ON MIXED BOUNDARY VALUE PROBLEM IN ELECTROSTATICS

by B. M. SINGH, 30/3, *Gandhi Colony, Muzaffar Nagar* (*U. P.*)

(Communicated by B. R. SETH, F.N.A.)

(*Received* 11 *August* 1972)

The problem of determining the electrostatic field due to a parallel plate condenser which is situated inside an earthed hollow cylinder is considered. The discs of the condenser are equal in radius but at different potentials. The problem is reduced to a pair of dual relations involving Fourier-Bessel series equations. By making use of the theory of Fourier-Bessel series equations, the problem is finally reduced to that of solving simultaneous Fredholm integral equations of second kind. The simultaneous Fredholm integral equations are solved by iterative process so as to obtain expressions for capacities of the discs.

1. INTRODUCTION

The problem of determining the electrostatic field due to an electrified disc which is situated inside an earthed coaxial infinitely long hollow cylinder has been treated by Collins (1961), Cooke and Tranter (1959), Rusia (1968) and Sneddon (1962). Assuming axial symmetry of the potential function, Collins (1961) solved it with the help of dual integral equations reaching finally at single Fredholm integral equation of second kind; while using the method of dual series equations, Cooke and Tranter (1959) reduced the problem to the solution of a system of algebraic equations. Finally, they derived the expression for the capacity of the disc. Sneddon (1962) arrived at a similar Fredholm integral equation of the second kind as given by Collins (1961) by making use of dual integral equations. Recently, Rusia (1968) has generalized the above problem by considering the asymmetric potential function on the disc.

In this problem, we shall study the potential associated with electrostatic field of a parallel plate condenser lying within an earthed coaxial infinitely long hollow cylinder. The discs of the condenser are equal in radius but at different potentials. The problem leads to the solution of simultaneous dual integral equations. These equations are then reduced to a system of simultaneous Fredholm integral equations of second kind. However, in this case, when $a > 1$, $k > 1$, $k/a \ll 1$, $\sigma = 1/ak \ll 1$, it is possible to solve them iteratively, where a and k denote the radius of the cylinder and distance between the two discs respectively. The solution leads to the evaluation of the capacities of the discs.

2. The Problem, Fundamental Equation and Boundary Conditions

We shall now consider the problem of determining the electrostatic field produced due to two parallel coaxial equal discs at different potentials. We shall use cylindrical coordinates (ρ, θ, z) for the purpose. Let O and O′ be the centres of the two discs. O is the origin of the system. Suppose that the discs $0 \leqslant \rho < 1, z = 0$ and $0 \leqslant \rho < 1, z = k$, are at prescribed potentials $F_2(\rho)$ and $F_1(\rho)$ respectively. The functions, $F_1(\rho)$ and $F_2(\rho)$, are axially symmetric.

We divide the whole space into three regions : (1) $0 < z < k, 0 < \rho < a$, (2) $k < z < \infty, 0 < \rho < a$ and (3) $z < 0, 0 < \rho < a$. The potential fields are denoted in these regions by V_2, V_1 and V_3 respectively as shown in Fig. 1. Here V_1, V_2 and V_3 are functions of ρ and z. The following are the boundary conditions :

$$V_1 = V_2, z = k, \quad 0 \leqslant \rho < a. \qquad \ldots(1)$$

$$V_2 = V_3, z = 0, \quad 0 \leqslant \rho < a. \qquad \ldots(2)$$

On the common boundaries, we have

$$V_1 = F_1(\rho), 0 \leqslant \rho < 1, z = k. \qquad (3)$$

$$V_3 = F_2(\rho), 0 \leqslant \rho < 1, z = 0. \qquad \ldots(4)$$

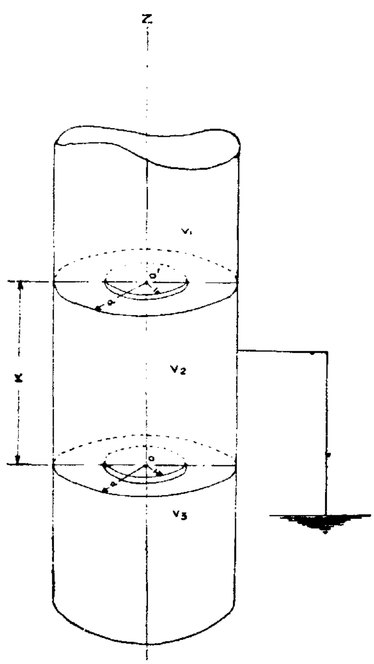

Fig. 1

In addition, on the section of the common boundaries that are not on the plates of the condenser, the normal components of the potentials must be continuous. Hence we have

