A NOTE ON THE DISTURBANCES IN A PIEZOELECTRIC MATERIAL UNDER THE INFLUENCE OF AN ELECTRON STREAM AND WITH A SEMI-CONDUCTING BOUNDARY LAYER

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The present paper is an attempt to investigate disturbances in a piezoelectric medium characterized by dissipative mechanical parameters. The paper brings out distinctive features between the dissipative and non-dissipative cases.

1. INTRODUCTION

A large number of studies concerning disturbances in piezoelectric medium have been undertaken in recent years, vide Sinha (1965), Ray (1969) and Chakravarty (1970). But all the studies have been confined to materials with non-dissipative mechanical parameters. It would, therefore, be worthwhile to investigate how piezoelectric materials respond to dissipative effects. Even though the task becomes too complicated, it would be interesting to study disturbances if they are excited by electron stream. The present note is an effort of this nature. The study for non-dissipative case has been done by Kaliski (1965a, b). The analysis presented here proceeds on the lines shown by Kaliski (1965a, b). It goes further in depth by studying the different amplifications of the disturbances in all the aspects. The distinctive features of such problems have been brought out in this problem. Some comparisons with non-dissipative cases have been referred to.

STATEMENT OF THE PROBLEM, FUNDAMENTAL EQUATIONS

We consider a piezo-quartz plate with a semi-conducting boundary layer and its mechanical parameter is characterized by dissipative properties. The excitation takes place by means of an electron stream. Our problem is to investigate the propagation of electro-mechanical waves in the material in its longitudinal direction. We consider the quartz plate with x_3 cut and width in the x_2-direction. The waves are taken to propagate longitudinally in the x_1-direction. The fundamental equations are those of electricity and of mechanics of continua, supplemented by constitutive equations.

The equation of longitudinal vibration (mechanical) of the plate is given by

$$\rho \ddot{u} = \sigma_{,1} \quad \text{...(1.1)}$$

where u is the mechanical displacement along x_1, σ the normal stress and ρ the density of the material.

The constitutive equations of the material for a piezoquartz plate are given by

$$\sigma = \left(c_0 + c_1 \frac{\partial}{\partial t} \right) u - eE \quad \text{...(1.2)}$$

$$D = \epsilon E + 4\pi e u.$$

where c_0 is the elastic parameter, c_1 the mechanical parameter owing to the dissipative property, e the piezoelectric constant and ϵ the dielectric constant.

The current in the semi-conducting layer is given by

$$j_n = q n_c \mu_n E_c + q D_n n_{c,1}$$

$$j_p = q p_c \mu_p E_c - q D_p p_{c,1}$$

where j_n is electron current density, j_p the hole current density, E_c total electric field in the conduction and, p_c the number of holes, μ_n the mobility of electrons, μ_p the mobility of holes, q the electron charge and D_n, D_p the diffusion constants

where

$$D_n = \frac{KT}{q} \mu_n, \quad D_p = \frac{KT}{q} \mu_p.$$

As in Kaliski (1965b), in our problem also, we shall confine ourselves to extrinsic semi-conductor of n type or to intrinsic semi-conductor of considerable domination of mobility of electrons μ_n over that of holes so that effects connected with holes are negligible for the disturbed field.

In what follows, we have, therefore, for the electrons

$$j = q n_c \mu E_c + q D_n n_{c,1}$$

where in general, n_c can be represented by

$$n_c = n_0 + fn$$

n_0 being undisturbed constant density of electrons in the conduction band, n the perturbing densities of electrons, f a coefficient less than 1, characterizing the spatial charge, $\bar{E}_s$ the disturbing electric field in the semi-conductor, E the disturbed electric field in the quartz plate and α the interaction constant between these two fields so that

$$\frac{\tilde{E}_s}{\tilde{E}} = \alpha.$$

Therefore, the total electric field in the semi-conductor is given by

$$E_c = E_0 + \triangle E$$

where E_0 is constant outer field, $\triangle E$ the perturbed field can be represented as

