THE INVERSION OF A CONVOLUTION TRANSFORM

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Using the methods of operational calculus the inversion integral for the integral transformation involving $K_\beta(x, n)$ (Erdelyi 1955, p. 215) as the kernel has been obtained.

1. Introduction

Following the methods of operational calculus the inversion integrals for the integral equations with suitable functions as the kernel have been attempted by many authors. This paper also aims to obtain an analogous result with a new kernel.

The solutions of the differential equation

$$\frac{d^n y}{dx^n} + y = 0$$

give rise to the trigonometric functions $K_\beta(x. n)$ of order n which can be represented as (Erdelyi 1955, p. 215)

$$K_\beta(x, n) = \sum_{r=0}^{\infty} \frac{(-1)^r x^{nr+\beta-1}}{(nr+\beta-1)!}. \qquad \ldots \quad (1.1)$$

If λ is an nth root of -1, then

$$K_\beta(\lambda x, n) = \lambda^{\beta-1} h_\beta(x, n) \qquad \ldots \quad (1.2)$$

where all $h_\beta(x, n)$ satisfy the differential equation

$$\frac{d^n y}{dx^n} - y = 0.$$

By specializing we get

$$h_1(x) = E_n(x^n) \qquad \ldots \quad (1.3)$$

and

$$h_\beta(x) = x^{\beta-1} E_{n,\beta}(x^n) \qquad \ldots \quad (1.4)$$

where $E_n(x)$ and $E_{n,\beta}\ (x)$ are the Mittag Leffler's and generalized Mittag Leffler's function (Erdelyi 1955, p.p 206, 210) respectively.

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The present paper is devoted to establish an inversion integral with $K_\beta(x, n)$ as the kernel.

The Laplace transform of the function $f(t)$ is given by

$$\int_0^\infty e^{-pt} f(t)dt = F(p), \quad \text{Re } p > 0 \qquad \dots \quad (2.1)$$

provided the above integral exists. We shall denote (2.1) symbolically by

$$f(t) \doteqdot F(p).$$

The following results are known (Erdelyi 1954, pp. 129, 131, 133) and will be used in the sequel.

$$f^n(t) \doteqdot p^n F(p) - p^{n-1} f(0) - p^{n-2} f'(0) - \dots - f^{n-1}(0) \qquad \dots \quad (2.2)$$

$$\int_0^t f_1(u)\, f_2(t-u)\, du \doteqdot g_1(p).g_2(p) \qquad \dots \quad (2.3)$$

where

$$f_1(t) \doteqdot g_1(p)$$

and

$$f_2(t) \doteqdot g_2(p)$$

$$t^n \doteqdot \frac{\Gamma(n+1)}{p^{n+1}}. \qquad \dots \quad (2.4)$$

If

$$f(t) \doteqdot F(p)$$

then

$$e^{-at} f(t) \doteqdot F(p+a). \qquad \dots \quad (2.5)$$

From the results Erdelyi (1955, p. 216), we get

$$K_\beta(x, n) \doteqdot \frac{p^{n-\beta}}{p^n + 1}, \qquad \text{Re } p > 0, \quad i = 1, 2, \dots, n) \qquad \dots \quad (2.6)$$

Theorem—If

(i) the functions $f(x)$ and its first β derivatives are continuous in $0 \leqslant x < x_1 < \infty$ and

(ii) $f^m(0) = 0$ for $0 \leqslant m \leqslant (\beta - 1)$

then

$$\int_0^x K_\beta\,[(x-t), n]\; g(t)dt = f(x) \qquad \dots \quad (3.1)$$

has the solution

$$g(x) = f^\beta(x) - [\,\Gamma(n-\beta)]^{-1} \int_0^x (x-t)^{n-\beta-1} f(t)\, dt \qquad \dots \quad (3.2)$$

Proof: Let $f(t) \doteqdot F(p)$ and $g(t) \doteqdot G(p)$. Then from (2.2) we get

$$f^{\beta}(t) \doteqdot p^{\beta}F(p). \qquad \ldots \quad (3.3)$$

Now taking the Laplace transform of (3.1) in the light of (2.3), (2.6) and rearranging the result we obtain,

$$G(p) = p^{\beta}F(p) + \frac{1}{p^{n-\beta}} F(p).$$

Laplace inversion in view of (2.3) and (3.3) thus establishes the theorem.

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