$$\frac{\partial V_1}{\partial z} = \frac{\partial V_2}{\partial z}, \quad 1 < \rho < a, z = k \qquad \ldots(5)$$

$$\frac{\partial V_2}{\partial z} = \frac{\partial V_3}{\partial z}, \quad 1 < \rho < a, z = 0. \qquad \ldots(6)$$

Due to earthing of the cylinder, we have

$$V_1 = V_2 = V_3 = 0, \rho = a. \qquad \ldots(7)$$

Finally, V_1, V_2 and V_3 must satisfy the Laplace's equation

$$\nabla^2 V_l = 0, l = 1, 2, 3, \qquad \ldots(8)$$

and

$$\left.\begin{array}{l} V_1 \to 0, z \to \infty \\ V_3 \to 0, z \to -\infty. \end{array}\right\} \qquad \ldots(9)$$

3. Reduction of the Problem to a System of Simultaneous Dual Series Equations

$$V_1 = \sum_{n=0}^{\infty} (\lambda_n)^{-1} a_n \exp(-\lambda_n(z-k)) J_0 (\lambda_n \rho), z > k, 0 < \rho < a. \qquad \ldots(10)$$

$$V_2 = \sum_{n=0}^{\infty} (\lambda_n)^{-1} [c_n \sinh \lambda_n z + d_n \sinh \lambda_n (k-z)] J_0 (\lambda_n \rho),$$
$$0 < z < k, 0 < \rho < a \qquad \ldots(11)$$

$$V_3 = \sum_{n=0}^{\infty} (\lambda_n)^{-1} b_n \exp (\lambda_n z) J_0 (\lambda_n \rho), z < 0, 0 < \rho < a. \qquad \ldots(12)$$

where a_n, b_n, c_n and d_n are unknown constants. The above functions satisfy the conditions (8) and (9). The harmonic functions V_1, V_2 and V_3 will satisfy the condition (7) provided λ_1, λ_2 ..... are the positive zeros of $J_0 (\lambda a)$.

From (1) and (2) we immediately obtain two relations:

$$\left.\begin{array}{l} c_n = \dfrac{a_n}{\sinh (\lambda_n k)} \\ d_n = \dfrac{b_n}{\sinh (\lambda_n k)}. \end{array}\right\} \qquad \ldots(13)$$

The conditions (3), (4), (5) and (6) are then equivalent to a system of simultaneous dual series equations:

$$\sum_{n=0}^{\infty} (\lambda_n)^{-1} a_n J_0 (\lambda_n \rho) = F_1 (\rho), \quad 0 \leqslant \rho < 1 \qquad \ldots(14)$$

$$\sum_{n=0}^{\infty} (\lambda_n)^{-1} b_n J_0 (\lambda_n \rho) = F_2 (\rho), \quad 0 \leqslant \rho < 1 \qquad \ldots(15)$$

$$\sum_{n=0}^{\infty} \left[(1 + \coth k \lambda_n) a_n - \frac{b_n}{\sinh \lambda_n k} \right] J_0 (\lambda_n \rho) = 0, \; 1 < \rho < a \qquad \ldots(16)$$

$$\sum_{n=0}^{\infty} \left[(1 + \coth k \lambda_n) b_n - \frac{a_n}{\sinh \lambda_n k} \right] J_0 (\lambda_n \rho) = 0, \; 1 < \rho < a. \qquad (17)$$