$$\triangle E_0 = E_s - \alpha \tilde{E}.$$

The electric field in the semi-conductor is given by

$$qn_c = \rho_{ec}.$$

The velocity of the electron is represented by

$$-\rho_{ec} v_c = \mu \rho_{ec} E_c \qquad \text{...(1.3)}$$

$$\rho_{ec} = \rho_{0e} + f\rho_e,\ V_c = V_0 + v,\ E_c = E_0 + \triangle E$$

$$-V_0 = \mu E_0$$

$$-v = -\mu \triangle E = -\mu(\tilde{E}_s - \alpha \tilde{E}). \qquad \text{...(1.4)}$$

The continuity equation of charges in the semi-conductor is given by

$$\operatorname{div} j - \dot{\rho}_{ec} = 0 \qquad \text{...(1.5)}$$

or,

$$f\dot{\rho}_e + f v_0 \rho_{e,1} + \rho_{0e} v_1 - f D_n \rho_{e,11} = 0.$$

The continuity equation of induction in the semi-conductor is given by

$$\operatorname{div} D_s = 4\pi\rho_e \text{ or, } \epsilon_s E_{s,1} = -4\pi\rho_e. \qquad \text{...(1.6)}$$

The divergence condition in the quartz plate is given by

$$\operatorname{div} D = \epsilon \tilde{E}_{,1} \qquad \text{...(1.7)}$$

where $\tilde{E}$ is the sourceless field of action of the quartz plate of the electron stream in the semi-conductor.

From (1.7) and on the basis of (1.2), we find

$$E_{,1} = \frac{1}{1+\alpha}\left(E_{s,1} - \alpha \frac{4\pi e}{\epsilon} u_{,11}\right)$$

$$= \frac{1}{1+\alpha}\left(\frac{4\pi e}{\epsilon}\eta + \alpha \frac{4\pi e}{\epsilon} u_{,11}\right)$$

where η is the ratio of the cross-sectional area of the semi-conducting layer to that of the quartz plate.

Substituting this value of E in (1.1) and (1.2) and eliminating ρ_e, we obtain

$$\left(f \frac{\partial}{\partial t} + fV_0 \frac{\partial}{\partial x} + \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} - fD_n \frac{\partial^2}{\partial x^2} \right) \left\{ \ddot{u} - a_0^2 u_{,11} - a_1^2 \left(\frac{\partial u}{\partial t} \right)_{,11} \right\}$$

$$+ \rho_{0e} \left(\frac{4\pi e}{1+\alpha} \right)^2 \frac{\mu\alpha\eta}{\rho\epsilon\epsilon_s} u_{,11} = 0 \qquad \ldots(1.8)$$

This constitutes the fundamental equation of the problem.

2. Solution of the Problem

Let us seek the solution of the problem in the form given by

$$u = U \exp i(wt - Kx). \qquad \ldots(2.1)$$

Substituting (1.7) in (1.6) we get

$$fw^3 - \left(3fKV_0 + i \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} + ifD_nK^2 + ia_1^2 K^2 f\right) w^2$$

$$+ \Big(3fK^2V_0^2 + i \frac{8\pi\mu\eta}{(1+\alpha)\,\epsilon_s} KV_0 + 2ifD_nK^3V_0 - a_0^2 K^2 f$$

$$+ ia_1^2 K^3 fV_0 - a_1^2 K^2 \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} - fD_n a_1^2 K^4 \Big) w$$

$$+ \Big\{ K^3V_0^3 f + i \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} K^2 V_0^2 + ifD_nK^4V_0^2$$

$$- a_0^2 K^3 f V_0 - i \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} a_0^2 K^2 - ia_0^2 fD_nK^4$$

$$+ i\rho_{0e} \left(\frac{4\pi e}{1+\alpha} \right)^2 \frac{\mu\alpha\eta}{\rho\epsilon\epsilon_s} \Big\} = 0 \qquad \ldots(2.2)$$

This is the frequency equation. To make it amenable for solution, we have to make some restrictions on the values of w for given K, and on the values of K for given w.

