EVOLUTION OF A DISCONTINUITY IN A GAS OF VARIABLE DENSITY FLOWING IN A CHANNEL OF VARIABLE CROSS-SECTION

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(Communicated by F. C. Auluck, F.N.A.)

(*Received* 18 *August* 1972)

The problem of formation of a shock wave in a channel flow is considered by superposing unsteady perturbations over equilibrium gas-dynamic channel flow. We have considered two cases for equilibrium configuration: (i) when density monotonically decreases as we approach the throat of the converging-diverging nozzle from either side, and (ii) when density monotonically increases as we approach the throat of the nozzle from either side.

1. INTRODUCTION

In this paper we consider the problem of a shock wave in a channel flow by considering superposition of unsteady perturbations over equilibrium gas-dynamic channel flow. We assume that the equation of a channel, which we will take as converging-diverging nozzle, is

$$A = A^{*} (1 + sx^{2}) \qquad \ldots(1.1)$$

where $x = 0$ corresponds to throat of nozzle and s is a constant and for a given throat area the curvature of the generating curve of the nozzle is proportional to s. In converging-diverging nozzle the throat area is minimum and hence $s > 0$.

When a shock-free flow is set up in such an equilibrium gas-dynamic channel flow, and the pressure at the entry, or the exit, of the nozzle is then lowered, or raised (in such a way that a shock wave is not formed immediately), then 'wave-fronts' are formed which separate the region of equilibrium flow from the region of unsteady flow.

The set of governing equations of the problem form a system of hyperbolic partial differential equations in two independent variables. In such a system discontinuities of certain derivatives propagate along the characteristics and their growth during propagation is governed effectively by an ordinary differential equation.

Here we follow the method given by Jeffrey and Taniuti (1964) and corrected by Prasad and Tagare (1971) in which they have shown that one of the equations [eqn. (2.4.20)] of Jeffrey and Taniuti (1964) is not correct and requires a modification. The main distinctive feature of this method is that the location of the point, where the shock wave is formed, may be determined from the local considerations, without having to resort to a complete solution. Here discontinuity appears across the wave-front say $\phi(x, t) = 0$, because the forward facing characteristics or the backward facing characteristics of the compression waves intersect at the wave-front. However, this intersection point need not necessarily occur along the wave-front, $\phi(x, t) = 0$ but can occur within the area of influence, say $\phi < 0$, also. Here $\phi > 0$ always corresponds to undistributed region corresponding to equilibrium channel flow. When intersection point occurs within the area of influence it leads to the formation of "internal shock wave". In our analysis of evolution of discontinuity discussed here, we assume that either the "internal" shock wave is absent or even if it appears it does not influence the flow in the neighbourhood of the leading and trailing front of the disturbance.

This method can be extended to discuss the formation of shock wave over steady channel flow in a converging-diverging duct. This problem was first considered by Meyer (1952) By using different method Asano (1971) has considered the wave propagation and formation of shock wave in a quasi-steady flow in a duct of varying cross-section.

In § 2 and § 3 we have given method to determine a position $x = x_c$ and critical time $t = t_c$ when shock is first formed. In § 4 we have applied that method to find the position of a shock wave formation when density distribution in equilibrium position is given by

$$g(x) = \rho^* (1 + rx^2) \quad .. (1.2)$$

where ρ^* is the value of ρ at the throat and r is a parameter.

2. Formulation of the Problem

Defining the matrices

$$U(x,t) = \begin{bmatrix} \rho \\ u \\ a^2 \end{bmatrix}, \quad B = \begin{bmatrix} u & \rho & 0 \\ \dfrac{a^2}{\gamma} & u & \dfrac{1}{\gamma} \\ 0 & a^2(\gamma - 1) & u \end{bmatrix}$$

and

$$C = \begin{bmatrix} \dfrac{\rho u A'}{A} \\ 0 \\ \dfrac{(\gamma - 1)\, a^2 u A'}{A} \end{bmatrix} \quad .. (2.1)$$

we can write one dimensional motion of a perfect gas, neglecting any dissipative mechanism, in a channel of slowly varying cross-sectional area as

$$U_t + BU_x + C = 0 \qquad \ldots(2.2)$$

where dash denotes differentiation with respect to x, ρ the mass density, u the particle velocity, a the isentropic sound speed, γ the ratio of specific heats and A the area of cross-section of a channel given by (1.1). Let us assume that, because of some disturbance at the exit (or entrance) of the channel, say a local pressure fluctuation, an unsteady wave motion propagates upstream (downstream) in the medium in equilibrium gas-dynamic channel flow. We can take the initial configuration to be

$$U_0 = \begin{bmatrix} g(x) \\ 0 \\ \dfrac{\gamma p_0}{(gx)} \end{bmatrix} \qquad \ldots(2.3)$$

where $g(x)$ is an arbitrary function of x and p_0 is the constant pressure of the medium.