4. Reduction of Simultaneous Dual Series Equations to Simultaneous Fredholm Integral Equations

Dual relations involving Fourier-Bessel series are considered by Sneddon and Srivastav (1964). Making use of the method of Sneddon and Srivastav (1964) we assume that

$$\sum_{n=0}^{\infty} \left[(1 + \coth k \lambda_n) a_n - \frac{b_n}{\sinh \lambda_n k} \right] J_0 (\lambda_n \rho)$$

$$= - \frac{1}{\rho} \frac{d}{d\rho} \int_{\rho}^{1} \frac{t \, g(t) \, dt}{(t^2 - \rho^2)^{1/2}}, \quad 0 \leqslant \rho < 1 \qquad \ldots(18)$$

$$\sum_{n=0}^{\infty} \left[(1 + \coth k\lambda_n) b_n - \frac{a_n}{\sinh \lambda_n k} \right] J_0 (\lambda_n \rho)$$

$$= - \frac{1}{\rho} \frac{d}{d\rho} \int_{\rho}^{1} \frac{t \, h(t) \, dt}{(t^2 - \rho^2)^{1/2}}, \quad 0 \leqslant \rho < 1 \qquad \ldots(19)$$

where $g(t)$ and $h(t)$ are unknown functions, but it is presumed that the integrals involving $g(t)$ and $h(t)$ exist. Using the Fourier-Bessel theorem and performing integration by parts in the integrals, and with the help of the well-known integral

$$\int_0^t \frac{J_0(\rho\,\lambda_n)\,d\rho}{(t^2-\rho^2)^{1/2}} = (t\,\lambda_n)^{-1}(1-\cos t\,\lambda_n) \qquad \text{..(20)}$$

we can easily get from (18) and (19) that

$$a_n = \frac{1}{a^2 J_1^2(\lambda_n a)}\left[\int_0^1 g(t)\cos t\,\lambda_n\,dt + e^{-k\lambda_n}\int_0^1 h(t)\cos t\lambda_n\,dt\right] \qquad \text{...(21)}$$

$$b_n = \frac{1}{a^2 J_1^2(\lambda_n a)}\left[\int_0^1 h(t)\cos t\lambda_n\,dt + e^{-k\lambda_n}\int_0^1 g(t)\cos t\lambda_n\,dt\right]. \qquad \text{...(22)}$$

If we substitute the value of a_n and b_n in (14) and (15), and interchanging the order of integration and summation, we obtain the relations

$$\int_0^1 g(t)\,dt\sum_{n=0}^{\infty}\frac{(\lambda_n)^{-1}J_0(\lambda_n\rho)\cos t\lambda_n}{a^2 J_1^2(\lambda_n a)} + \int_0^1 h(t)\,dt$$

$$\times\sum_{n=0}^{\infty}\frac{e^{-k\lambda_n}(\lambda_n)^{-1}J_0(\lambda_n\rho)\cos t\lambda_n}{a^2 J_1^2(\lambda_n a)} = F_1(\rho),\ 0\leqslant\rho<1 \qquad \text{...(23)}$$

$$\int_0^1 h(t)\,dt\sum_{n=0}^{\infty}\frac{(\lambda_n)^{-1}J_0(\lambda_n\rho)\cos t\lambda_n}{a^2 J_1^2(\lambda_n a)} + \int_0^1 g(t)\,dt$$

$$\times\sum_{n=0}^{\infty}\frac{(\lambda_n)^{-1}e^{-k\lambda_n}J_0(\lambda_n\rho)\cos t\lambda_n}{a^2 J_1^2(\lambda_n a)} = F_2(\rho),\ 0\leqslant\rho<1. \qquad \text{...(24)}$$

From Sneddon (1966, pp. 34–35) it follows :

$$\frac{2}{a^2}\sum_{n=0}^{\infty}\frac{(\lambda_n)^{-1}J_0(\lambda_n\rho)\cos t\lambda_n}{J_1^2(a\,\lambda_n)}$$

$$= \int_0^{\infty} J_0(\rho x)\cos tx\,dx - \frac{2}{\pi}\int_0^{\infty}\frac{K_0(ay)}{I_0(ay)}I_0(\rho y)\cosh ty\,dy. \qquad \text{...(25)}$$