The disturbances in the following cases have been studied along these lines.

For small values of w, we get the above equation as a quadratic equation which, when solved gives,

$$w = \frac{3fKV_0 + i \dfrac{4\pi\mu\eta}{(1+\alpha)\epsilon_s} \rho_{0e} + ifD_nK^2 + ia_1^2 K^2 f \pm w_m}{2f}$$

where

$$w_m = \left\{ 3fK^2V_0^2 + i\frac{8\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\,\rho_{0e}\,KV_0 + \dots)^2 - 4f(K^3V_0^3 + \dots)\right\}^{1/2}.$$

The value of this w_m differs from the w_m which we find in the case for the electron stream. Unlike the case for the electron stream, the factor f and D_n due to semi-conducting layer appear in this case.

Thus the solution may be written as

$$u = U_1 \exp\left[-\frac{1}{2f}\left\{\frac{4\pi\mu\eta}{(1+\alpha)\epsilon_s}\,\rho_{0e} + fD_nK^2 + a_1^2\,K^2f\right\}\right] t$$

$$\times \exp i\left(\frac{3KV_0}{2f}\right) t \,\exp i\left(\frac{w_m}{2f}\,t - Kx\right)$$

$$+ U_2 \exp\left[-\frac{1}{2f}\left\{\frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\,\rho_{0e} + fD_nK^2 + a_1^2\,K^2f\right\} t\right.$$

$$\times \exp i\left(\frac{3KV_0}{2f}\right) t \,\exp - i\left(\frac{w_m}{2f}\,t + Kx\right).$$

This solution brings out a wave character of the solution in which amplitudes are clearly time decaying in nature and are propagating in forward and backward directions. The decay factor involves diffusion constant due to semi-conducting layer and it increases the damping factor.

A point that distinguishes this sort of wave propagation from that in the case of electron stream is that dissipative parameter appears in the decay factor as an additive term and accentuates the process of decay while in the case of flow of electron stream, it appears as a multiplying factor. Another striking factor of this case, unlike the case in which piezoelectric material subjected to an electron flow, is that the decay factor does not exhibit the coupling of the electron stream and the dissipative characteristics.

Obviously, the solution exists for

$$\frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\,\rho_{0e} + fD_nK^2 + a_1^2\,K^2f > 0.$$

The presence of f and D_n due to semi-conducting layer changes this factor from that of the case of an external electron stream.

For large values of w, we get

$$w = \frac{(\alpha + 3fKV_0w_m) + i\left\{\beta \pm w_m\left(\frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\rho_{0e} + fD_nK^2 + a_1^2\,K^2f\right)\right\}}{\gamma}$$

where α, β, γ, δ are the quantities involving V_0, K, ρ_{0e}, fD_n characterising the electron stream, semi-conducting layer and the density. Of these, the expressions β and w_m are important and are given by

$$\beta = \left(-3fK^2V_0^2 + fD_n a_1^2 K^4 + a_1^2 K^2 \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} + a_0^2 K^2 f \right)$$

$$\times \left(\frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} + fD_n K^2 + a_1^2 K^2 f \right)$$

$$w_m = \{(3fK^2V_0^2 + \ldots)^2 - 4\,(3fKV_0 + \ldots)\,(K^3V_0^3 + \ldots)\}^{1/2}.$$

Thus, the solution is given by

$$u = U_1 \exp - \frac{1}{\gamma} \left\{ \beta + w_m \left(\frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} + fD_n K^2 + a_1^2 K^2 f \right) \right\} t$$

$$\times \exp i \left(\frac{\alpha}{\gamma} \right) t \exp i \left(\frac{3KfV_0}{\gamma} w_m t - Kx \right)$$

$$+ U_2 \exp - \frac{1}{\gamma} \left\{ \beta - w_m \left(\frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} + fD_n K^2 + a_1^2 K^2 f \right) \right\} t$$

$$\times \exp i \left(\frac{\alpha}{\gamma} \right) t \exp - i \left(\frac{3KfV_0}{\gamma} w_m t + Kx \right).$$

As before, here amplitudes continue to be time decaying in nature. Further, diffusion constants being coupled with parameters of the electron stream and the dissipative effect (which are present in β and w_m) accelerate the process of decay. In the case of the flow of the electron stream, we find the coupling of the electron stream and of dissipation.

