MAGNETOHYDRODYNAMIC FLOW OF A RAREFIED GAS NEAR A TIME-VARYING ACCELERATED PLATE

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The flow of an electrically conducting rarefied gas due to the time-varying motion of an infinite flat plate has been studied in the presence of a uniform magnetic field. The magnetic lines of force are taken to be fixed relative to the fluid. General expressions of the velocity and the skin friction have been compared by means of some graphs and tables.

INTRODUCTION

The flow of a viscous incompressible and electrically conducting fluid in the presence of an external magnetic field past an impulsively moving infinite plate has been discussed by Rossow (1957, 1958). Kakutani (1958) and Ong and Nicholls (1959) respectively extended the problem to the case of an oscillating infinite plate with and without the induced magnetic field by the current. Gupta (1960), Soundalgekar (1965) and Mohapatra (1971) have studied similar problems in the case of time-varying accelerated flat plate. In all these problems, the flow of the normal density fluids was considered. In the present age of high altitude flights, the study of rarefied gases is receiving attention of a number of researchers. In case of the slightly rarefied gases, the physical aspect of the problem can be analysed by solving the Navier-Stokes equations under the first order velocity slip boundary conditions at the boundaries. Such an hydrodynamic attempt was made by Schaaf for an impulsively moving flat plate under first order velocity slip boundary conditions and has been discussed in the book by Eckert and Drake (1959). Soundalgekar *et al.* (1968*a*, *b*) studied the same problem under the influence of a magnetic field.

The object of this paper is to discuss the flow of an electrically conducting rarefied gas past an accelerated plate under the influence of a magnetic field due to the time-varying motion (of the nature $Ae^{at}t^n$) of an infinite flat plate. The magnetic lines of force are taken to be fixed relative to the fluid. A few particular cases are discussed.

FORMULATION OF THE PROBLEM

In this problem, x-axis is taken along the flat plate and in the direction of its motion and y-axis is taken perpendicular to it. It is assumed that the magnetic field lines are perpendicular to the free stream velocity and the magnetic permeability μ_e is constant throughout the field.

The equation of motion is

$$\frac{\partial \vec{V}}{\partial t}+(\vec{V}.\nabla)\vec{V}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\vec{V}+\frac{1}{\rho}\vec{J}\times\vec{B} \quad ...(1)$$

where $\vec{V}$ is the velocity of the fluid, p is the pressure, ρ the density, ν the kinematic coefficient of viscosity, $\vec{J}$ the current density and $\vec{B}=\mu_e\vec{H}$, $\vec{B}$ being the electromagnetic induction and $\vec{H}$ the magnetic field. It is also assumed that the induced magnetic field produced by the motion of the electrically conducting fluid is negligible. We can take the electric field $\vec{E}=0$, since no applied or polarization voltage exists. When these assumptions are made, the components of electromagnetic induction are given by

$$B_x=0,\ B_y=\mu_e H_0=B_0 \quad \text{(constant)} \quad ...(2)$$

and the components of ponderomotive force are

$$F_x=-\frac{\sigma}{\rho}B_0^2 u,\ F_y=0 \quad ...(3)$$

where σ is the electrical conductivity of the fluid. The equation of continuity is

$$div\ \vec{V}=0. \quad ...(4)$$

As the plate is infinite in length, all variables in this problem are functions of y and t only.

Hence from eqns. (1)-(3), we have

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \quad ...(5)$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial y} \quad ...(6)$$

and Maxwell equations become redundant in view of the simplifying assumptions made above. From eqn. (6) we have

$$p=\text{constant} \quad ...(7)$$

everywhere.

The problem is to be solved under the boundary conditions

$$u=0 \text{ everywhere for } t\leqslant 0 \quad ...(8)$$

$$\left.\begin{array}{l} u = At^n e^{at} + \xi u \dfrac{\partial u}{\partial y} \text{ at } y=0, \\ u=0 \text{ as } y \to \infty \end{array}\right\} t>0 \qquad \ldots(9)$$

where $\xi u = \left(\frac{2-\beta}{\beta}\right) l$; l being the mean free path given by $\left(\sqrt{\frac{\pi}{8}}/0.499\right) \mu \left(\sqrt{RT}/P\right)$; β being the Maxwell's reflection coefficient and R gas constant.

Solution of Equations

We shall solve eqn. (5) by the method of Laplace Transforms. If the Laplace Transform of $u(y, t)$ is defined as $\bar{u}(y, p)$, then

$$\bar{u}(y, p) = \int_0^\infty e^{-pt} u(y, t)\, dt, \ (p>0) \qquad \ldots(10)$$

and eqn. (5) is transformed to

$$\frac{d^2\bar{u}}{dy^2} = \frac{p+m}{\nu} \bar{u} \qquad \ldots(11)$$

where $m = \dfrac{\sigma B_0^2}{\rho}$.

