$$a_1 = \frac{\lambda_2}{2 \sinh M}, \qquad a_2 = 2(R_1+R_2)\chi_0, \qquad k = \sinh M - M \cosh M$$

$$\lambda_1 = 2(R_1+R_2)/R_w, \qquad \lambda_2 = 2(R_2-R_1)/R_w.$$

The solution for angular velocity is

$$g_{-1,0} = \frac{\lambda_1 \cosh Mz}{2 \cosh M} + \frac{\lambda_2 \sinh Mz}{2 \sinh M} + 2(R_1+R_2)\chi_0 \left(1 - \frac{\cosh Mz}{\cosh M}\right)$$

where

$$2\chi_0 = \int_{-1}^{+1} g_{-1,0}\, dz = \frac{\lambda_1 \tanh M}{M+2(R_1+R_2)(\tanh M - M)}$$

Similarly,

$$g_{-1,1} = E_1 \cosh Mz + F_1 \sinh Mz + b_1 z^2 \cosh Mz + b_2 z \sinh Mz$$
$$+ b_3 z^2 \sinh Mz + b_4 z \cosh Mz + 2(R_1+R_2)\chi_1 + b_5 \qquad \dots (32)$$

where,

$$b_1 = \frac{AM^2}{8}(2a_2 - \lambda_1), \qquad b_2 = \frac{AM}{8}(3\lambda_1 - 10a_2),$$

$$b_3 = -\frac{A}{8} M^2 \lambda_2 \coth M, \qquad b_4 = -\frac{3b_3}{M},$$

$$b_5 = A(a_0 - a_2 \cosh M),$$

$$E_1 = -[b_1 + b_2 \tanh M + \{b_5 + 2(R_1+R_2)\chi_1\} \operatorname{sech} M],$$

$$F_1 = [b_4 \coth M - b_3],$$

$$2\chi_1 = \int_{-1}^{+1} g_{-1,1} dz = \frac{2}{M+2(R_1+R_2)(\tanh M - M)} \left[\frac{b_2(2M - \sinh 2M)}{2M \cosh M} + \right.$$
$$\left. + (M - \tanh M)\left(b_5 - \frac{2b_1}{M^2 \cosh M}\right)\right]$$

The differential equations governing the system II are

$$f''''_{0,0} - M^2 f''_{0,0} = h_{0,0} \qquad \dots (33a)$$

$$f''''_{0,1} - M^2 f''_{0,1} = h_{0,1} - M^2(f_{-1,0} f''_{0,0} + g_{-1,0} g_{0,0}) \qquad \dots (33b)$$

$$g''_{0,0} - M^2 g_{0,0} = 0 \qquad \dots (33c)$$

$$g''_{0,1} - M^2 g_{0,1} = M^2(f'_{0,0} g_{-0,1} - f_{-1,0} g'_{0,0}) \qquad \dots (33d)$$

The differential equations governing the system III are

$$f'''_{1,0} - M^2 f'_{1,0} = -2h_{1,0} - f'^2_{0,0} - g^2_{0,0} - g_{-1,0} g_{1,0} \qquad \dots (34a)$$

$$g''_{1,0} - M^2 g_{1,0} = 0 \qquad \dots (34b)$$

$$g''_{1,1} - M^2 g_{1,1} = M^2(f_{1,0} g'_{-1,0} - f_{-1,0} g'_{1,0} + g_{-1,0} f'_{1,0} - f'_{-1,0} g_{1,0}) \qquad \dots (34c)$$

The solutions for systems II and III are obtained similarly as in the previous case. Since the expressions are lengthy, they are not given. However, the expressions are numerically evaluated and the typical velocity profiles are presented including the contributions from systems II and III.