As in Sneddon (1966, pp. 34–35), we can easily derive the following result:

$$\frac{2}{a^2}\sum_{n=0}^{\infty}\frac{(\lambda_n)^{-1} J_0(\rho\lambda_n)\cos t\lambda_n\, e^{-k\lambda_n}}{J_1^2(a\lambda_n)} = \int_0^{\infty} J_0(\rho x)\cos(tx)\, e^{-kx}\,dx$$

$$-\frac{2}{\pi}\int_0^{\infty}\frac{k_0(ay)}{I_0(ay)} I_0(\rho y)\cosh\, ty\cos ky\,dy. \quad \text{.. (26)}$$

Making use of (25) and (26) and the integral from Sneddon (1966, p. 27), the following two relations are obtained:

$$\int_0^{\rho}\frac{g(t)\,dt}{(\rho^2-t^2)^{1/2}} = \frac{2}{\pi}\int_0^1 g(t)\,dt\int_0^{\infty}\frac{k_0(ay)}{I_0(ay)} I_0(\rho y)\cosh ty\,dy$$

$$-\frac{2}{\pi}\int_0^1 h(t)\,dt\int_0^{\infty}\frac{k_0(ay)}{I_0(ay)} I_0(\rho y)\cosh ty\cos ky\,dy$$

$$-\int_0^1 h(t)\,dt\int_0^{\infty} J_0(\rho x)\cos xt\, e^{-kx}dx + 2F_1(\rho),\ 0\leqslant\rho<1 \quad \text{...(27)}$$

and

$$\int_0^{\rho}\frac{h(t)dt}{(\rho^2-t^2)^{1/2}} = \frac{2}{\pi}\int_0^1 h(t)\,dt\int_0^{\infty}\frac{k_0(ay)}{I_0(ay)} I_0(\rho y)\cosh ty\,dy$$

$$-\frac{2}{\pi}\int_0^1 g(t)dt\int_0^{\infty}\frac{k_0(ay)}{I_0(ay)} I_0(\rho y)\cosh ty\cos ky\,dy$$

$$-\int_0^1 g(t)dt\int_0^{\infty} e^{-kx} J_0(\rho x)\cos tx\,dx + 2F_2(\rho),\ 0\leqslant\rho<1. \quad \text{.. (28)}$$

Equations (27) and (28) are two Abel type equations. Regarding the right-hand sides of these equations as known functions of ρ and solving these equations with the help of one given by Sneddon(1966, pp. 40–41), we find that

$$g(t) = \frac{4}{\pi}\frac{d}{dt}\int_0^t \frac{\rho F_1(\rho)d\rho}{(t^2-\rho^2)^{1/2}} + \frac{4}{\pi^2}\int_0^1 g(u)\,du \int_0^\infty \frac{k_0(ay)}{I_0(ay)} \cosh yt \cosh yu\, dy$$

$$+\frac{4}{\pi^2}\int_0^\infty h(u)du \int_0^\infty \frac{k_0(ay)}{I_0(ay)} \cosh yu \cosh ty \cos ky\, dy$$

$$-\frac{2}{\pi}\int_0^1 h(u)\,du \int_0^\infty e^{-kx} \cos ux \cos xt\, dx,\ 0\leqslant t<1 \qquad \ldots(29)$$

and

$$h(t) = \frac{4}{\pi}\frac{d}{dt}\int_0^t \frac{\rho F_2(\rho)d\rho}{(t^2-\rho^2)^{1/2}} + \frac{4}{\pi^2}\int_0^1 g(u)du$$

$$\times\int_0^\infty \frac{k_0(ay)}{I_0(ay)} \cosh yu \cosh ty \cos ky\, dy - \frac{2}{\pi}\int_0^1 g(u)du$$

$$\times\int_0^\infty e^{-kx}\cos ux \cos xt\, dx + \frac{4}{\pi^2}\int_0^1 h(u)\ du \int_0^\infty \frac{k_0(ay)}{I_0(ay)}$$

$$\times \cosh (yu) \cosh (yt)\, dy,\ 0\leqslant t<1. \qquad \ldots(30)$$

For obtaining (29) and (30), we used the two well-known results

$$\frac{d}{dt}\int_0^t \frac{\rho I_0(\rho y)d\rho}{(t^2-\rho^2)^{1/2}} = \cosh yt \qquad \ldots(31)$$

$$\frac{d}{dt}\int_0^t \frac{\rho J_0(\rho y)d\rho}{(t^2-\rho^2)^{1/2}} = \cos yt. \qquad \ldots(32)$$