The transformed boundary conditions are from (9)

$$\left.\begin{array}{l} \bar{u} = \dfrac{A\Gamma(n+1)}{(p-a)^{n+1}} + \xi u \dfrac{d\bar{u}}{dy} \text{ at } y=0 \\ \bar{u}=0 \quad \text{as } y \to \infty. \end{array}\right\} \qquad \ldots(12)$$

Solving eqn. (11) with the conditions (12), we have

$$\bar{u}(y, p) = \frac{A\Gamma(n+1) e^{-y\sqrt{\frac{p+m}{\nu}}}}{\left(1+\xi u \sqrt{\frac{p+m}{\nu}}\right)(p-a)^{n+1}} \qquad \ldots(13)$$

Inverting (13) we get

$$u(y, t) = \frac{A\Gamma(n+1)}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{pt - y\sqrt{\frac{p+m}{\nu}}} \times \frac{dp}{\left(1+\xi u \sqrt{\frac{p+m}{\nu}}\right)(p-a)^{n+1}}. \qquad \ldots(14)$$

Let $b=\frac{y}{\sqrt{\nu}}$; $b'=\frac{\xi u}{\sqrt{\nu}}$; $p+m=Z^2$, $\alpha=m+a$, $b_1=\frac{1}{b'}$.

Therefore (14) reduces to

$$u(y, t)=A\ \Gamma(n+1)\ b_1\ I\,(y, t, \alpha, n) \qquad \ldots(15)$$

where

$$I(y, t, \alpha, n)=\frac{1}{2\pi i}\int_{Br_3} e^{(z^2-m)t-bz}\frac{2z\,dz}{(z+b_1)\ (z^2-\alpha)^{n+1}} \qquad \ldots(16)$$

Br_3 being the Bromwich path.

Now we are to obtain the values of I(y, t, α, n) for different integral values of n.

Let $$I(y, t, \alpha, 0)=\mathrm{F}(\alpha)=\frac{1}{2\pi i}\int_{Br_3} e^{(z^2-m)t-bz}\frac{2z\,dz}{(z+b_1)(z^2-\alpha)}$$

$$=\frac{1}{2\pi i}\left[\frac{2b_1}{a-b_1^2}\int_{Br_3} e^{(z^2-m)t-bz}\frac{dz}{z+b_1}+\frac{1}{b_1-\sqrt{\alpha}}\int_{Br_3} e^{(z^2-m)t-bz}\frac{dz}{z+\sqrt{\alpha}}\right.$$

$$\left.+\frac{1}{b_1+\sqrt{\alpha}}\int_{Br_3} e^{(z^2-m)t-bz}\frac{dz}{z-\sqrt{\alpha}}\right]. \qquad \ldots(17)$$

We know that

$$\frac{1}{\pi i}\int_{Br_3}\frac{e^{z^2t-bz}}{z-\sqrt{\alpha}}\,dz=e^{\alpha t-b\sqrt{\alpha}}\ \mathrm{erfc.}\ \frac{b-2t\sqrt{\alpha}}{2\sqrt{t}}$$

and

$$\left.\frac{1}{\pi i}\int_{Br_3}\frac{e^{z^2t-bz}}{z+\sqrt{\alpha}}\,dz=e^{\alpha t+b\sqrt{\alpha}}\ \mathrm{erfc.}\ \frac{b+2t\sqrt{\alpha}}{2\sqrt{t}}.\right\} \qquad \ldots(18)$$

From (17) and (18)

$$I(y, t, \alpha, 0)=\frac{b_1}{\alpha-b_1^2}e^{b_1^2t+bb_1-mt}\ \mathrm{erfc.}\frac{b+2b_1t}{2\sqrt{t}}$$

$$+\frac{1}{2\,(b_1-\sqrt{\alpha})}\ e^{at+b\sqrt{\alpha}}\ \mathrm{erfc.}\ \frac{b+2t\sqrt{\alpha}}{2\sqrt{t}}+\frac{1}{2(b_1+\sqrt{\alpha})}e^{at-b\sqrt{\alpha}}$$

$$\times\ \mathrm{erfc.}\ \frac{b-2t\sqrt{\alpha}}{2\sqrt{t}}. \qquad \ldots(19)$$

Again we observe that

$$nI(y,\ t,\ \alpha,\ n)=\frac{d}{d\alpha}\ I(y,\ t,\ \alpha,\ n-1)+tI(y,\ t,\ \alpha,\ n-1),\ \text{for } n\geqslant 1. \qquad \ldots(20)$$

From (19) and (20) we have

$$I(y,\ t,\ \alpha,\ 1)=-\frac{b_1}{(\alpha-b_1^2)^2}e^{b_1^2 t+mt+bb_1}\ \text{erfc.}\frac{b+2b_1 t}{2\sqrt{t}}$$

$$+e^{\alpha t+b\sqrt{\alpha}}\text{erfc.}\frac{b+2t\sqrt{\alpha}}{2\sqrt{t}}\left(\frac{b}{4\sqrt{\alpha}\,(b_1-\sqrt{\alpha}\,)}+\frac{1}{4\sqrt{\alpha}\ (b_1-\sqrt{\alpha}\,)^2}\right.$$

$$\left.+\frac{t}{2(b_1-\sqrt{\alpha}\,)}\right)-e^{\alpha t-b\sqrt{\alpha}}\ \text{erfc.}\frac{b-2t\sqrt{\alpha}}{2\sqrt{t}}\left(\frac{1}{4\sqrt{\alpha}\ (b_1+\sqrt{\alpha}\,)^2}\right.$$

$$\left.+\frac{b}{4\sqrt{\alpha}\ (b_1+\sqrt{\alpha}\,)}-\frac{t}{2(b_1+\sqrt{\alpha}\,)}\right)-\sqrt{\frac{t}{\pi}}\ e^{-b^2/4t-mt}\frac{1}{b_1^2-\alpha}\,. \qquad \ldots(21)$$

Similarly we can find $I(y,\ t,\ \alpha,\ 2)$ and, in general, $I(y,\ t,\ \alpha,\ n)$ for different integral values of n.