Let us dimensionalize eqn. (2.2) by $\bar{x} = \dfrac{x}{|x_1|}$,

$$\bar{\rho} = \frac{\rho}{\rho^*},\ \bar{u} = \frac{u}{a^*},\quad \bar{t} = \frac{a^*}{|x_1|}t,\quad \bar{A} = \frac{A}{A^*} \qquad \ldots(2.4)$$

$\bar{a} = \dfrac{a}{a^*}$ where (*) denotes the value of the nozzle and $x = x_1$ is the position where initial vector $U(x, 0)$ has a discontinuity which propagates along a wave-front $\phi(x, t) = 0$.

Equation (2.2) can be written as

$$\bar{U}_t + \bar{B}\bar{U}_x + \bar{C} = 0$$

where $\bar{U}$, $\bar{B}$ and $\bar{C}$ are same as U, B, C in eqn. (2.1) except the quantities in U, B,C are replaced by corresponding dimensionless quantities $\bar{\rho}$, $\bar{u}$, $\bar{a}^2$ and $\bar{A}$. Throughout our analysis we will delete this bar and hence consider eqn. (2.2) itself in dimensionless form x and quantities in ρ, u and a^2 are all dimensionless and functions of dimensionless variables x and t. Discontinuity is initiated at $t = 0$ and $x = \pm 1$. It is initiated in the converging part of the nozzle at $t = 0$ and $x = -1$ and in the diverging part of the nozzle at $t = 0$ and $x = 1$.

Since the eigenvalues $\lambda^{(1)}, \lambda^{(2)}, \lambda^{(3)}$ of B are real and are given by

$$\lambda^{(1)} = u + a, \qquad \lambda^{(2)} = u - a, \qquad \lambda^{(3)} = u$$

and the associated left-eigenvectors

$$l^{(1)} = [a,\ \gamma\rho,\ \rho/a], \qquad l^{(2)} = [a,\ -\gamma\rho,\ \rho/a]$$

and

$$l^{(3)} = [a, 0, -\rho/a(\gamma-1)] \qquad \text{... (2.5)}$$

are linearly independent, the quasi-linear system of equations is totally hyperbolic.

If we adopt the family of curves

$$\phi(x, t) = \text{constant}, \qquad t' = \text{constant} \qquad \text{...(2.6)}$$

as alternative coordinates to x and t, where ϕ and t are introduced through the equations

$$t' = t \qquad \text{...(2.7)}$$

and

$$\phi_t + \lambda^{\phi}\phi_x = 0, \qquad \text{...(2.8)}$$

where $\phi(x, t) = 0$ is the equation of advancing wave-front with the initial condition chosen for ϕ so that $\phi>0$ ahead of the wave-front and $\phi<0$ behind the wave-front (see Figures in Prasad and Tagare 1971). $\varepsilon = 1$ corresponds for a forward facing wave and $\varepsilon = -1$ corresponds for backward facing wave. The curves $\phi =$ constant become characteristics across which the solution U is assumed to be continuous but across which the normal derivative of U, U_ϕ, is discontinuous. Then using notation $\left[\chi\right]_{\phi=0+}^{\phi=0-}$ to denote the jump $(\chi)_{\phi=0-} - (\chi)_{\phi=0+}$ of a quantity χ (scalar or vector across $\phi = 0$, the following jump relations were defined:

U is continuous : $\left[\dot{U}\right]_{\phi=0+}^{\phi=0-} = 0$

$U_{t'}$ is continuous : $\left[U_{t'}\right]_{\phi=0+}^{\phi=0-} = 0$

U_ϕ is discontinuous : $\left[U_\phi\right]_{\phi=0+}^{\phi=0-} \equiv \pi(t') \neq 0$

and

x_ϕ is discontinuous : $\left[x_\phi\right]_{\phi=0+}^{\phi=0-} \equiv X(t') \neq 0$ ...(2.9)

Since x_ϕ is discontinuous across $\phi = 0$, the assumption by Jeffrey and Taniuti (1964) that the jumps $[x_\phi l_x^{(\phi, k)}] = 0$ and $[x_\phi \lambda_x^{(\phi, k)}] = 0$ across the characteristic are not correct. However, while re-examining eqns. (2.4.19), (2.4.20) and (2.4.12) of Jeffrey and Taniuti (1964), Prasad and Tagare (1971) have considered variation $l_0^{(\phi, k)}$ and $b_0^{(\phi, k)}$ with x into account and concentrated on equation (2.4.20) and modified it. However, they have not taken into account variation of $\lambda_0^{(\phi)}$ with x. In fact both eqns. (2.4.20) and (2.4.12) of Jeffrey and Taniuti (1964) require modifications and correct set of equations for π and X is :

$$-l_0^{(j)}U_{\phi_0}X + l_0^{(j)}\pi\, x_{\phi_0} = 0,\ j = r_{\phi+1},\ldots,n \text{ and } \lambda^{(j)} \neq \lambda^{(\phi)} \qquad \ldots(2.10)$$

$$l_0^{(\phi,k)}\pi_{t'} + [(\nabla_u l^{(\phi,k)})_0\pi]'\, U_{0t'} + (\nabla_u b^{(\phi,k)})_0\,\pi + (l_{0x}^{(\phi,k)}X)\, U_{0t'} + b_{0x}^{(\phi,k)}X = 0, \qquad k = 1, 2\ldots, r_\phi \qquad \ldots(2.11)$$

and

$$X_{t'} = (\nabla_u \lambda^{(\phi)})_0\,\pi \qquad \ldots(2.12)$$

where the prime in the second term of eqn. (2.11) denotes the transpose operation.

