OPERATIONAL READINESS OF A COMPLEX SYSTEM HAVING SHELF-LIFE OF THE COMPONENTS UNDER PRE-EMPTIVE REPEAT REPAIR DISCIPLINE

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Complex system comprised of two types of components (say I and II), has been considered. Type I consists of N components connected in series while Type II consists of components connected in stand-by redundancy. It has been assumed that the stand-by component may fail in its shelf-life. The operational readiness of such a system has been investigated under pre-emptive repeat repair discipline, using supplementary variable technique and Laplace transforms. In the end, some particular cases have been discussed when the system is comprised of components connected in parallel redundancy.

Introduction

Reliability characteristics of a system are often improved by providing components in stand-by redundancy (Srivastava *et al.* 1971), assuming that (i) the switching over device is hundred per cent reliable and (ii) the stand-by components always work, when sensed for operation. However, in many practical situations, it has been observed that the stand-by component may itself fail in its shelf-life before being switched for actual operation. Consider for example, a battery connected in stand-by redundancy may fail, before being sensed for the actual operation, in its shelf-life.

In considering this situation, it has been assumed that failure rates in case of operation (λ') and shelf-life (α) are different. The advantage of such an assumption is: if it is considered that these failure rates are equal, i.e., $\lambda'=\alpha$, the behaviour of the system may also be studied when it is comprised of components connected in parallel redundancy (Gaver 1963).

With this in view, the behaviour of a complex system having two types of components has been investigated. Type I consists of N components connected in series where failure of any one component brings about the complete breakdown of the system. Type II is comprised of two identical components (say A and B), one of which is in stand-by redundancy (B). It has been assumed that the stand-by component (B) may also fail in its shelf-life before being switched for the operation. The characteristics of the components of type II are such that on failure of both the components, the system works in a state of reduced efficiency. It has been assumed

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that repair of type II is only undertaken when both the components have failed. The failure and repair of the components of both types follow exponential and general time distributions, respectively. Further, the repair of the components of type I and type II has been carried out under pre-emptive repeat repair discipline (Govil 1970) because of the fact that the components of type I are essential components in the operation of the system, as such priority (White and Christic 1956) has been given for repair to this type of components.

Pre-emptive Repeat Repair Discipline

In this discipline, a failed component of priority type is taken under repair immediately on failure. In other words, the component of this type pre-empts in the repair facility even when it was engaged in the repair of the components of type II. However, when the components of type II which had been removed from repair facility due to the priority, are taken up for repair again and no consideration is given to its repair already carried out. It only means that the repair of the components of type II, when restarts, is considered as a fresh failure.

The various state probabilities of the system have been obtained by using supplementary variable technique and Laplace transforms (Widder 1946). In the end, some particular cases have also been discussed when $\alpha=\lambda'$, i.e., the stand-by components do not fail in its shelf-life.

NOTATIONS

λ_i $\equiv$ constant failure rate of the ith component of type I.

λ $\equiv \sum_{i=1}^{N} \lambda_i$

λ' $\equiv$ constant failure rate of the components of type II.

α $\equiv$ constant failure rate of the stand-by (due to the shelf-life) component of type II.

$S_i(x)$ $\equiv$ the general probability density function of ith component of type I

$S'(y)$ $\equiv$ the general probability density function of the components of type II.

$\eta_i(x)\Delta$ $\equiv$ the first order probability that the ith component of type I will be repaired in the interval $(x, x+\Delta)$, conditioned that it was not repaired up to time x.

$\phi(y)\triangle$ $\equiv$ the first order probability that the components of type II will be repaired in the interval $(y, y+\triangle)$, conditioned that they were not repaired up to time y.

According to Davis (1952), the relation between $S_i(x)$ *and* $\eta_i(x)$ may be given as:

$$S_i(x)=\eta_i(x)\exp\left[-sx-\int_0^x \eta_i(x)\,dx\right].$$

Similar result holds good for $S'(y)$ and $\phi(y)$.

DEFINE

$p_{0,0,0}(t)$ $\equiv$ the probability that at time t, all the components of type I and type II are working in normal efficiency.

$p_{0,A,0}(t)$ $\equiv$ the probability that at time t, component (A) has failed and after sensing, stand-by component (B) is working and all the components of type I are also working.

$p_{0,0,B}(t)$ $\equiv$ the probability that at time t, the stand-by component (B) has already failed in its shelf-life whereas component (A) of type II and all the components of type I are working.

$p_{0,A,B}(y,t)\triangle$ $\equiv$ the probability that at time t, the system is working in reduced efficiency state due to the failure of both the components of type II and the elapsed repair time lies in the interval $(y, y+\triangle)$.