The shearing stress at the wall is

$$\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}. \qquad \ldots(22)$$

From (15), (16) and (22) we obtain

$$Ab_1\mu\Gamma(n+1)\left[\frac{\partial}{\partial y}\ I(y,\ t,\ \alpha,\ n)\right]_{y=0}$$

$$=-\frac{\mu Ab_1\Gamma(n+1)}{\sqrt{\nu}}\left[\frac{1}{2\pi i}\int_{Br_3} e^{(z^2-m)t-bz}\frac{2z^2dz}{(z+b_1)(z^2-\alpha)^{n+1}}\right]_{y=0}$$

$$=-\frac{\mu Ab_1\Gamma(n+1)}{\sqrt{\nu}}\left[\frac{1}{2\pi i}\int_{Br_3} e^{(z^2-m)t}\ \frac{2zdz}{(z^2-\alpha)^{n+1}}-\frac{b_1}{2\pi i}\int_{Br_3} e^{(z^2-m)t}\ \frac{2zdz}{(z+b_1)(z^2-\alpha)^{n+1}}\right]$$

$$=-\frac{\mu Ab_1\Gamma(n+1)}{\sqrt{\nu}}\left[I_0(t,\ \alpha,\ n)-b_1\ I(0,\ t,\ \alpha,\ n)\right] \qquad \ldots(23)$$

where $I_0(t,\ \alpha,\ n)=\frac{1}{2\pi i}\int_{Br_3} e^{(z^2-m)t}\ \frac{2zdz}{(z^2-\alpha)^{n+1}}\,.$ $\qquad \ldots(24)$

As for $I(y, t, \alpha, n)$ here we have

$$nI_0(t, \alpha, n) = \frac{dI_0}{d\alpha}(t, \alpha, n-1) + tI_0(t, \alpha, n-1), \text{ for } n \geqslant 1. \quad \ldots(25)$$

Again from (18) with $b=0$ we get

$$I_0(t, \alpha, 0) = e^{at}. \quad \ldots(26)$$

From eqns. (19), (20), (25) and (26) we can evaluate both the integrals in (23), for different integral values of 'n'.

Particular Cases

Now we shall consider a few particular cases:

(1) *When* at$\neq$0 *and* n$=$0

From eqns. (15) and (19)

$$u(y, t) = Ab_1\, I(y, t, \alpha, 0)$$

$$= Ab_1\Big[\frac{b_1}{\alpha - b_1^2}\, e^{b_1^2 t - mt + bb_1} \text{ erfc. } \frac{b+2b_1 t}{2\sqrt{t}} + \frac{1}{2(b_1 - \sqrt{\alpha})}\, e^{at + b\sqrt{\alpha}}\Big)$$

$$\times \text{erfc. } \frac{b+2t\sqrt{\alpha}}{2\sqrt{t}} + \frac{1}{2(b_1+\sqrt{\alpha}\,)}\, e^{at - b\sqrt{\alpha}} \text{ erfc. } \frac{b-2t\sqrt{\alpha}}{2\sqrt{t}}\Big]$$

which can be written as

$$\frac{u(y, t)}{A} = \frac{1}{h_1^2(a_0^2+at)-1}\, e^{\frac{1}{h_1^2} - a_0^2 + \eta/h_1} \text{ erfc. } \Big(\eta/2 + \frac{1}{h_1}\Big)$$

$$+ \frac{1}{2(1-h_1\sqrt{a_0^2+at})}\, e^{at+\eta\sqrt{a_0^2+at}} \text{ erfc. } (\eta/2 + \sqrt{a_0^2+at})$$

$$+ \frac{1}{2(1+h_1\sqrt{a_0^2+at})}\, e^{at-\eta\sqrt{a_0^2+at}} \text{ erfc. } (\eta/2 - \sqrt{a_0^2+at}) \quad \ldots(27)$$

where $\quad h_1 = \dfrac{\xi u}{\sqrt{\nu t}} = \dfrac{b'}{\sqrt{t}},\ a_0 = \sqrt{mt},\ \eta = \dfrac{b}{\sqrt{t}}\,.$

The shearing stress at the wall is from (23)

$$= -\frac{\mu A b_1}{\sqrt{\nu}}[I_0(t, \alpha, 0) - b_1 I(0, t, \alpha, 0)]\cdot$$