On $\phi(x, t) = 0$, we have $\dfrac{d}{dt'} = \dfrac{d}{dt}$ and

$$\left(\frac{dx}{dt}\right)_{\phi=0-} = \lambda_0^{(\phi)} = \varepsilon a_0 = \varepsilon\sqrt{\frac{\gamma p_0}{g(x)}} \qquad \ldots(2.13)$$

In eqns. (2.10), (2.11), (2.12) and (2.13) the suffix zero signifies that the associated expression is to be evaluated on the side of the wave-front belonging to the region $\mathcal{R}$ into which the wave-front is advancing; r_ϕ is multiplicity of the eigenvalue $\lambda^{(\phi)}$; $l^{(j)}$ is the jth left eigenvector of B corresponding to the eigenvalue $\lambda^{(j)}$ and $b^{(\phi,k)} = l^{(\phi,k)}\,C$.

Equations (2.10) is same as eqn. (2.4.19) of Jeffrey and Taniuti (1964), while eqns. (2.4.20) and (2.4.12) of Jeffrey and Taniuti are not correct The correct form of eqn. (2.4.20) is eqn. (2.11) and (2.4.12) is eqn. (2.12). In most of the problems in fluid dynamics, B depends only on $u_1\ldots\ldots u_n$ and we find that $l_{0x}^{(\phi,k)} \equiv 0$ and $\lambda_{0x}^{(\phi)} \equiv 0$. Thus one of the extra terms in (2.11) and (2.12) vanish. In the cases of spherical, cylindrical and plane symmetric motions in a compressible medium or the case of a channel flow of variable cross-section, $b_{0x}^{(\phi,k)}$ contains the particle velocity in its numerator and so this terms also vanishes if we take the initial configuration to be an equilibrium configuration When $l_{0x}^{(\phi,k)} = \lambda_{0x}^{(\phi)} = b_{0x}^{(\phi,k)} = 0$, eqns. (2.11) and (2.12) are the same as eqns. (2.4.20) and (2.4.12) of Jeffrey and Taniuti (1964).

Since $U_x = \dfrac{U_\phi}{x_\phi}$, U will cease to be continuous and hence shock wave will be formed if other $(x_\phi)_{\phi=0-} = 0$ or the magnitude of $(U_\phi)_{\phi=0+}$ becomes infinitely large. From the initial configuration $(x_\phi)_{\phi=0+}$ and $(U_\phi)_{\phi=0+}$ are known functions of x and do not depend on the disturbance.

3. Basic Equations for Determination of x_c and t_c

Setting $\pi = \begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix}$, with $\pi_1 = \Big[\rho_\phi\Big]_{\phi=0+}^{\phi=0-}$, $\pi_2 = \Big[u_\phi\Big]_{\phi=0+}^{\phi=0-}$ and $\pi_3 = \Big[(a^2)_\phi\Big]_{\phi=0+}^{\phi=0-}$

and substituting (2.1) in equations (2.10)-(2.12), we get

$$\pi_1 - \frac{\varepsilon\sqrt{\gamma}\, g^{3/2}(x)}{\sqrt{p_0}}\pi_2 + \frac{g^2(x)}{\gamma(\gamma-1)p_0}\pi_3 = 0, \quad \ldots(3.1)$$

$$-\frac{\gamma g'(x)}{(\gamma-1)} X + \pi_1 - \frac{g^2(x)}{\gamma(\gamma-1)p_0}\pi_3 = 0, \quad \ldots(3.2)$$

$$\frac{d\pi_1}{dx} + \varepsilon\sqrt{\frac{\gamma}{p_0}}\, g^{3/2}(x)\frac{d\pi_2}{dx} + \frac{g^2(x)}{\gamma p_0}\frac{d\pi_3}{dx} - \frac{g'(x)}{g(x)}\pi_1$$
$$+ \frac{2\varepsilon s x g^{3/2}(x)}{1+sx^2}\sqrt{\frac{\gamma}{p_0}}\pi_2 + \frac{g(x)g'(x)}{\gamma p_0}\pi_3 = 0, \quad \ldots(3.3)$$

and

$$\frac{dX}{dx} = \pi_3\frac{g(x)}{2\gamma p_0} + \pi_2\frac{\varepsilon g^{1/2}(x)}{\sqrt{\gamma p_0}}. \quad \ldots(3.4)$$