4. Velocity Distribution

The radial, azimuthal and axial velocity components u^*, v^* and w are respectively given by

$$u^* = \frac{u}{r} = \frac{1}{2} \frac{M^2}{N} f'_{-1} + R_e^*[f'_0 + R_e^* f'_1 + 0(R_e^*)^2]$$

$$v^* = \frac{v}{r} = \frac{1}{2} \frac{M^2}{N} g'_{-1} + R_e^*[g_0 + R_e^* g_1 + 0(R_e^*)^2]$$

$$w = -\frac{M^2}{N} f_{-1} + [2R_e^{*2} f_1 + 0(R_e^*)^3].$$

The velocity components are numerically computed for different values of R_e^*, R_w, M and $K = 1$, 0, -1 and the results are plotted in figures (2) to (7). Typical behaviour is observed for other values of these parameters. In figure (2) the azimuthal velocity profiles are drawn for the case $K = 1$. We find that for small values of M the velocity is parabolic and the angular velocity is negative in almost the entire space between the disks except for a small region near the disk. For a given value of R_w, R_e^* and K the increase in magnetic field flattens the velocity profiles and the velocity becomes uniform in central region and the velocity gradients are confined to narrow region near the disks. In figure (3), the angular velocity profiles for the case $K = 0$ are drawn. We observe that the angular velocity is always positive. The boundary layer develops near the disks and a uniform rotation appears in

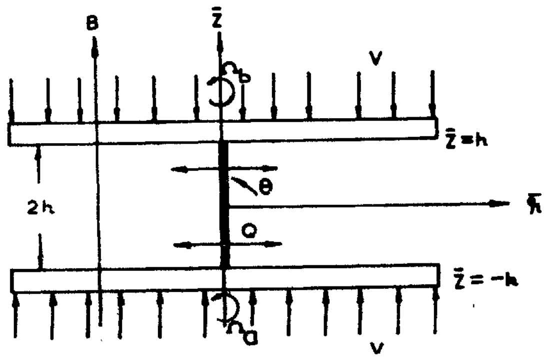

Fig. 1. Physical model.

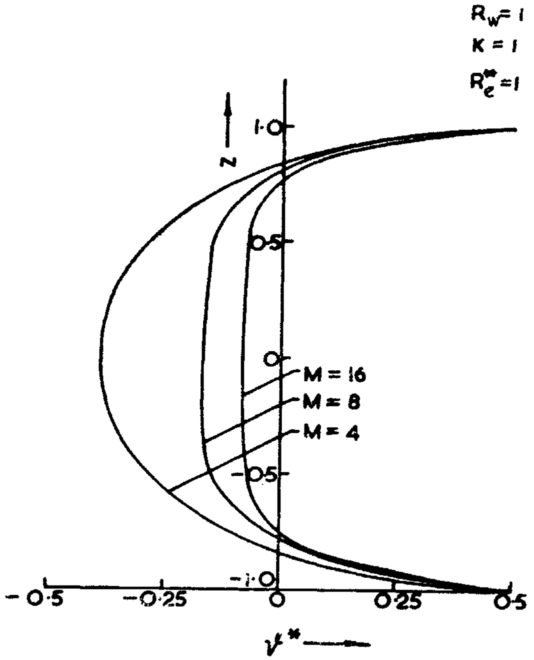

Fig. 2. Azimuthal velocity profiles when both disks are rotating in the same direction.

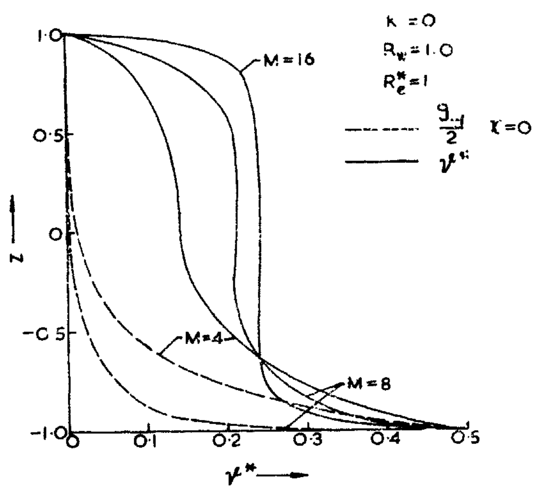

Fig. 3. Azimuthal velocity profiles when one disk is rotating and the other is stationary.