Hence, the skin friction is from (19) and (26)

$$\tau_0=\frac{e^{at}h_1(a_0^2+at)}{h_1^2(a_0^2+at)-1}+e^{at}\operatorname{erf.}(\sqrt{a_0^2+at})\frac{\sqrt{a_0^2+at}}{1-h_1^2(a_0^2+at)}$$

$$+\frac{1}{h_1\{1-h_1^2(a_0^2+at)\}}e^{\frac{1}{h_1^2}-a_0^2}\operatorname{erfc.}\left(\frac{1}{h_1}\right) \qquad \ldots(28)$$

If $at=0$, we get from (27) and (28)

$$\frac{u(y,t)}{A}=\frac{1}{h_1^2a_0^2-1}e^{\frac{1}{h_1^2}-a_0^2+\eta/h_1}\operatorname{erfc.}\left(\eta/2+\frac{1}{h_1}\right)+\frac{1}{2(1-h_1a_0)}e^{\eta a_0}$$

$$\times\, erfc.\,(\eta/2+a_0)+\frac{1}{2(1+h_1a_0)}e^{-\eta a_0}\operatorname{erfc.}(\eta/2-a_0) \qquad ..(29)$$

and

$$\tau_0=\frac{h_1a_0^2}{h_1^2a_0^2-1}+\frac{a_0\, erf.\,(a_0)}{1-h_1^2a_0^2}+\frac{1}{h_1\{1-h_1^2a_0^2\}}e^{\frac{1}{h_1^2}-a_0^2}\operatorname{erfc.}\left(\frac{1}{h_1}\right) \qquad ..(30)$$

(2) *When* at $\neq 0$ *and* n$=1$

From eqns. (15) and (21)

$$u(y,t)=Ab_1I(y,t,\alpha,1)$$

$$=Ab_1\Bigg[\frac{-b_1}{(\alpha-b_1^2)^2}e^{b_1^2t-mt+bb_1}\operatorname{erfc.}\frac{b+2b_1t}{2\sqrt{t}}+e^{at+b\sqrt{\alpha}}\operatorname{erfc.}\frac{b+2t\sqrt{\alpha}}{2\sqrt{t}}$$

$$\times\left(\frac{b}{4\sqrt{\alpha}\,(b_1-\sqrt{\alpha})}+\frac{1}{4\sqrt{\alpha}\,(b_1-\sqrt{\alpha})^2}+\frac{t}{2(b_1-\sqrt{\alpha})}\right)-e^{at+b\sqrt{\alpha}}$$

$$\times\operatorname{erfc.}\frac{b-2t\sqrt{\alpha}}{2\sqrt{t}}\left(\frac{1}{4\sqrt{\alpha}\,(b_1+\sqrt{\alpha})^2}+\frac{b}{4\sqrt{\alpha}\,(b_1+\sqrt{\alpha})}-\frac{t}{2(b_1+\sqrt{\alpha})}\right)$$

$$-\sqrt{\frac{t}{\pi}}\,e^{-b^2/4t-mt}\frac{1}{b_1^2-\alpha}\Bigg].$$

This can be written in the form

$$\frac{u(y,t)}{At} = e^{t+\eta\sqrt{a_0^2+at}}\,\text{erfc.}\,(\eta/2+\sqrt{a_0^2+at})\left[\frac{1}{2(1-h_1\sqrt{a_0^2+at})}\right.$$

$$+\frac{\eta}{4\sqrt{a_0^2+at}\,(1-h_1\sqrt{a_0^2+at})}+\left.\frac{h_1}{4\sqrt{a_0^2+at}\,(1-h_1\sqrt{a_0^2+at})^2}\right]$$

$$+\,e^{at-\eta\sqrt{a_0^2+at}}\,\text{erfc.}\,(\eta/2-\sqrt{a_0^2+at})\left[\frac{1}{2(1+h_1\sqrt{a_0^2+at})}\right.$$

$$-\frac{h_1}{4\sqrt{a_0^2+at}\,(1+h_1\sqrt{a_0^2+at})^2}-\left.\frac{\eta}{4\sqrt{a_0^2+at}\,(1+h_1\sqrt{a_0^2+at}}\right]$$

$$-\frac{h_1^2}{\{h_1^2(a_0^2+at)-1\}^2}\times e^{(1/h_1^2)-a_0^2+\eta/h_1}\,\text{erfc.}\left(\eta/2+\frac{1}{h_1}\right)$$

$$-\frac{h_1}{\sqrt{\pi}}\cdot\frac{e^{-\eta^2/4-a_0^2}}{1-h_1^2(a_0^2+at)}. \qquad ..(31)$$

The shearing stress at the wall is from (23)

$$=-\frac{\mu A b_1}{\sqrt{\nu}}\left[I_0(t,\alpha,1)-b_1 I(0,t,\alpha,1)\right]\cdot \qquad ..(32)$$