$p_{f_i,0,0}(x,t)\triangle$ $\equiv$ the probability that at time t, the system has failed due to the failure of ith component of type I and the elapsed repair time lies in the interval $(x, x+\triangle)$, whereas both the components of type II are in working order.

$p_{f_i,A,0}(x,t)\triangle$ $\equiv$ the probability that at time t, the system is in the failed state due to the failure of ith component of type I and the elapsed repair time lies in the interval $(x, x+\triangle)$, whereas 'A' component of type II has failed and after sensing, stand-by componont (B), is operating.

$p_{f_i,0,B}(x,t)\triangle$ $\equiv$ the probability that at time t, the system is in the failed state due to the failure of ith component of type I and the elapsed repair time lies in the interval $(x, x+\triangle)$, whereas the stand-by component (B) of type II has failed in its shelf-life.

$q_{f_i, A, B}(x, t)\triangle \equiv$ the probability that at time t, the system is in the failed state due to the failure of ith component of type I and both the components of type II are already in failed state and the elapsed repair time of the ith component of type I lies in the interval $(x, x+\triangle)$ at the instant when it pre-empted in the repair facility and the failed components of type II which had been in service, are awaiting repair.

Formulation of the Model

Following the continuity arguments, we get the following differential equations governing the behaviour of the system:

$$\left[\frac{\partial}{\partial t}+\lambda+\lambda'+\alpha\right]p_{0,0,0}(t)=\sum_{i=1}^{N}\int_0^\infty p_{f_i,0,0}(x,t)\eta_i(x)dx + \int_0^\infty p_{0,A,B}(y,t)\,\phi(y)\,dy \qquad \ldots(1)$$

$$\left[\frac{\partial}{\partial t}+\lambda+\alpha\right]p_{0,A,0}(t)=\lambda' p_{0,0,0}(t)+\sum_{i=1}^{N}\int_0^\infty p_{f_i,A,0}(x,t)\,\eta_i(x)\,dx \qquad \ldots(2)$$

$$\left[\frac{\partial}{\partial t}+\lambda+\lambda'\right]p_{0,0,B}(t)=\alpha p_{0,0,0}(t)+\sum_{i=1}^{N}\int_0^\infty p_{f_i,0,B}(x,t)\,\eta_i(x)\,dx \qquad \ldots(3)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\lambda+\phi(y)\right]p_{0,A,B}(y,t)=0 \qquad \ldots(4)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\eta_i(x)\right]p_{f_i,0,0}(x,t)=0 \qquad \ldots(5)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\eta_i(x)\right]p_{f_i,A,0}(x,t)=0 \qquad \ldots(6)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\eta_i(x)\right]p_{f_i,0,B}(x,t)=0 \qquad \ldots(7)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\eta_i(x)\right]q_{f_i,A,B}(x,t)=0. \qquad \ldots(8)$$

The above equations are to be solved under the following boundary conditions :

$$p_{0,A,B}(0,t)=\alpha\, p_{0,A,0}(t)+\lambda' p_{0,0,B}(t)+\sum_{i=1}^{N}\int_0^\infty q_{f_i\,A,B}(x,t)\,\eta_i(x)\,dx \quad ...(9)$$

$$p_{f_i,0,0}(0,t)=\lambda_i\, p_{0,0,0}(t) \quad ...(10)$$

$$p_{f_i,A,0}(0,t)=\lambda_i\, p_{0,A,0}(t) \quad ...(11)$$

$$p_{f_i,0,B}(0,t)=\lambda_i\, p_{0,0,B}(t) \quad ...(12)$$

$$q_{f_i,A,B}(0,t)=\lambda_i\int_0^\infty p_{0,A,B}(y,t)\,dy. \quad ...(13)$$

Assuming, that initially the system is working in normal efficiency, i.e., $p_{0,0,0}(0)=1$, so that other state probabilities are zero.