5. Solution of Simultaneous Fredholm Integral Equations

We shall now find the iterative solution of eqns. (29) and (30). Here we have found the solution of only a particular case of the problem, though by numerical process the solution of the general case of the problem can also be obtained.

We shall only consider the case in which $a>1$, $k>1$, $k/a\ll 1$, and $\sigma\ll 1$, where

$$\sigma = \frac{1}{ak}\ .$$

The following experssions are expanded in powers of σ

$$\frac{2}{\pi}\int_0^\infty e^{-kx}\cos ux \cos xt\, dx = \frac{k}{\pi}\left[\frac{1}{k^2+(u+t)^2}+\frac{1}{k^2+(u-t)^2}\right]$$

$$= \frac{2}{\pi}\left[a\sigma - a^3\sigma^3\,(u^2+t^2)+a^5\sigma^5\,(u^4+t^4+6t^2u^2)+O(\sigma^7)+\ldots\right],\ k>1 \quad \ldots(33)$$

$$\int_0^\infty \frac{k_0(ay)}{I_0(ay)}\cosh yt \cosh uy\, dy$$

$$= \left[k\sigma M_0 + \frac{M_2}{2}k^3\sigma^3(t^2+u^2) + \frac{M_4\sigma^5k^5}{24}\,(t^4+u^4+6t^2u^2) + O(\sigma^7) + -\right],\ a>1 \quad \ldots(34)$$

$$\int_0^\infty \frac{k_0(ay)}{I_0(ay)}\cosh uy \cosh ty \cos ky dy$$

$$= k\sigma M_0 + \frac{\sigma^3k^3}{2}M_2\left[u^2+t^2-k^2\right] + \frac{\sigma^5M_4k^5}{24}\left[u^4+t^4+6t^2u^2 - 6k^2\left(u^2+t^2-\frac{k^2}{6}\right)+0(\sigma^7)+\ldots\right],\ a>1,\ \frac{k}{a}\ll 1, \quad \ldots(35)$$

where

$$M_n = \int_0^\infty \frac{r^n k_0(r)dr}{I_0(r)},\ (n = 0, 2, 4, \ldots). \quad \ldots(36)$$

Cooke and Tranter (1959) calculated the numerical values of the integrals. The values are:

$M_0 = 1.36768,\ M_2 = 0.64689,$
$M_4 = 2.06986.$

Suppose that the solutions of eqns. (29) and (30) can be expressed as

$$g(t) = g_0(t)+\sigma g_1(t)+\sigma^2 g_2(t)+\sigma^3 g_3(t)+\cdots \quad \ldots(37)$$

and

$$h(t) = h_0(t)+\sigma h_1(t)+\sigma^2 h_2(t)+\sigma^3 h_3(t)+\ldots \quad \ldots(38)$$

Let

$$F_1(\rho) = V_0,\ F_2(\rho) = u_1$$

where V_0 and u_1 are constants.

Substituting the expressions for $g(t)$ and $h(t)$ from (37) and (38) in (29) and (30), and equating the like powers of σ, we get

$$g_0(t) = \frac{4V_0}{\pi}, \qquad h_0(t) = \frac{4u_1}{\pi},$$

$$g_1(t) = \frac{16kM_0}{\pi^3}(V_0+u_1) - \frac{8u_1 a}{\pi^2},$$

$$h_1(t) = \frac{16kM_0}{\pi^3}(V_0+u_1) - \frac{8V_0 a}{\pi^2},$$

$$g_2(t) = \frac{128kM_0}{\pi^5}(V_0+u_1)\left(M_0 k - \frac{\pi a}{4}\right) + \frac{16V_0 a^3}{\pi^3},$$

$$h_2(t) = \frac{128kM_0}{\pi^5}(V_0+u_1)\left(M_0 k - \frac{\pi a}{4}\right) + \frac{16u_1 a^2}{\pi^3}.$$