By eliminating π_1 and π_3 from two algebraic equations and two differential equations, we get

$$\frac{d\pi_2}{dx} = -\pi_2\left(\frac{g'(x)}{4g(x)} + \frac{sx}{1+sx^4}\right) \quad \ldots(3.5)$$

and

$$\frac{dX}{dx} + \frac{g'(x)}{2g(x)} X = \frac{(\gamma+1)}{2}\varepsilon\sqrt{\frac{g(x)}{\gamma p_0}}.\pi_2. \quad \ldots(3.6)$$

For $\phi(x, 0) = \varepsilon(x-\varepsilon_1)$, we have $\phi_x = \varepsilon$ at $t = 0$ for $\phi>0$ as well as for $\phi<0$ so that

$$X(\varepsilon_1) = 0. \quad \ldots(3.7)$$

where $\varepsilon_1 = -1$ in converging part of the nozzle and $\varepsilon_1 = 1$ in diverging part of the nozzle. Thus for forward facing wave the discontinuity is initiated at $t = 0$ in converging part and so $\varepsilon_1 = -1$ and $\varepsilon = 1$ and for backward facing wave the discontinuity is initiated in diverging part and so $\varepsilon_1 = 1$ and $\varepsilon = -1$.

Since $u \equiv 0$ for $\phi>0$, we also have

$$\pi(\varepsilon_1) = \mathbf{u}_\phi = \varepsilon\mathbf{u}_x \quad \ldots(3.8)$$

where

$$\mathbf{u}_x = \lim_{x\to\varepsilon_1} \{(\mathrm{u}_x)_{\phi=0-}\}.$$

Solving (3.5) and (3.6) with initial conditions (3.7) and (3.8), we have

$$\pi_2(x) = \varepsilon\mathbf{u}_x\left\{\frac{g(\varepsilon_1)}{g(x)}\right\}^{1/4}\left(\frac{1+s}{1+sx^2}\right)^{1/2} \quad \ldots(3.9)$$

and

$$X(x) = \frac{(\gamma+1)\mathbf{u}_x g^{1/4}(\varepsilon_1)\,(1+s)^{1/2}}{2\sqrt{\gamma p_0 g(x)}}\,. \qquad \ldots(3.10)$$

We can also show that

$$(x_\phi)_{\phi=0+} = \frac{1}{(\phi_x)_{\phi=0+}} = \varepsilon\sqrt{\frac{g(\varepsilon_1)}{g(x)}}, \qquad \ldots(3.11a)$$

and

$$\mathbf{u}_t = -\,\varepsilon \mathbf{u}_x \sqrt{\gamma p_0/g(\varepsilon_1)}. \qquad \ldots(3.11b)$$

At the point $x = x_c$, where a discontinuity appears, we have

$$X(x_c) = -\,(x_\phi)_{\phi=0+} = -\varepsilon\sqrt{\frac{g(\varepsilon_1)}{g(x_c)}}. \qquad \ldots(3.12)$$

From (3.10), (3.11*b*) and (3.12), we have at $x = x_c$

$$\int_{\varepsilon_1}^{x_c} g^{3/4}(x)(1+sx^2)^{-1/2}\,dx = \beta/\mathbf{u}_t \qquad \ldots(3.13)$$

where

$$\beta = \frac{2a_0^2(\varepsilon_1)g^{3/4}(\varepsilon_1)}{(\gamma+1)(1+s)^{1/2}} > 0 \qquad \ldots(3.14)$$

and $a_0(\varepsilon_1)$ is the value of a_0 at $t = 0$, $x = \varepsilon_1$. Equation (3.13) gives the value of x_c in terms of ε_1, the initial thermodynamic state at $x = \varepsilon_1$ and the intensity of the initial disturbance (which can be measured in terms of $\mathbf{u}_t$) for both forward facing and backward facing wave. When (3.13) is satisfied i.e., when $(x_\phi(t_c))_{\phi=0-}$ becomes a zero, a singularity of the transformation appears at $x = x_c$, $t = t_c$ due to intersection of characteristics.

We also note that $\mathbf{u}_t > 0$ at the leading front of either a forward facing compression wave or backward facing expansion wave and $\mathbf{u}_t < 0$ at the leading front of either a forward facing expansion wave or backward facing compression wave.

t_c is determined from the equation

$$t_c = \varepsilon\int_{\varepsilon_1}^{x_c} \frac{\sqrt{g(x)}}{\sqrt{\gamma p_0}}\,dx. \qquad \ldots(3.15)$$

When the area of cross-section of channel varies very slowly so that s being a very small parameter the higher powers of s can be neglected as compared with s, eqn. (3.13) reduces to

$$\int_{\varepsilon_1}^{x_e} g^{3/4}(x)\left[1-\frac{sx^2}{2}\right]dx = \frac{\beta}{u_i}. \quad \ldots(3.16)$$

4. Discussion of Numerical Results when Density Configuration $g(x) = \rho^*(1+rx^2)$

Let $g(x) = \rho^*(1+rx^2)$ ...(4.1)

where r is some parameter and ρ^* is the critical value of ρ at the throat, $x = 0$, of converging-diverging nozzle.