Taking Laplace transforms of eqn. (1) through (13) and using initial conditions mentioned above, we get

$$[s+\lambda+\lambda'+\alpha]\,\bar{p}_{0,0,0}(s)=1+\sum_{i=1}^{N}\int_0^\infty \bar{p}_{f_i,0,0}(x,s)\,\eta_i(x)\,dx + \int_0^\infty \bar{p}_{0,A,B}(y,s)\,\phi(y)\,dy \quad ...(14)$$

$$[s+\lambda+\alpha]\,\bar{p}_{0,A,0}(s)=\lambda'\,\bar{p}_{0,0,0}(s)+\sum_{i=1}^{N}\int_0^\infty \bar{p}_{f_i,A,0}(x,s)\,\eta_i(x)\,dx \quad ...(15)$$

$$[s+\lambda+\lambda']\,\bar{p}_{0,0,B}(s)=\alpha\,\bar{p}_{0,0,0}(s)+\sum_{i=1}^{N}\int_0^\infty \bar{p}_{f_i,0,B}(x,s)\,\eta_i(x)\,dx \quad ...(16)$$

$$\left[\frac{\partial}{\partial y}+s+\lambda+\phi(y)\right]\bar{p}_{0,A,B}(y,s)=0 \quad ...(17)$$

$$\left[\frac{\partial}{\partial x}+s+\eta_i(x)\right]\bar{p}_{f_i,0,0}(x,s)=0 \quad ...(18)$$

$$\left[\frac{\partial}{\partial x}+s+\eta_i(x)\right]\bar{p}_{f_i,A,0}(x,s)=0 \quad ...(19)$$

$$\left[\frac{\partial}{\partial x} + s + \eta_i(x)\right] \bar{p}_{f_i, 0, B}(x, s) = 0 \qquad \ldots(20)$$

$$\left[\frac{\partial}{\partial x} + s + \eta_i(x)\right] \bar{q}_{f_i, A, B}(x, s) = 0 \qquad \ldots(21)$$

$$\bar{p}_{0, A, B}(0, s) = \alpha\, \bar{p}_{0, A, 0}(s) + \lambda' \bar{p}_{0, 0, B}(s) + \sum_{i=1}^{N} \int_0^{\infty} \bar{q}_{f_i, A, B}(x, s)\, \eta_i(x)\, dx \qquad \ldots(22)$$

$$\bar{p}_{f_i, 0, 0}(0, s) = \lambda_i\, \bar{p}_{0, 0, 0}(s) \qquad \ldots(23)$$

$$\bar{p}_{f_i, A, 0}(0, s) = \lambda_i\, \bar{p}_{0, A, 0}(s) \qquad \ldots(24)$$

$$\bar{p}_{f_i, 0, B}(0, s) = \lambda_i\, \bar{p}_{0, 0, B}(s) \qquad \ldots(25)$$

$$\bar{q}_{f_i, A, B}(0, s) = \lambda_i \int_0^{\infty} \bar{p}_{0, A, B}(y, s)\, dy \qquad \ldots(26)$$

Integrating relation (17) through (21) by parts, we have

$$\bar{p}_{0, A, B}(y, s) = \bar{p}_{0, A, B}(0, s) \exp\left[-(s+\lambda)\, y - \int_0^y \phi(y)\, dy\right] \qquad \ldots(27)$$

$$\bar{p}_{f_i, 0, 0}(x, s) = \bar{p}_{f_i, 0, 0}(0, s) \exp\left[-sx - \int_0^x \eta_i(x)\, dx\right] \qquad \ldots(28)$$

$$\bar{p}_{f_i, A, 0}(x, s) = \bar{p}_{f_i, A, 0}(0, s) \exp\left[-sx - \int_0^x \eta_i(x)\, dx\right] \qquad \ldots(29)$$

$$\bar{p}_{f_i, 0, B}(x, s) = \bar{p}_{f_i, 0, B}(0, s) \exp\left[-sx - \int_0^x \eta_i(x)\, dx\right] \qquad \ldots(30)$$

$$\bar{q}_{f_i, A, B}(x, s) = \bar{q}_{f_i, A, B}(0, s) \exp\left[-sx - \int_0^x \eta_i(x)\, dx\right] \qquad \ldots(31)$$

Using relations (24), (25), (29) and (30) in relations (15) and (16), respectively and epitomising, we get

$$\bar{p}_{0,\,A,\,0}(s) = A\,\lambda'\,\bar{p}_{0,\,0,\,0}(s)$$

where

$$A = \left[s + \sum_{i=1}^{N} \lambda_i \{ 1 - \bar{S}_i(s) \} + \alpha \right]^{-1} \qquad \ldots(32)$$

and

$$\bar{p}_{0,\,0,\,B}(s) = B\,\alpha\,\bar{p}_{0,\,0,\,0}(s)$$

where

$$B = \left[s + \sum_{i=1}^{N} \lambda_i \{ 1 - \bar{S}_i(s) \} + \lambda' \right]^{-1} \qquad \ldots(33)$$