From (25) and (26)

$$I_0(t,\alpha,1)=te^{at}. \qquad ..(33)$$

Using eqns. (21), (32) and (33), the skin friction

$$\tau_0=e^{at}\left[\frac{h_1(a_0^2+at)}{h_1^2(a_0^2+at)-1}-\frac{h_1}{\{1-h_1^2(a_0^2+at)\}^2}\right]$$

$$+e^{at}\,\text{erf.}\sqrt{a_0^2+at}\left[\frac{1+h_1^2(a_0^2+at)}{2\sqrt{a_0^2+at}\{1-h_1^2(a_0^2+at)\}^2}\right.$$

$$+\left.\frac{\sqrt{a_0^2+at}}{1-h_1^2(a_0^2+at)}\right]+\frac{h_1 e^{\frac{1}{h_1^2}-a_0^2}\;\text{erfc.}\left(\frac{1}{h_1}\right)}{\{h_1^2(a_0^2+at)-1\}^2}$$

$$+\frac{1}{\sqrt{\pi}}\,e^{-a_0^2}\,\frac{1}{1-h_1^2(a_0^2+at)}\cdot \qquad ..(34)$$

If $at=0$ we get from (31) and (34)

$$\frac{u(y,t)}{At}=e^{\eta a_0}\,\text{erfc.}\,(\eta/2+a_0)\left[\frac{1}{2(1-h_1a_0)}+\frac{\eta}{4a_0(1-h_1a_0)}\right.$$
$$\left.+\frac{h_1}{4a_0(1-h_1a_0)^2}\right]+e^{-\eta a_0}\,\text{erfc.}\,(\eta/2-a_0)\left[\frac{1}{2(1+h_1a_0)}\right.$$
$$\left.-\frac{h_1}{4a_0(1+h_1a_0)^2}-\frac{\eta}{4a_0(1+h_1a_0)}\right]-\frac{h_1^2}{(h_1^2.\,a_0^2-1)^2}\,e^{\frac{1}{h_1^2}-a_0^2+\eta/h_1}\,\text{erfc.}\left(\eta/2+\frac{1}{h_1}\right)$$
$$-\frac{h_1}{\sqrt{\pi}}\,\frac{e^{-\eta^2/4-a_0^2}}{1-h_1^2a_0^2} \qquad \ldots(35)$$

and

$$\tau_0=\frac{h_1a_0^2}{h_1^2a_0^2-1}-\frac{h_1}{(1-h_1^2a_0^2)^2}+\text{erf.}\,(a_0)\left[\frac{1+h_1^2a_0^2}{2a_0(1-h_1^2a_0^2)^2}+\frac{a_0}{1-h_1^2a_0^2}\right]$$
$$+\frac{h_1e^{(1/h_1^2)-a_0^2}\,\text{erfc.}\left(\frac{1}{h_1}\right)}{(h_1^2a_0^2-1)^2}+\frac{1}{\sqrt{\pi}}\,\frac{e^{-a_0^2}}{1-h_1^2a_0^2}\,. \qquad \ldots(36)$$

(3) *When* at $\neq 0$ *and* n=2

From eqns. (15), (20) and (21) we have

$$u(y,t)=2Ab_1\,I(y,t,\alpha,2)$$
$$=Ab_1\left[\frac{2b_1}{(\alpha-b_1^2)^3}\,e^{(b_1^2-m)t+bb_1}\,\text{erfc.}\,\frac{b+2b_1t}{2\sqrt{t}}+e^{at+b\sqrt{\alpha}}\,\text{erfc.}\,\frac{b+2t\sqrt{\alpha}}{2\sqrt{t}}\times\right.$$
$$\left\{\frac{bt}{2\sqrt{\alpha}(b_1-\sqrt{\alpha})}+\frac{t}{2\sqrt{\alpha}(b_1-\sqrt{\alpha})^2}+\frac{t^2}{2(b_1-\sqrt{\alpha})}\right.$$
$$\left.+\frac{\{b(b_1-\sqrt{\alpha})+1\}^2}{8\alpha(b_1-\sqrt{\alpha})^3}+\frac{1}{4\alpha(b_1-\sqrt{\alpha})^3}-\frac{b_1}{8\alpha^{3/2}(b_1-\sqrt{\alpha})^3}-\frac{b}{8\alpha^{3/2}(b_1-\sqrt{\alpha})}\right\}$$
$$+e^{at-b\sqrt{\alpha}}\,\text{erfc.}\,\frac{b-2t\sqrt{\alpha}}{2\sqrt{t}}\left\{\frac{t^2}{2(b_1+\sqrt{\alpha})}-\frac{t}{2(\sqrt{\alpha}(b_1+\sqrt{\alpha})^2}-\frac{bt}{2\sqrt{\alpha}(b_1+\sqrt{\alpha})}\right.$$
$$\left.+\frac{\{b(b_1+\sqrt{\alpha})+1\}^2}{8\alpha(b_1+\sqrt{\alpha})^3}+\frac{b_1}{8\alpha^{3/2}(b_1+\sqrt{\alpha})^3}+\frac{1}{4\alpha(b_1+\sqrt{\alpha})^3}+\frac{b}{8\alpha^{3/2}(b_1+\sqrt{\alpha})}\right\}$$
$$+e^{-\frac{b}{4t}-mt}\left\{\frac{-bb_1\sqrt{t}}{2\alpha\sqrt{\pi}(b_1^2-\alpha)}+\frac{\sqrt{t}}{2\alpha\sqrt{\pi}(b_1-\alpha)}\right.$$
$$\left.\left.-\frac{\sqrt{t}(b_1^2+\alpha)}{\alpha\sqrt{\pi}(b_1^2-\alpha)^2}-\frac{t^{3/2}}{\sqrt{\pi}(b_1^2-\alpha)}\right\}\right]$$