Case 1 : $r>0$

Here density monotonically decreases as we approach the throat $x = 0$ from either side of the nozzle. [$x>0$ corresponds to diverging part of the nozzle and $x<0$ corresponds to convering part of the nozzle (see Fig. 1)]. At throat $x = 0$, density is minimum.

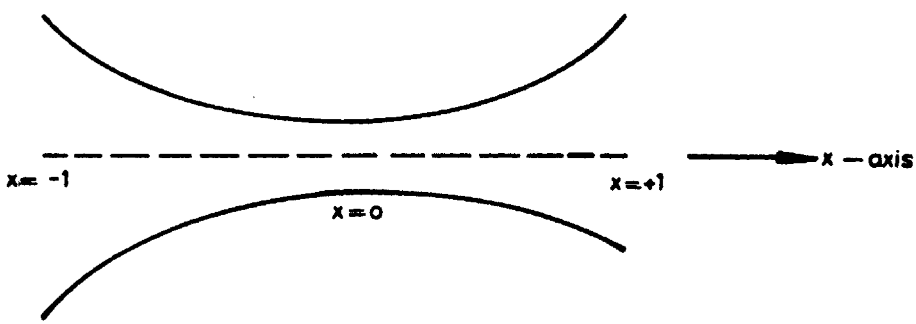

FIG. 1. Converging diverging nozzle $A=A_0\,(1+sx^2)$

Case 2 : $0<r<-1$

Here density monotonically increases as we approach the throat $x = 0$ from either side of the nozzle. At throat $x = 0$, density is maximum.

Tables I-IV give us the idea of formation of discontinuity for various values of wave-strength and for three different values of the parameter s which corresponds to a curvature of the converging-diverging nozzle when a compression wave is initiated at $x = -1$ initially on equilibrium nozzle flow. Tables I and II correspond to case 1 when density of the compressible fluid in equilibrium configuration decreases as we approach the throat $x=0$ from either side of the nozzle. Tables III and IV correspond to case 2 when density of the compressible fluid in equlibrium configuration increases as we approach the throat $x = 0$ from either side.

We draw the following conclusions: (i) All compression wave-fronts show the tendency of shock formation. The shock wave will be formed in the nozzle if the strength of the compression wave exceeds some critical strength otherwise it will be formed outside the nozzle (as length of the nozzle is finite), (ii) for the same wave-length of the compression wave in both cases the critical distance x_c where shock is first formed from x_1 (where the disturbance on equilibrium nozzle flow leading to compression wave is initiated at $t = 0$) increases as the curvature of the nozzle increases and (iii) all expansion wave-fronts show a tendency opposed to shock formation.