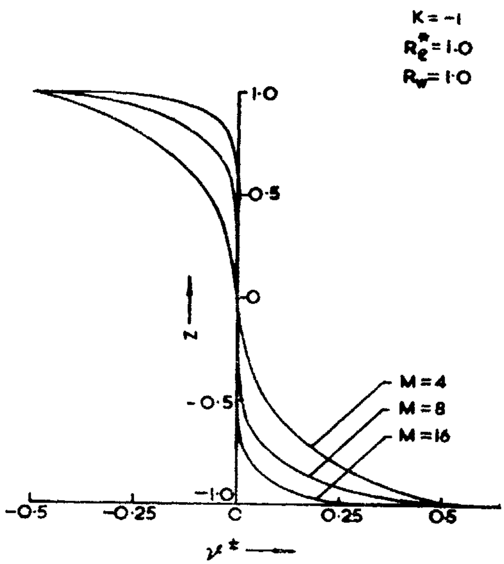

Fig. 4. Azimuthal velocity profiles when the disks are rotating in opposite directions.

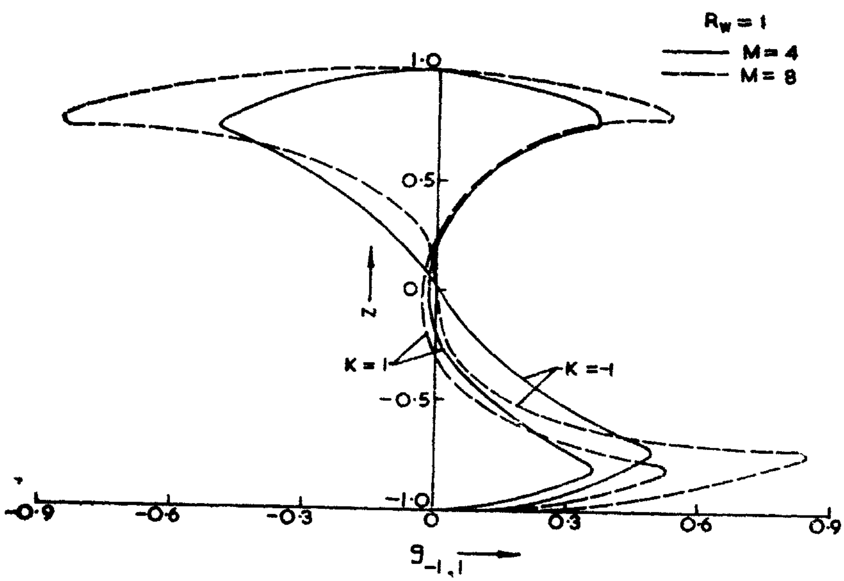

Fig. 5. Azimuthal velocity perturbation.

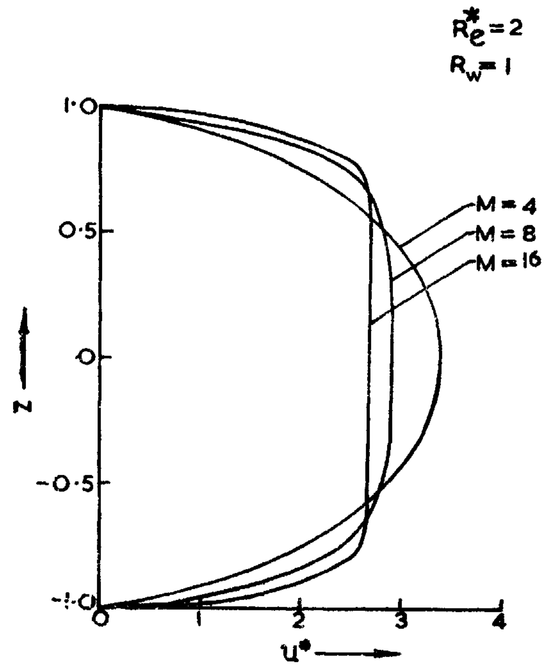

Fig. 6. Radial velocity profiles.