It is clear that $g_0(t)$, $h_0(t)$, $g_1(t)$, $h_1(t)$, $g_2(t)$ and $h_2(t)$ are constants. They are not the function of t. Hence

$$g_3(t) = C_0 + t^2\left[\frac{8M_2k^3}{\pi^3}(V_0+u_1) + \frac{8a^3u_1}{\pi^2}\right]$$

where

$$C_0 = \frac{512M_0k}{\pi^7}\left[(V_0+u_1)\left(M_0k - \frac{\pi a}{4}\right)\left(2kM_0 - \frac{\pi a}{2}\right)\right] - \frac{8M_2k^5u_1}{\pi^3}$$

$$+ \frac{64a^2kM_0}{\pi^5}(V_0+u_1) - \frac{32a^3u_1}{\pi^4} + \frac{1}{3}\left[\frac{8M_3k^3}{\pi^3}(V_0+u_1) + \frac{8a^3u_1}{\pi^2}\right]$$

$$h_3(t) = R_0 + t^2\left[\frac{8M_2k^3}{\pi^3}(V_0+u_1) + \frac{8a^3V_0}{\pi^2}\right]$$

in which

$$R_0 = \frac{512M_0k}{\pi^7}\left[(V_0+u_1)\left(M_0k - \frac{\pi a}{4}\right)\left(2M_0k - \frac{\pi a}{2}\right)\right]$$

$$- \frac{8M_2k^5V_0}{\pi^3} + \frac{64a^2kM_0}{\pi^5}(V_0+u_1) - \frac{32V_0a^3}{\pi^4}$$

$$+ \frac{1}{3}\left[\frac{8M_2k^3}{\pi^3}(V_0+u_1) + \frac{8a^3V_0}{\pi^2}\right],$$

$$g_4(t) = t^2\left[\frac{16M_0k}{\pi^3}(V_0+u_1)\left(\frac{2a^3}{\pi} + \frac{4M_2k^3}{\pi^2}\right)\right.$$

$$\left. - \frac{16}{\pi^4}M_2k^3a(V_0+u_1) - \frac{16V_0a^4}{\pi^3}\right] + C_1$$

and

$$C_1 = \frac{2M_2k^3g_1(t)}{3\pi^2} + \left(\frac{4}{\pi^2}kM_0 - \frac{2a}{\pi}\right)\left\{R_0 + \tfrac{1}{3}\left(\frac{8M_2k^3}{\pi^3}(V_0+u_1) + \frac{8a^3V_0}{\pi^2}\right)\right\}$$

$$+ \frac{4kM_0}{\pi^2}\left[C_0 + \tfrac{1}{3}\left\{\frac{8M_2k^3}{\pi^3}(V_0+u_1) + \frac{8a^3u_1}{\pi^2}\right\}\right]$$

$$+ \left[\frac{2}{\pi^2}M_2k^3(\tfrac{1}{3}-k^2) + \frac{2a^3}{3\pi}\right]h_1(t),$$

$$h_4(t) = t^2\left[\frac{16M_0k}{\pi^3}(V_0+u_1)\left(\frac{2a^3}{\pi} + \frac{4M_2k^3}{\pi^2}\right)\right.$$

$$\left. - \frac{16M_2k^3a}{\pi^4}(V_0+u_1) - \frac{16u_1a^4}{\pi^3}\right] + R_1,$$

in which

$$R_1 = \frac{2M_2k^3}{3\pi^2}h_1(t) + \left(\frac{4}{\pi^2}kM_0 - \frac{2a}{\pi}\right)\left\{C_0 + \tfrac{1}{3}\left(\frac{8M_2k^3}{\pi^3}(V_0+u_1) + \frac{8a^3u_1}{\pi^3}\right)\right\}$$

$$+ \frac{4M_0k}{\pi^2}\left[R_0 + \tfrac{1}{3}\left\{\frac{8M_2k^3}{\pi^3}(V_0+u_1) + \frac{8a^3V_0}{\pi^2}\right\}\right]$$

$$+ \left[\frac{2M_2k^3}{\pi^2}(\tfrac{1}{3}-k^2) + \frac{2a^3}{3\pi}\right]g_1(t).$$

C_0, C_1, R_0 and R_1 are constants and not the function of t.