Figures 2 and 3 correspond to case 1 and Figs. 4 and 5 correspond to case 2

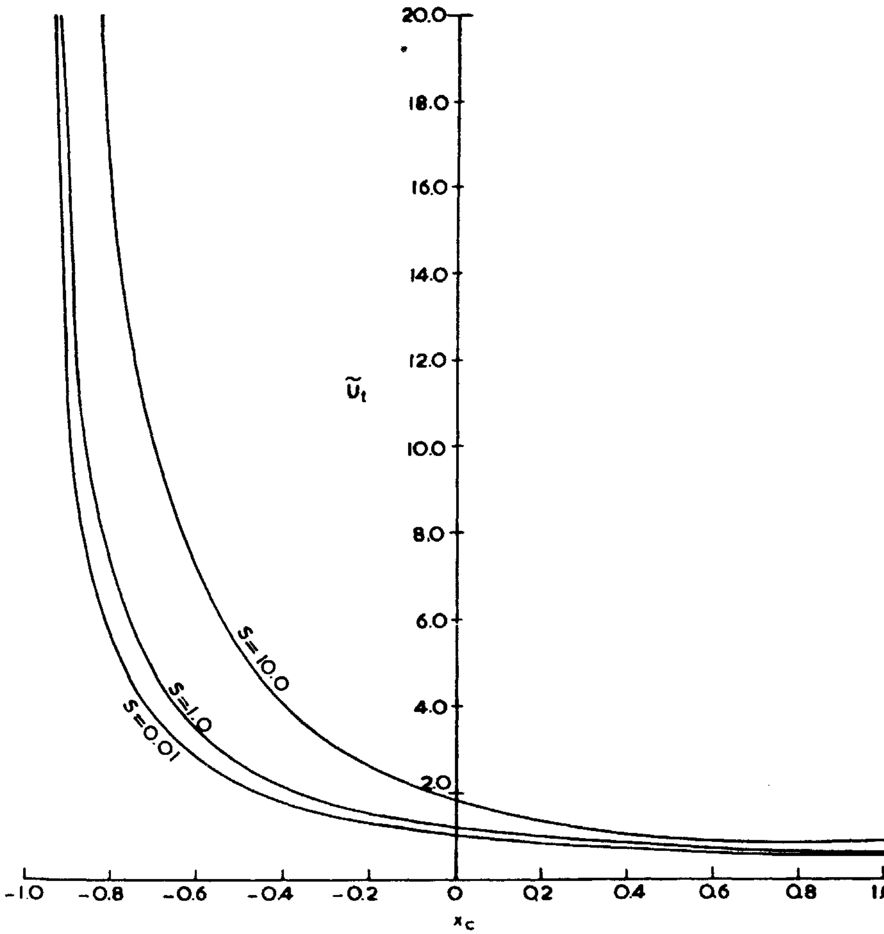

FIG. 2. Location of shock wave at $x = x_c$ for different values of wave-strength u_t.

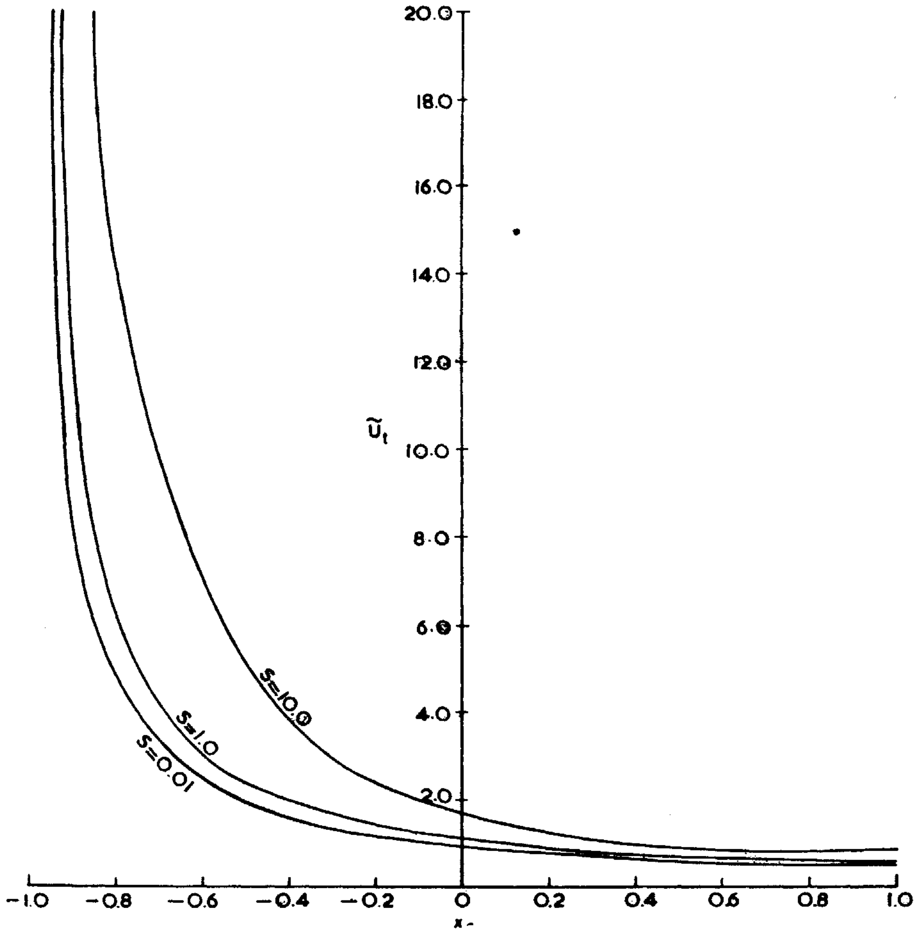

FIG. 3. Location of shock wave at $x = x_0$ for different values of wave-strength $\mathbf{u}_t$.

