EFFECT OF SUCTION ON UNSTEADY FREE CONVECTION FLOW PAST A VERTICAL FLAT PLATE

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(Communicated by F. C. Auluck, F.N.A.)

(*Received* 25 *July* 1973)

Unsteady laminar free convection flow past a vertical infinite flat plate subjected to time-dependent suction is considered when the plate temperature varies as some power of time. Series solutions for velocity and temperature are obtained in terms of known functions when the Prandtl number of the fluid is close to unity. Effect of suction/Prandtl number on them is discussed numerically.

1. INTRODUCTION

Rao (1961) and Nanda and Sharma (1962) investigated the unsteady laminar free convection flow past a vertical flat plate when there is a normal velocity of suction at the plate. They have carried out their analyses for fluids when the Prandtl number is unity; the surface temperature was taken as some power of t (time) and the suction velocity proportional to $t^{-1/2}$. In this paper we have analysed the case when the Prandtl number is close to unity. Similar problem, in the absence of suction, has recently been solved by Pop (1969).

Expressions for temperature and velocity profiles in the form of series, are obtained in terms of known functions and effect of suction/Prandtl number on them is studied numerically.

2. MATHEMATICAL ANALYSIS

The dimensionless boundary layer equations which describe the unsteady free convection flow past a vertical infinite flat plate are (Nanda and Sharma 1962)

$$\frac{\partial v}{\partial y} = 0 \qquad ...(2.1)$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G \qquad ...(2.2)$$

$$\frac{\partial G}{\partial t} + v\frac{\partial G}{\partial y} = \frac{1}{\sigma}\cdot\frac{\partial^2 G}{\partial y^2} \qquad ...(2.3)$$

where $\bar{y} = y/L$, $t = \nu \bar{t}/L^2$, $u = \bar{u}L/\nu$, $v = \bar{v}L/\nu$, $G = g\beta L^3(\bar{T} - \bar{T}_\infty)/\nu^2$, $p = p\bar{L}/\rho\nu^2$ and $(\bar{u}, \bar{v})$ are the components of velocity in the $\bar{x}$ (along the plate) and $\bar{y}$ (normal to the plate) directions respectively, σ the Prandtl number and rest of the notations have their usual meanings.

From (2.1) it is clear that v is a function of time only. Hence we consider v in the form of

$$v = -v_0(t) = -V_0 t^{-1/2} \qquad \text{...(2.4)}$$

where V_0 (> 0) represents the velocity of suction. Substituting the value of v from (2.4) in (2.2) and (2.3), we get

$$\frac{\partial u}{\partial t} - V_0 t^{-1/2} \cdot \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G \qquad \text{...(2.5)}$$

$$\frac{\partial G}{\partial t} - V_0 t^{-1/2} \cdot \frac{\partial G}{\partial y} = \frac{1}{\sigma} \cdot \frac{\partial^2 G}{\partial y^2} \cdot \qquad \text{...(2.6)}$$

Equations (2.5) and (2.6) are to be solved under the following set of initial and boundary conditions

$$\left.\begin{aligned} &u(y, 0) = G(y, 0) = 0 \text{ for } y \geqslant 0 \\ &u(0, t) = u(\infty, t) = G(\infty, t) = 0 \\ &G(0, t) = at^{\alpha} \end{aligned}\right\} \qquad \text{...(2.7)}$$

where a and α ($\geqslant 0$) are constants.

3. Solution of the Problem

Following Pop (1969), the expressions for temperature and velocity functions are given by

$$G = at^{\alpha} \sum_{n=0}^{\infty} (1 - \sigma)^n \theta_n(\eta) \qquad \text{...(3.1)}$$

$$u = at^{\alpha+1} \sum_{n=0}^{\infty} (1 - \sigma)^n u_n(\eta) \qquad \text{...(3.2)}$$

where θ_n and u_n are unknown functions and $\eta = \frac{1}{2} yt^{-1/2}$. Also we can write the expression for Prandtl number as

$$\frac{1}{\sigma} = 1 + (1 - \sigma) + (1 - \sigma)^2 + \dots . \qquad \text{...(3.3)}$$

Substituting these values of G, u and σ in eqns. (2.5) and (2.6) and equating the coefficients of different powers of $(1 - \sigma)$, up to $(1 - \sigma)^2$, we get

$$u_n'' + 2(\eta + V_0)\, u_n' - 4(\alpha + 1)\, u_n = -\, 4\theta_n;\ (n = 0, 1, 2) \quad ...(3.4)$$

and

$$\left.\begin{aligned} \theta_0'' + 2(\eta + V_0)\, \theta_0' - 4\alpha\theta_0 &= 0 \\ \theta_1'' + 2(\eta + V_0)\, \theta_1' - 4\alpha\theta_1 &= -\, \theta_0'' \\ \theta_2'' + 2(\eta + V_0)\, \theta_2' - 4\alpha\theta_2 &= -\, (\theta_0'' + \theta_1'') \end{aligned}\right\} \quad ...(3.5)$$

where the primes denote differentiation with respect to η. The boundary conditions (2.7) reduce to

$$u_n(0) = u_n(\infty) = 0;\ (n = 0, 1, 2) \quad ...(3.6)$$

$$\left.\begin{aligned} &\theta_0(0) = 1,\ \theta_0(\infty) = 0, \\ &\theta_n(0) = \theta_n(\infty) = 0; \qquad (n = 1, 2) \end{aligned}\right\} \quad ...(3.7)$$