Let us now turn to the solution of the frequency equation for given w. In such a case, eqn. (2.2) can be written as

$$\{ifD_n(a_2^0 - V_0^2) - fD_n a_1^2 w\} K^4 + K^3\{2ifD_n V_0 w + ia_1^2 V_0 w - V_0^3 + a_0^2 f V_0\}$$

$$+ K^2 \left\{ 3wV_0^2 - i \frac{4\pi\mu\eta\,\rho_{0e}}{(1+\alpha)\,\epsilon_s} V_0^2 - ifD_n w^2 - a_0^2 fw \right.$$

$$+ i \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e}\, a_0^2 - ia_0^2 fw^2 - a_1^2 \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} w$$

$$\left. - i\,\rho_{0e} \left(\frac{4\pi e}{1+\alpha} \right)^2 \frac{\mu\alpha\eta}{\rho\epsilon\epsilon_s} \right\}$$

$$- K \left\{ 3w^2 f V_0 - i \frac{8\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} \right\} + \left\{ fw^3 - i \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s} \rho_{0e} w^2 \right\} = 0.$$

For small K, we get by solving the equation

$$K = \frac{(\alpha \pm D_n a_1^2\, wK_m) + i\,\{\beta \pm fD_n(a_0^2 - V_0^2)\, K_m\}}{2\gamma}$$

where K_m, α, β, γ are the quantities as stated above. Thus, the wave solution is given by

$$u = U_1 \;\exp - \left\{\frac{\beta + K_m f D_n(a_0^2 - V_0^2)}{2\gamma}\right\} x$$

$$\exp\; - i\, \frac{(V_0^3 f - a_0 f V_0)}{2\gamma}\, fD_n a_1^2\, wx \exp i \left(\frac{wf - K_m fD_n a_1^2\, w}{2\gamma}\, x\right)$$

$$+ U_2 \exp - \left\{\frac{\beta - K_m fD_n(a_0^2 - V_0^2)}{2\gamma}\right\} x$$

$$\exp - i\, \frac{(V_0^3 f - a_0 f\, V_0)}{2\gamma}\, fD_n a_1^2\, wx \exp i \left(wf + \frac{K_m fD_n a_1^2\, w}{2\gamma}\, x\right).$$

This clearly does not reveal any feature distinctively new from the earlier one. It would be worthwhile to look for expression of group velocity, for that gives an idea about the nature of the propagation.

From (2.2) we obtain the relation for the group velocity as

$$\Bigg[\; 3fw^2 - \left(3fKV_0 + i\, \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\, \rho_{0e} + ifD_nK^2 + ia_1^2\, K^2 f\right) 2w$$

$$+ 3fK^2V_0^2 + i\, \frac{8\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\, \rho_{0e}KV_0 + 2ifD_nK^3V_0 - a_0^2\, K^2 f$$

$$+ ia_1^2\, K^3 fV_0 - a_1^2 K^2\, \frac{4\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\, \rho_{0e} - fD_n a_1^2\, K^4 \Bigg] \frac{dw}{dK}$$

$$= (3fV_0 + 2ifD_nK + 2ia_1^2\, fK)w^2 - w\, \Bigg(6V_0^2\, fK + i\,\frac{8\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\, \rho_{0e}V_0$$

$$+ 6ifD_nK^2V_0 - 2a_0^2\, fK + 3ia_1^2\, K^2 fV_0 - a_1^2\, \frac{8\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\, \rho_{0e}\, K - 4fD_n a_1^2\, K^3\Bigg)$$

$$+ \Bigg(3\, K^2 V_0^3 f + i\, \frac{8\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\, \rho_{0e}KV_0^2 + 4ifD_nK^3V_0^2 + 3a_0^2\, K^2 fV_0$$

$$- i\, \frac{8\pi\mu\eta}{(1+\alpha)\,\epsilon_s}\, \rho_{0e}a_0^2\, K - 4ia_0^2\, fD_nK^3 + 2i\rho_{0e}\left(\frac{4\pi e}{1+\alpha}\right)^2 \frac{\mu\alpha\eta}{\rho_e\epsilon_s}\Bigg)$$