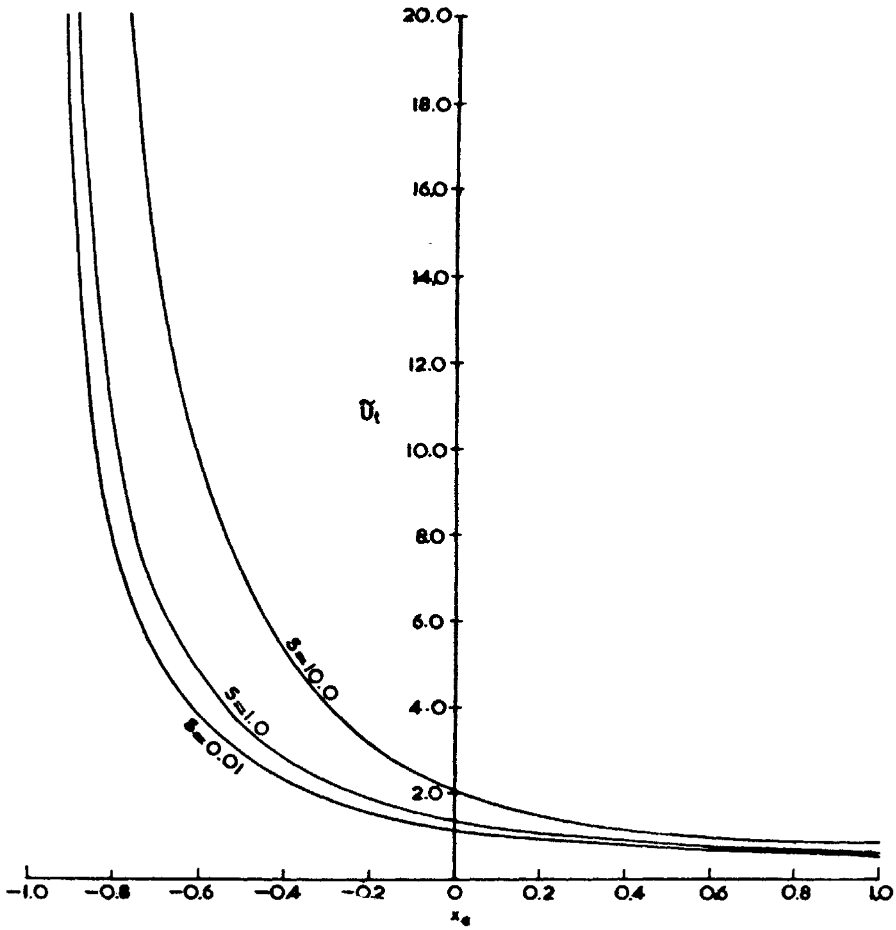

FIG. 4. Location of shock wave at $x = x_c$ for different values of wave-strength u_f.

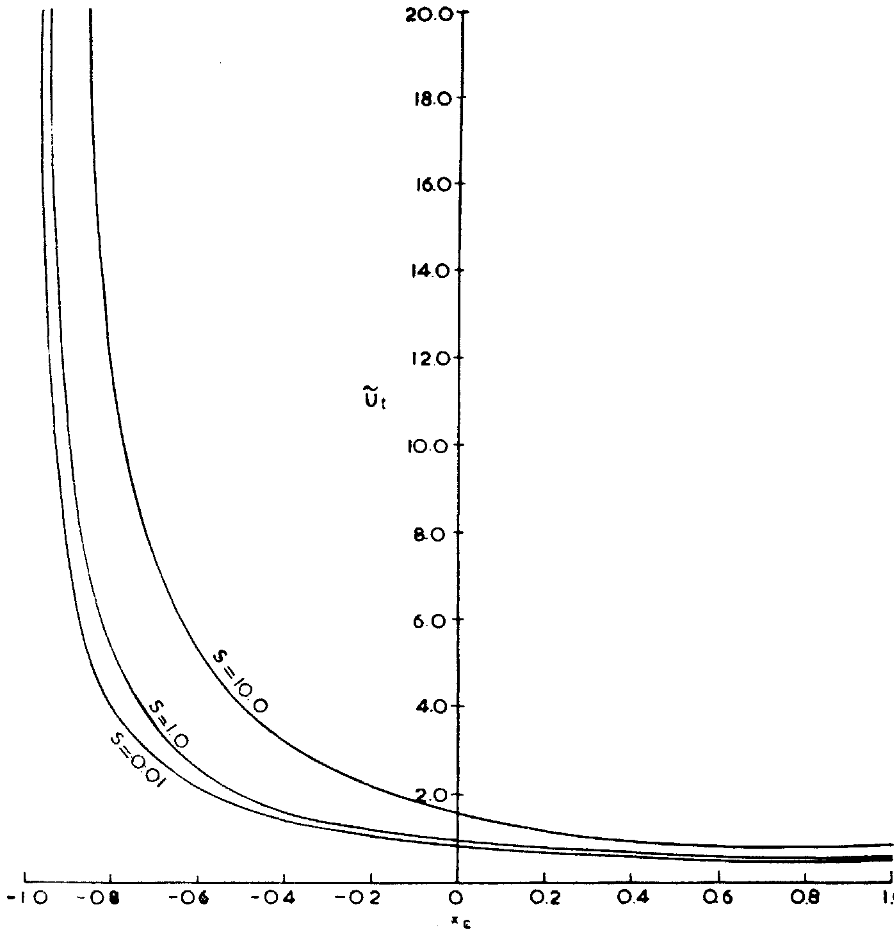

FIG. 5. Location of shock wave at $x=x_0$ for different values of wave-strenghth u_t.