which can be written as

$$\frac{u(y,t)}{At^2} = 2e^{\frac{1}{h_1^2}-a_0^2+\eta/h_1} \text{ erfc.}\left(\eta/2+\frac{1}{h_1}\right)\frac{h_1^4}{\{h_0^2(a_0^2+at)-1\}^3}$$

$$+e^{at+\eta\sqrt{a_0^2+at}} \text{ erfc. } (\eta/2+\sqrt{a_0^2+at})\Big[\frac{1}{2(1-h_1\sqrt{a_0^2+at})}$$

$$+\frac{\eta}{2\sqrt{a_0^2+at}\,(1-h_1\sqrt{a_0^2+at})}+\frac{h_1}{2\sqrt{a_0^2+at}\;(1-h_1\sqrt{a_0^2+at})^2}$$

$$-\frac{h_1}{8(a_0^2+at)^{3/2}(1-h_1\sqrt{a_0^2+at})^3}-\frac{\eta}{8(a_0^2+at)^{3/2}(1-h_1\sqrt{a_0^2+at})}$$

$$+\frac{\eta h_1}{4(a_0^2+at)(1-h_1\sqrt{a_0^2+at})^2}+\frac{\eta^2}{8(a_0^2+at)(1-h_1\sqrt{a_0^2+at})}$$

$$+\frac{3h_1{}^2}{8(a_0^2+at)(1-h_1\sqrt{a_0^2+at})^3}\Big]$$

$$+e^{at-\eta\sqrt{a_0^2+at}}\text{erfc.}\left(\eta/2-\sqrt{a_0^2+at}\right)$$

$$\Big[\frac{1}{2(1+h_1\sqrt{a_0^2+at})}-\frac{h_1}{2\sqrt{a_0^2+at}\;(1+h_1\sqrt{a_0^2+at})^2}$$

$$-\frac{\eta}{2\sqrt{a_0^2+at}(1+h_1\sqrt{a_0^2+at})}+\frac{h_1}{8(a_0^2+at)^{3/2}(1+h_1\sqrt{a_0^2+at})^3}$$

$$+\frac{\eta^2}{8(a_0^2+at)(1+h_1\sqrt{a_0^2+at})}+\frac{\eta h_1}{4(a_0^2+at)(1+h_1\sqrt{a_0^2+at})^2}$$

$$+\frac{3h_1{}^2}{8(a_0^2+at)(1+h_1\sqrt{a_0^2+at})^3}+\frac{\eta}{8(a_0^2+at)^{3/2}(1+h_1\sqrt{a_0^2+at})}\Big]$$

$$-\frac{1}{\sqrt{\pi}}\frac{e^{-\eta^2/4-a_0{}^2}}{1-h_1{}^2(a_0^2+at)}\Big[h_1-\frac{h_1}{2(a_0^2+at)}+\frac{\eta}{2(a_0^2+at)}$$

$$+\frac{h_1+h_1^3(a_0^2+at)}{(a_0^2+at)\{1-h_1^2(a_0^2+at)\}}\Big]. \qquad \dots(37)$$

If $at=0$ we get from (37)

$$\frac{u(y,\,t)}{At^2}=2e^{1/h_1^2-a_0^2+\eta/h_1}\ erfc.\ (\eta/2+\frac{1}{h_1})h_1^4/(h_1^2a_0^2-1)^3$$

$$+e^{\eta a_0}\text{erfc.}(\eta/2+a_0)\Bigg[\frac{1}{2(1-h_1a_0)}+\frac{\eta}{2a_0(1-h_1a_0)}+$$

$$+\frac{h_1}{2a_0(1-h_1a_0)^2}-\frac{h_1}{8a_0^3(1-h_1a_0)^3}-\frac{\eta}{8a_0^3(1-h_1a_0)}$$

$$+\frac{\eta h_1}{4a_0^2(1-h_1a_0)^2}+\frac{\eta^2}{8a_0^2(1-h_1a_0)}+\frac{3h_1^2}{8a_0^2(1-h_1a_0)^3}\Bigg]$$

$$+e^{-\eta a_0}\ \text{erfc.}(\eta/2-a_0)\Bigg[\frac{1}{2(1+h_1a_0)}-\frac{h_1}{2a_0(1+h_1a_0)^2}$$

$$-\frac{\eta}{2a_0(1+h_1a_0)}+\frac{h_1}{8a_0^3(1+h_1a_0)^3}+\frac{\eta^2}{8a_0^2(1+h_1a_0)}$$

$$+\frac{\eta h_1}{4a_0^2(1+h_1a_0)^2}+\frac{3h_1^2}{8a_0^3(1+h_1a_0)^3}+\frac{\eta}{8a_0^3(1+h_1a_0)}\Bigg]$$

$$-\frac{1}{\sqrt{\pi}}\frac{e^{-\eta^2/4-a_0^2}}{1-h_1^2a_0^2}\Bigg[h_1-\frac{h_1}{2a_0^2}+\frac{\eta}{2a_0^2}+\frac{h_1+h_1^2a_0^2}{a_0^2(1-h_1^2a_0^2)}\Bigg]. \qquad \ldots(38)$$

CONCLUSIONS

Figures 1 and 2 show the effect of the magnetic field strength, a_0 on the velocity field. The parameter a_0 can be looked upon as the non-dimensional Hartmann's number in the unsteady motion. These two figures show that for constant value of 'at' and 'h_1' an increase in the magnetic field strength decreases the velocity at any point of the fluid. Again we observe that with the same difference in the value of a_0 the decrease in the velocity at any point is more rapid as a_0 increases.