Solution of eqns. (3.5) satisfying the boundary conditions (3.7), may be written as

$$\left.\begin{aligned} \theta_0(\eta) &= \frac{Hh_{2\alpha}(\sqrt{2}\,\xi)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} \\ \theta_1(\eta) &= \tfrac{1}{2} \cdot \frac{Hh_{2\alpha-2}(\sqrt{2}\,V_0)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} \left[\frac{Hh_{2\alpha}(\sqrt{2}\,\xi)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} - \frac{Hh_{2\alpha-2}(\sqrt{2}\,\xi)}{Hh_{2\alpha-2}(\sqrt{2}\,V_0)}\right] \\ \theta_2(\eta) &= \tfrac{1}{2} \cdot \frac{Hh_{2\alpha-2}(\sqrt{2}\,V_0)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} \left\{1 + \tfrac{1}{2} \cdot \frac{Hh_{2\alpha-2}(\sqrt{2}\,V_0)}{Hh_{2\alpha}(\sqrt{2}\,V_0)}\right\} \\ &\quad \times \left[\frac{Hh_{2\alpha-2}(\sqrt{2}\,\xi)}{Hh_{2\alpha-2}(\sqrt{2}\,V_0)} - \frac{Hh_{2\alpha}(\sqrt{2}\,\xi)}{Hh_{2\alpha}(\sqrt{2}\,V_0)}\right] \\ &\quad - \tfrac{1}{8} \cdot \frac{Hh_{2\alpha-4}(\sqrt{2}\,V_0)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} \left[\frac{Hh_{2\alpha-4}(\sqrt{2}\,\xi)}{Hh_{2\alpha-4}(\sqrt{2}\,V_0)} - \frac{Hh_{2\alpha}(\sqrt{2}\,\xi)}{Hh_{2\alpha}(\sqrt{2}\,V_0)}\right] \end{aligned}\right\} \quad ...(3.8)$$

where $\xi = \eta + V_0$ and function $Hh_{2\alpha}(\sqrt{2}\,\xi)$ is defined by Jeffreys and Jeffreys (1962); in particular $Hh_0(\sqrt{2}\,\xi) = (\pi/2)^{1/2}\, Erfc\,(\xi)$.

Now substituting the values of $\theta_n(\eta)$ from (3.8) into (3.4) and solving the equations under the boundary conditions (3.6), we get

$$u_0(\eta) = \frac{Hh_{2\alpha}(\sqrt{2}\,\xi)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} - \frac{Hh_{2\alpha+2}(\sqrt{2}\,\xi)}{Hh_{2\alpha+2}(\sqrt{2}\,V_0)}$$

$$u_1(\eta) = \tfrac{1}{4} \cdot \frac{Hh_{2\alpha-2}(\sqrt{2}\,V_0)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} \left[2\, \frac{Hh_{2\alpha}(\sqrt{2}\,\xi)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} - \frac{Hh_{2\alpha+2}(\sqrt{2}\,\xi)}{Hh_{2\alpha+2}(\sqrt{2}\,V_0)} - \frac{Hh_{2\alpha-2}\,(\sqrt{2}\,\xi)}{Hh_{2\alpha-2}\,(\sqrt{2}\,V_0)}\right]$$

$$u_2(\eta) = \tfrac{1}{2} \cdot \frac{Hh_{2\alpha-2}(\sqrt{2}\,V_0)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} \left\{1 + \tfrac{1}{2} \cdot \frac{Hh_{2\alpha-2}(\sqrt{2}\,V_0)}{Hh_{2\alpha}(\sqrt{2}\,V_0)}\right\}$$

$$\times \left[\tfrac{1}{2} \cdot \frac{Hh_{2\alpha-2}(\sqrt{2}\,\xi)}{Hh_{2\alpha-2}(\sqrt{2}\,V_0)} - \frac{Hh_{2\alpha}(\sqrt{2}\,\xi)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} + \tfrac{1}{2} \cdot \frac{Hh_{2\alpha+2}(\sqrt{2}\,\xi)}{Hh_{2\alpha+2}(\sqrt{2}\,V_0)}\right]$$

$$- \tfrac{1}{4} \cdot \frac{Hh_{2\alpha-4}(\sqrt{2}\,V_0)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} \left[\tfrac{1}{6} \cdot \frac{Hh_{2\alpha-4}(\sqrt{2}\,\xi)}{Hh_{2\alpha-4}(\sqrt{2}\,V_0)}\right.$$

$$\left. - \tfrac{1}{2} \cdot \frac{Hh_{2\alpha}(\sqrt{2}\,\xi)}{Hh_{2\alpha}(\sqrt{2}\,V_0)} + \tfrac{1}{3} \cdot \frac{Hh_{2\alpha+2}(\sqrt{2}\,\xi)}{Hh_{2\alpha+2}(\sqrt{2}\,V_0)}\right].$$

In order to get a physical understanding of the problem, numerical calculations are carried out for the temperature and the velocity profiles and results are shown graphically for stepwise variations in the plate temperature ($\alpha = 0$). For the temperature distribution, Fig. 1 shows that at a given position increase in the suction results in a decrease in temperature. Figure 2 shows that for fixed Prandtl number, the velocity at any point decreases with increase in suction velocity or in other words suction increases the width of the viscous boundary layer.