TABLE I

Location of shock wave for different values of wave-strength

$(r = 0.1, \beta = 1, \epsilon = 1, x_1 = -1)$

x_e	u_t for $s = 0\,01$	u_t for $s = 1$	u_t for $s = 10$
−0.9	10.8342	14.8588	34.1297
−0.8	5.3908	7.2150	16.1290
−0.7	3.5778	4.6707	10.1317
−0.6	2.6709	3.4037	7.1225
−0.5	2.1272	2.6469	5.3107
−0.4	1.7649	2.1464	4.0984
−0.3	1.5062	1.7928	3.2321
−0.2	1.3123	1.5316	2.5880
−0.1	1.1616	1.3323	2.1030
0	1.0411	1.1765	1.7443
0.1	0.9426	1.0524	1.4883
0.2	0.8606	0.9523	1.3106
0.3	0.8085	0.8704	1.1862
0.4	0.7318	0.8028	1.0966
0.5	0.6804	0.7463	1.0294
0.6	0.6355	0.6988	0.9771
0.7	0.5959	0.6583	0.9352
0.8	0.5607	0.6237	0.9006
0.9	0.5293	0.5937	0.8714
1	0.5010	0.5625	0.8465

TABLE II

Location of shock wave for different values of wave-strength

($r = 0.5, \beta = 1, \varepsilon = 1, x_1 = -1$)

x_c	$\mathbf{u}_t$ for $s = 0.01$	$\mathbf{u}_t$ for $s = 1$	$\mathbf{u}_t$ for $s = 10$
−0.9	16.2866	22.3714	51.2820
−0.8	7.8616	10.5263	23.4742
−0.7	5.0684	6.6138	14.2857
−0.6	3.6819	4.6816	9.7371
−0.5	2.8571	3.5411	7.0422
−0.4	2.3121	2.7956	5.2743
−0.3	1.0268	2.5758	4.0355
−0.2	1.6410	1.8972	3.1358
−0.1	1.4211	1.6121	2.4759
0	1.2472	1.3924	1.9992
0.1	1.1066	1.2197	1.6661
0.2	0.9908	1.0820	1.4382
0.3	0.8940	0.9708	1.2802
0.4	0.8121	0.8798	1.1671
0.5	0.7446	0.8047	1.0825
0.6	0.6837	0.7419	1.0166
0.7	0.6306	0.6890	0.9636
0.8	0.5839	0.6440	0.9198
0.9	0.5428	0.6152	0.8828
1	0.5062	0.5716	0.8511

TABLE III

Location of shock wave for different values of wave-strength

$(r = -0.1, \beta = 1, \varepsilon = 1, x_1 = -1)$

x_c	u_t for $s = 0.01$	u_t for $s = 1$	u_t for $s = 10$
−0.9	9.3809	12.8866	26.3158
−0.8	4.7059	6.3052	14.1044
−0.7	3.1466	4.1152	8.9286
−0.6	2.3674	3.0221	6.3331
−0.5	1.9000	2.3691	4.7642
−0.4	1.5886	1.9368	3.7106
−0.3	1.3659	1.6310	2.9551
−0.2	1.1992	1.4047	2.3906
−0.1	1.0695	1.2318	1.9631
0	0.9658	1.0965	1.6450
0.1	0.8810	0.9887	1.4170
0.2	0.8104	0.9106	1.2582
0.3	0.7508	0.8304	1.1470
0.4	0.6996	0.7715	1.0670
0.5	0.6554	0.7224	1.0072
0.6	0.6167	0.6811	0.9608
0.7	0.5826	0.6460	0.9238
0.8	0.5524	0.6160	0.8935
0.9	0.5253	0.5901	0.8681
1	0.5010	0.5675	0.8466

TABLE IV

Location of shock wave for different values of wave-strength

($r = -0.5$, $\beta = 1$, $\epsilon = 1$, $x_1 = -1$)

x_c	u_t for $s = 0.01$	u_t for $s = 1$	u_t for $s = 10$
−0.9	7.5075	10.3093	23.6407
−0.8	3.8008	5.0942	11.3895
−0.7	2.5661	3.3568	7.2886
−0.6	1.9497	2.4913	5.2274
−0.5	1.5805	1.9743	3.9809
−0.4	1.3351	1.6324	3.1427
−0.3	1.1604	1.3906	2.5381
−0.2	1.0299	1.2121	2.0846
−0.1	0.9288	1.0761	1.7397
0	0.8486	0.9699	1.4819
0.1	0.7834	0.8858	1.2983
0.2	0.7295	0.8183	1.1692
0.3	0.6844	0.7635	1.0794
0.4	0.6461	0.7186	1.0156
0.5	0.6134	0.6818	0.9688
0.6	0.5853	0.6512	0.9333
0.7	0.5609	0.6258	0.9056
0.8	0.5397	0.6045	0.8837
0.9	0.5212	0.5867	0.8660
1	0.5050	0.5717	0.8516

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