Figures 1 and 2 also show the effect of the rarefaction constant h_1 on the velocity field. It is also shown that for constant values of a_0 and at, the velocity at any point of the fluid decreases as h_1 increases. When $h_1=0$, for all values of a_0 the plate velocity remains the same, whereas when h_1 is non-zero, the plate velocity is different for different values of a_0, the value of at remaining fixed.

Figures 3 and 4 show the velocity distributions for different values of at and fixed values of h_1 and a_0. With an increase in the value of at, the velocity increases, at any point of the fluid. With the same difference in the value of at, the increase in the value of the velocity at any point is more rapid as at increases.

Figure 5 shows the velocity for different values of n in the expression for the plate velocity $Ae^{at}t^n$, for fixed values of a_0, at and h_1. It is observed that the velocity decreases as n increases. And the fall in the velocity profile gradually decreases as n increases.

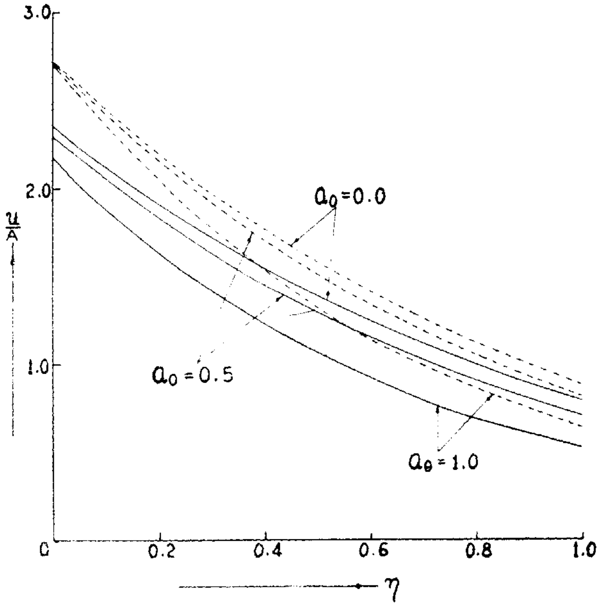

FIG. 1. Velocity distribution for different values of the magnetic number a_0 ($at=1.0$, $n=0$, $h_1=0.0$ ··············, $h_1=0.2$ ————)

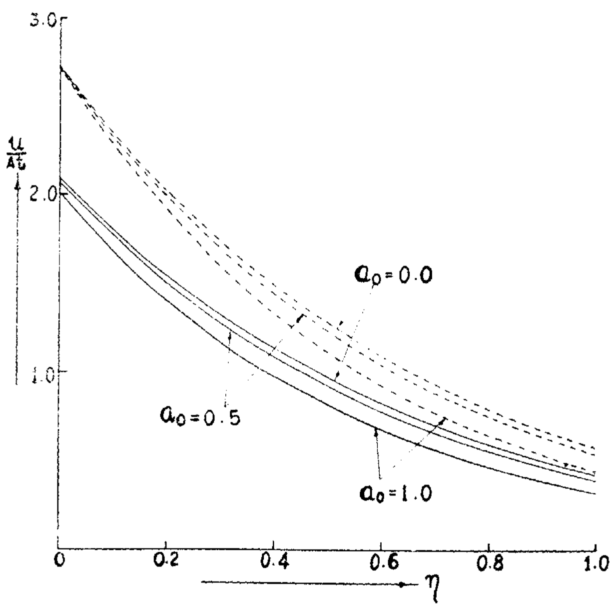

FIG. 2. Velocity distribution for different values of the magnetic number a_0 ($at=1.0$, $n=1$, $h_1=0.0$ ············, $h_1=0.2$ ————).

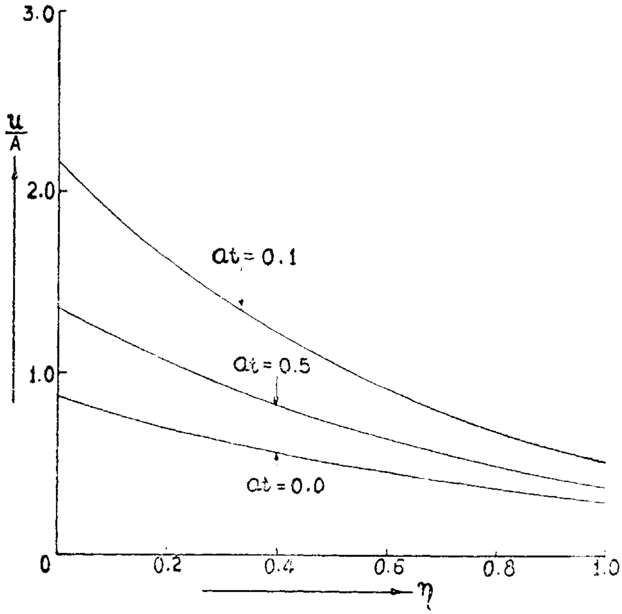

FIG. 3. Velocity distribution for different values of at ($a_0=1.0$, $h_1=0.2$, $n=0$).