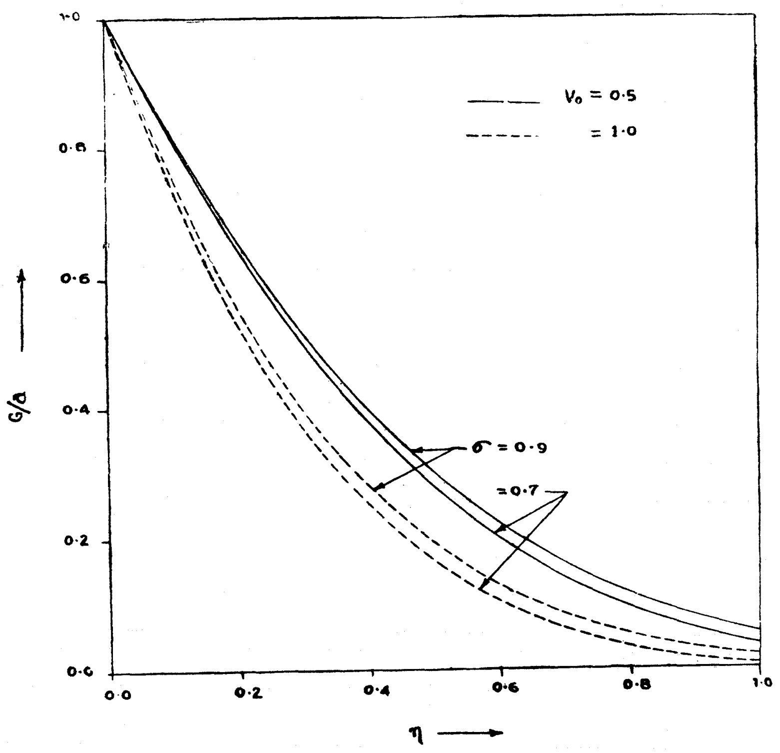

FIG. 1. Temperature profiles for $\alpha = 0$.

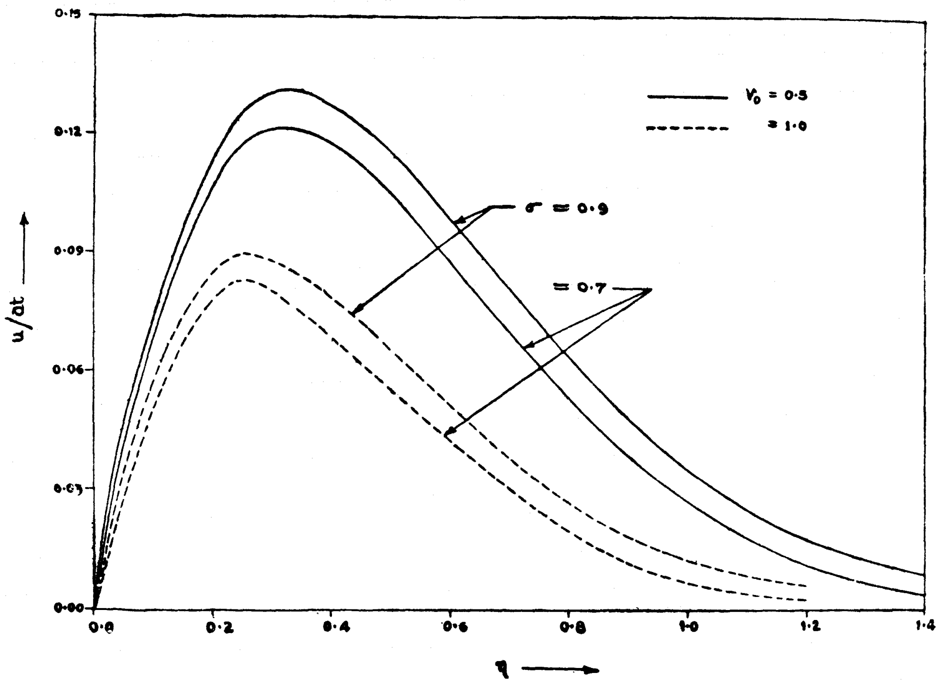

FIG. 2. Velocity profiles for $\alpha = 0$.

Further, the Prandtl number of the fluid is the governing parameter for the plate temperature. An increase in the Prandtl number leads to an increase in the temperature and the velocity of the fluid.

ACKNOWLEDGEMENT

The author is grateful to Dr. R. R. Aggarwal, Deputy Director, for his interest and encouragement and to the Director for the permission to publish this paper.

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