Substituting the small value of K and expanding binomially, we get

$$\frac{dw}{dK} \approx \frac{t_6}{V_6}\, \left\{1 + \left(\frac{t_5}{t_6} - \frac{V_5}{V_6}\right)\frac{1}{w} + \ldots\right\}$$

where $t_6, t_5 \ldots V_6, V_5 \ldots$ are constants involving material parameters. Here, the constants involve diffusion effects due to semi-conducting layer which is absent in the case of piezoelectric medium subjected to external electron stream. t_6, V_6 are constants which involve diffusion constant of the same indices and V_0 (parameter of the electron stream) of different indices. Thus, for large values of the frequency group velocity is independent of the diffusion effect and involves V_0 (parameter of the electron stream) of different indices. For large values of the frequency group velocity is independent of diffusion effect and it involves V_0.

For large values of the frequency, the above relation shows a sort of linear dependence of the group velocity on the frequency. For small values of w, we have by expanding binomially the group velocity as

$$\frac{dw}{dK} \approx \frac{t_0}{V_0}\left\{1 + \left(\frac{t_1}{t_0} - \frac{V_1}{V_0}\right) w + \ldots\right\}$$

t_0, V_0 are constants which involve diffusion constant of the same indices but parameters of the electron stream of different indices, t_1, V_1 also involve diffusion constants and parameter of the electron stream. Thus, for small values of the frequency diffusion constant appears in the group velocity. For small values of the frequency, the above relation shows a sort of linear dependence of the group velocity on the frequency.

For small values of K, we get by solving eqn. (1.8)

$$K = \frac{(\alpha \pm K_m\beta) + i\,(\gamma \pm K_m\delta)}{\mu}$$

clearly amplitudes are damped in both the cases. The dissipative effect influences the damping factor. We shall now find out group velocity for large values of K.

Substituting large K in the expression for group velocity and expanding binomially, we obtain

$$\frac{dw}{dK} \approx \frac{r_{10}}{q_{10}}\left\{1 + \left(\frac{r_9}{r_{10}} - \frac{q_9}{q_{10}}\right)\frac{1}{w} + \ldots\right\}$$

$r_{10}, r_9 \ldots, q_{10}, q_9$ are the constants.

For large frequency, group velocity involves diffusion constant and parameter of the electron stream. Also for large frequency, group velocity is independent of the frequency.

The picture is changed for small values of w.

For small values of w, we have by binomial expansion

$$\frac{dw}{dK} \approx \frac{r_1}{q_1}\left\{1 + \left(\frac{r_2}{r_1} - \frac{q_2}{q_1}\right) w + \ldots\right\}.$$

It is obvious that diffusion effects are not perceptible; further, they do not appear through r_1, q_1 but appear in r_2, q_2. Also for small values of the frequency diffusion constant appears in the group velocity.

The above relation shows a sort of linear dependence of the group velocity on the frequency for small values of the frequency.

Summing up all the foregoing cases, we arrive at the following conclusions :

(1) For large K, group velocity is independent of the frequency, while for small K group velocity depends linearly on the frequency.

(2) For large w, group velocity is independent of the diffusion constant and also of the frequency, while for small w, it involves diffusion constant and depends on the frequency linearly.

(3) For small values of K, and large w, group velocity is independent of the diffusion constant and frequency, while for large values of K and small w, the group velocity depends on the diffusion constant and on the frequency, too.

(4) The coupling of the electron stream and the dissipative effect appears for large values of w as well as for small values of K, but they are conspicuously absent for small values of w as well as for large values of K.

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