6. Some Approximate Results

Capacity of the disc $0 \leqslant \rho < 1$, $z = 0$ is equal to

$$\frac{1}{u_1}\int_0^1 h(t)dt = \frac{1}{u_1}\left[\frac{4u_1}{\pi} + \left\{\frac{16M_0k}{\pi^3}(V_0+u_1)\right.\right.$$

$$\left. - \frac{8V_0a}{\pi^2}\right\}\sigma + \sigma^2\left\{\frac{128kM_0}{\pi^5}(V_0+V_1)\left(M_0k - \frac{\pi a}{4}\right)\right.$$

$$\left. + \frac{16M_1a^2}{\pi^3}\right\} + \sigma^3\left\{R_0 + \frac{8M_2k^3}{3\pi^3}(V_0+u_1) + \frac{8a^3V_0}{3\pi^2}\right\}$$

$$+ \sigma^4\left\{R_1 + \frac{16M_0k}{3\pi^3}(V_0+u_1)\left(\frac{2a^3}{\pi} + \frac{4M_2k^3}{\pi^2}\right) - \frac{16M_2k^3a}{3\pi^4}\right.$$

$$\left.\left. \times (V_0+u_1) - \frac{16u_1a^4}{3\pi^3}\right\}\right] + O(\sigma^5) + \ldots$$

The capacity of the disc $0 \leqslant \rho < 1$, $z = k$ is equal to

$$\frac{1}{V_0}\int_0^1 g(t)\,dt = \frac{1}{V_0}\Bigg[\frac{4V_0}{\pi} + \sigma\left\{\frac{16kM_0}{\pi^3}(V_0+u_1) - \frac{8u_1 a}{\pi^2}\right\}$$

$$+\sigma^2\left\{\frac{128kM_0}{\pi^3}(V_0+u_1)\left(M_0 k - \frac{\pi a}{4}\right) + \frac{16V_0 a^2}{\pi^3}\right\}$$

$$+\sigma^3\left\{C_0 + \frac{8M_2k^3}{3\pi^3}(V_0+u_1) + \frac{8a^3u_1}{3\pi^2}\right\}$$

$$+\sigma^4\left\{\frac{16M_0k}{3\pi^3}(V_0+u_1)\left(\frac{2a^3}{\pi} + \frac{4M_2k^3}{\pi^2}\right)\right.$$

$$\left.-\frac{16M_2k^3a}{3\pi^4}(V_0+u_1) - \frac{16V_0a^4}{3\pi^3} + C_1\right\}\Bigg] + O(\sigma^5) + \dots$$

ACKNOWLEDGEMENT

The auther is thankful to Prof. P. M. Gupta for his interest during the preparation of this paper.

REFERENCES

Collins, W. D. (1961). Note on an electrified circular disc situated inside an earthed coaxial infinite hollow cylinder. *Proc. Camb. phil. Soc.*, **57**, 623–27.

Cooke, J. C., and Tranter, C. J. (1959). Dual Fourier-Bessel series. *Q. Jl Mech. appl. Math.*, **12**, 379–86.

Rusia, K. C. (1968). On certain asymmetric mixed boundary value problems of an electrified circular disc situated inside a coaxial infinite hollow cylinder. *Indian J. pure appl. Phys.*, **6**, No. 1, 44-46.

Sneddon, I. N. (1962). Note on an electrified circular disc situated inside a coaxial infinite hollow cylinder. *Proc. Camb. phil. Soc.*, **58**, 621–24.

Sneddon, I. N. (1966). Mixed Boundary Value Problems in Potential Theory. North-Holland Publishing Company, Amsterdam.

Sneddon, I. N., and Srivastav, R. P. (1964). Dual series relations I—Dual relations involving Fourier-Bessel series. *Proc. R. Soc. Edinb.*, **66** A, 151.