Making use of relations (26), (27), and (31) through (33) in relation (22) and simplifying, we have

$$\bar{p}_{0,\,A,\,B}(0, s) = \frac{\alpha\,\lambda'\,(A + B)\,\bar{p}_{0,\,0,\,0}(s)}{\left[1 - \sum_{i=1}^{N} \lambda_i\,\bar{S}_i(s) \left\{ \frac{1 - \bar{S}'(s + \lambda)}{(s + \lambda)} \right\} \right]} . \qquad \ldots(34)$$

The value of $\bar{p}_{0,\,0,\,0}(s)$ may be obtained with the help of relation (22), (27), (28) and (34) in (14), as

$$\bar{p}_{0,\,0,\,0}(s) = \left[s + \sum_{i=1}^{N} \lambda_i \left\{ 1 - \bar{S}_i(s) \right\} + \lambda' + \alpha - \frac{\lambda'\,\alpha\,(A + B)\,\bar{S}'(y)}{\left\{ 1 - \sum_{i=1}^{N} \lambda_i\,\bar{S}_i(s) \left(\frac{1 - \bar{S}'(s + \lambda)}{s + \lambda} \right) \right\}} \right]^{-1} . \qquad \ldots(35)$$

The other state probabilities may be obtained by integrating relation (27) through (31) and using boundary conditions:

$$\bar{p}_{f_i,\,0,\,0}(s) = \frac{\lambda_i}{s} \left[1 - \bar{S}_i(s) \right] \bar{p}_{0,\,0,\,0}(s) \qquad \ldots(36)$$

$$\bar{p}_{f_i,\,A,\,0}(s) = \frac{A\,\lambda_i\,\lambda'}{s} \left[1 - \bar{S}_i(s) \right] \bar{p}_{0,\,0,\,0}(s) \qquad \ldots(37)$$

$$\bar{p}_{f_i,\,0,\,B}(s) = \frac{B\,\lambda_i\,\alpha}{s}\left[1 - \bar{S}_i(s)\right]\bar{p}_{0,\,0,\,0}(s) \qquad \ldots(38)$$

and

$$\bar{q}_{f_i,\,A,\,B}(s) = \frac{\lambda_i}{s^2}\left[1 - \bar{S}_i(s)\right]\left[1 - \bar{S}'(s)\right]\bar{p}_{0,\,A,\,B}(s). \qquad \ldots(39)$$

The values of $\bar{p}_{3,\,0,\,0}(s)$ and $\bar{p}_{0,\,A,\,B,}(s)$ are given by the relations (35) and (34), respectively.

Particular Cases

Substituting $\alpha = \lambda'$ in relation (32) through (35), we get the following state probabilities when the system is working:

$$\bar{p}_{0,\,A,\,0}(s) = \bar{p}_{0,\,0\,0,\,B}(s) = A\,\lambda'\,\bar{p}_{0,\,0,\,0}(s)$$

where

$$A = B = \left[s + \sum_{i=1}^{N}\lambda_i\left\{1 - \bar{S}_i(s)\right\} + \lambda'\right] \qquad \ldots(40)$$

$$\bar{p}_{0,\,A,\,B}(s) = \frac{2A(\lambda')^2\left[1 - \bar{S}'(s+\lambda)\right]}{\left[s + \sum_{i=1}^{N}\lambda_i\left\{1 - \bar{S}_i(s)\left(1 - \bar{S}'(s+\lambda)\right)\right\}\right]} \qquad \ldots(41)$$

$$\bar{p}_{0,\,0,\,0}(s) = \left[s + \sum_{i=1}^{N}\lambda_i\left\{1 - \bar{S}_i(s)\right\} + 2\lambda' - \frac{2A(\lambda')^2\,\bar{S}'(y)(s+\lambda)}{s + \sum_{i=1}^{N}\lambda_i\left\{1 - \bar{S}_i(s)\left(1 - \bar{S}'(s+\lambda)\right)\right\}}\right]^{-1}. \qquad \ldots(42)$$

Similarly, the state probabilities may be obtained when the system is in the failed state by substituting, $\alpha = \lambda'$.

It may also be noted that,

$$\bar{p}_{\text{down}}(s) = \frac{1}{s} - \bar{p}_{\text{up}}(s)$$

or

$$\bar{p}_{\text{down}}(s) = \frac{1}{s} - \left[\bar{p}_{0,\,0,\,0}(s) - 2\,\bar{p}_{0,\,A,\,0}(s) + \bar{p}_{0,\,A,\,B}(s)\right]. \qquad \ldots(43)$$

On inversion; for certain particular values of the parameters, the behaviour of the system may be studied when it is comprised of the components connected in parallel redundancy, instead of stand-by redundancy.

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