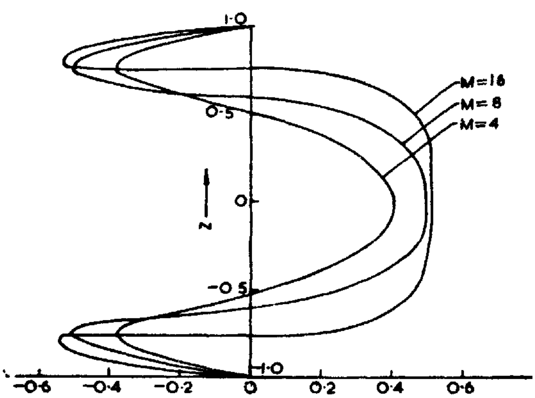

Fig. 7. Radial velocity perturbation.

the main body of the fluid. We find that the increase in magnetic field decreases boundary layer thickness. In figure (4) the angular velocity profiles for the case $K = -1$ are shown. It is observed that the angular velocity is antisymmetric about the z-axis and for large values of M, other parameters remaining constant, the angular velocity is zero in central region and has a finite value near the disks. This is in agreement with Stewartson (1953). In all these three cases the increase in injection Reynolds number R_w affects the angular velocity by increasing the magnitude. We also find that increase in source Reynolds number R_e^* affects the magnitude of the angular velocity slightly. If we compare the figures (5) and (7), with those of Khan we find that these velocity perturbations are completely modified by the effect of induced radial electric field and injection. In figure (6), the radial velocity u^* is shown for $R_e^* = 2$, $R_w = 1$ and for different values of M. The typical behaviour is observed for all the three cases, viz., $K = 1$, 0 and -1. We find that the radial velocity is parabolic for small value of M and as M increases the radial velocity exhibits the characteristic flattening. Radial velocity increases with increase in injection Reynolds number R_w and the source Reynolds number R_e^* while other parameters remain constant. The above analysis is also valid for suction Reynolds number $R_w < 1$.

5. Pressure Distribution

The pressure drop at the disk is

$$P^* = P(r, 1) - P(R, 1)$$

$$= -\frac{1}{4}\frac{M^2}{N}\left(h_{-1,0} + \frac{1}{N}h_{-1,1}\right)(R^2 - r^2)$$

$$-R_e\left[\left(h_{0,0} + \frac{1}{N}h_{0,1}\right)\log\frac{R}{r} + \frac{R_e}{R^2}h_{1,0}\left(1 - \frac{R^2}{r^2}\right)\right] \quad \dots (35)$$

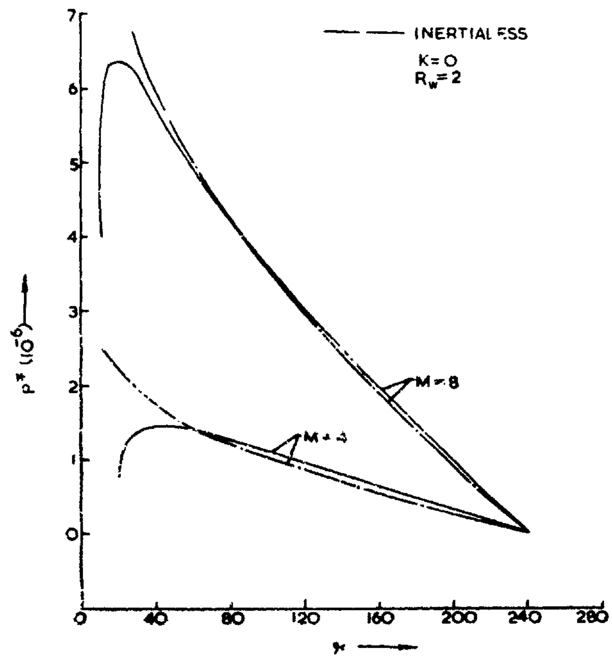

Fig. 8. Pressure drop in the radial direction.