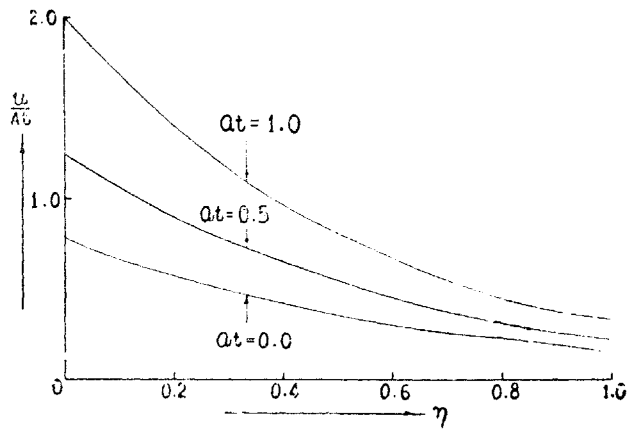

FIG 4. Velocity distribution for different values of at ($a_0=1.0$, $h_1=0.2$, $n=1$).

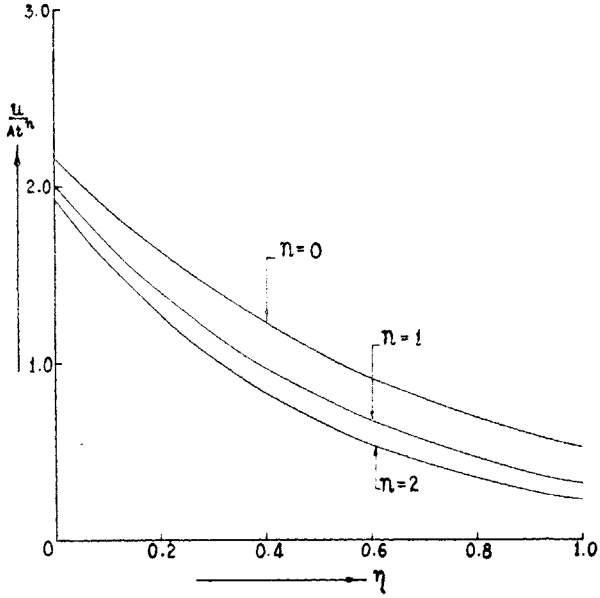

FIG. 5. Velocity distribution for different values of n ($a_0=1.0$, $at=1.0$, $h_1=0.2$).

Tables I and II show the effect of the magnetic field strength a_0 and the parameters at and h_1 on the skin-friction. Here we find that for all values of n, at and h_1 the skin friction increases as a_0 increases. For all values of a_0, h_1 and n, the skin friction increases with at, but for all values of at, a_0 and n we see that the skin friction decreases as h_1 increases.

TABLE I

Values of the skin friction ($n=0$)

a_0 \ at	0.0	0.5	1.0	1.5	h_1
	0.56420	1.36010	2.85500	5.59100	0.0
0.0	0.0	0.71704	2.03925	4.42602	0.1
	0.0	0.64389	1.81982	3.92274	0.2
	0.69760	1.552	3.13200	6.006	0.0
0.5	0.23583	0.99656	2.38314	4.86572	0.1
	0.21237	0.89220	2.11954	4.29707	0.2
	1.05020	2.05700	3.876	7.116	0.0
1·0	0.75019	1.62824	3.18944	5.93449	0.1
	0.66947	1.44309	2.80651	5.18412	0.2
	1.50850	2.74050	4.90600	8.68500	0.0
1.5	1.25233	2.29172	4.09730	7.21513	0.1
	1.09797	1.99472	3.54103	6.19286	0.2

TABLE II

Values of the skin friction ($n=1$)

at / a_0	0.0	0.5	1.0	1.5	h_1
	1.1284	2.1550	3.9999	7.2700	0.0
0.0	1.02836	1.92546	3.51207	6.29158	0.1
	0.92836	1.72154	3.11010	5.52067	0.2
	1.2140	2.2940	4.2090	7.5950	0.0
0.5	1.10024	2.0364	3.66785	6.52459	0.1
	0.98851	1.80681	3.23299	5.70018	0.2
	1.4715	2.6740	4.7940	8.4990	0.0
1.0	1.29202	2.31454	4.09245	7.16531	0.1
	1.14414	2.03094	3.56030	6.18400	0.2
	1.8305	3.2290	5.6540	9.8340	0.0
1.5	1.55312	2.70834	4.69098	8.08107	0.1
	1.34568	2.32861	4.00451	6.85270	0.2

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