where we assume that $P(R, 1)$ is a known pressure at the point $(R, 1)$.

The inertialess or creeping flow pressure drop is obtained by neglecting the contribution from inertia terms in equation (35) and is given by

$$P^* = -\frac{1}{4}\frac{M^2}{N}\, h_{-1,0}(R^2-r^2)-R_e h_{0,0}\log\frac{R}{r} \qquad \dots (36)$$

The pressure distribution is presented in figure (8) for

$$R_e = 3\times 10^4, \quad R = 240, \quad R_w = 2, \quad K = 0, \quad M = 4, 8.$$

and we observe that the inertia terms cause a decrease in pressure in the radial direction. The total pressure in the presence of magnetic field is higher than the corresponding hydrodynamic case. Therefore our results may find application in the design of magnetohydrodynamic porous bearing system in the sense that the increase in pressure may be utilized to increase the load carrying capacity of a bearing.

6. Skin Friction

The dimensionless shear stress at the upper disk is

$$\tau_0 = \left[\bar{\tau}_0\left(\frac{h^2}{\rho\nu^2}\right)\right]_{\bar{z}=h}^{*} = \left[-\mu\left(\frac{h^2}{\rho\nu^2}\right)\frac{\partial\bar{u}}{\partial\bar{z}}\right]_{\bar{z}=h} = -\left(\frac{\partial u}{\partial z}\right)_{z=1} \qquad \dots (37)$$

Equation (37) using expression for u given by (12) becomes

$$\tau_0 = -\left[\frac{1}{2}\, r\, \frac{M^2}{N}\left(f''_{-1,0}+\frac{f''_{-1,1}}{N}\right)+\frac{R_e}{r}\left(f''_{0,0}+\frac{f''_{0,1}}{N}\right)\right]_{z=1} \qquad \dots (38)$$

The inertialess shear stress at the upper disk which is obtained by neglecting the inertia terms in equation (38) is

$$(\tau_0)_{inertialess} = -\left[\frac{1}{2}\, r\, \frac{M^2}{N} f''_{-1,0}+\frac{R_e}{r} f''_{0,0}\right]_{z=1}$$

The shear stress ratio

$$\tau_0^* = \frac{\tau_0}{(\tau_0)_{inertialess}}$$

is evaluated for $R_e^* = 0, 1, 2, 3$, $R_w = 1, 2$, $M = 4, 8$, $K = 0$. The results are presented in Table I. We observe that the shear stress ratio increases with an increase in M for a given R_e^* and R_w. Further we observe that for a given M and K it decreases with increasing source Reynolds number R_e^* and injection Reynolds number R_w. We note that the incipient flow reversal (i.e., $\tau^* = 0$) occurs at very large value of R_e^*.

Table I

Shear stress ratio τ^*_0

$R_w = 1$		$K = 0$	
$M^2 = 16$		$M^2 = 64$	
R^*_e	τ^*_0	R^*_e	τ^*_0
0	0·9244	0	0·9505
1	0·8942	1	0·9415
2	0·8612	2	0·9319
3	0·8301	3	0·9222

$R_w = 2$		$K = 0$	
$M^2 = 16$		$M^2 + 64$	
R^*_e	τ^*_0	R^*_e	τ^*_0
0	0·8551	0	0·9028
1	0·8232	1	0·8934
2	0·7912	2	0·8837
3	0·7577	3	0·8740

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