UNSTEADY FLOW OF A DUSTY FLUID THROUGH A CIRCULAR PIPE

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In the present paper an attempt is made to study the laminar flow of an unsteady, incompressible viscous fluid with uniform distribution of dust particles through a circular pipe under the influence of pressure gradient (i) varying linearly with time, and (ii) decreasing exponentially with time. Analytical expressions for the velocities of fluid and dust particles are obtained. In the first case it is observed that the velocity of fluid particles increases and that of the dust particles decreases with the increase of dimensionless relaxation parameter $\sigma = \frac{\nu\tau}{R^2}$. The expressions for total flow flux for fluid and dust particles have been derived and the effect of σ upon them has also been discussed.

INTRODUCTION

Interest in problems of mechanics of systems with more than one phase has developed rapidly in recent years. Situations which occur frequently are concerned with the motion of liquid or gas which contains a distribution of solid particles. Such situations occur, for example, in the movement of dust laden air, in problems of fluidization, in the use of dust in gas-cooling systems to enhance heat transfer processes and in the process by which raindrops are formed by the coalescence of small droplets which might be considered as solid particles for the purpose of examining their movement prior to coalescence.

We developed interest in this subject from papers of Saffman (1962), Michael (1968) and Rao (1969). The present paper consists of two parts. In part A the flow of a dusty fluid through a circular pipe under pressure gradient varying linearly with time is discussed. Analytical expressions for the velocities of fluid and dust particles are obtained in dimensionless forms. These consist of two parts, one varying linearly with the parameter $T=\nu t/R^2$ and the other is the transient part which vanishes in the limit as t tends to infinity. It is also seen that the contribution of the transient part is insignificant for very large $T(\gg 1)$.

In part B the flow of a dusty viscous fluid in the same pipe under exponentially decreasing pressure gradient is studied. Expressions for the velocities of fluid and dust particles have been obtained taking

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}=a_0+\sum_{n=1}^{\infty} a_n e^{-nt}$$

The results obtained agree with those of Srivastava (1963). He has obtained the expression for the velocity of a viscous incompressible fluid in a circular pipe under the influence of pressure gradient decreasing exponentially with time.

1. Basic Equations

The equations of motion of a dusty, unsteady, viscous incompressible fluid are (Saffman (1962) :

$$\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}.\nabla)\mathbf{u}=-\frac{1}{\rho}\text{ grad } p+\nu\nabla^2\mathbf{u}+\frac{KN}{\rho}(\mathbf{v}-\mathbf{u}) \quad \text{...(1.1)}$$

$$\text{div } \mathbf{u}=0 \quad \text{...(1.2)}$$

$$m\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}.\nabla)\mathbf{v}\right]=K(\mathbf{u}-\mathbf{v}) \quad \text{...(1.3)}$$

$$\frac{\partial N}{\partial t}+\text{div}(N\mathbf{v})=0 \quad \text{...(1.4)}$$

where, **u** and **v** denote the local velocity vectors of fluid and dust particles respectively, ρ the density, p the static fluid pressure, ν the kinematic viscosity, N the number density of a dust particle, K the Stokes resistance coefficient (for spherical particles of radius ε, it is $6\pi\mu\varepsilon$), μ the fluid viscosity and m the mass of a dust particle.

For the present problem the velocity distribution of fluid and dust particles are defined respectively as:

$$u_1=0,\ v_1=0,\ w_1=w_1(r,t) \quad \text{...(1.5)}$$

$$u_2=0,\ v_2=0,\ w_2=w_2(r,t) \quad \text{...(1.6)}$$

$$N=N_0 \text{ (a constant)} \quad \text{...(1.7)}$$

where (u_1, v_1, w_1) and (u_2, v_2, w_2) are velocity components of the fluid and dust particles respectively.

The equations of motion then reduce to

$$\frac{\partial w_1}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w_1}{\partial r^2}+\frac{1}{r}\frac{\partial w_1}{\partial r}\right)+\frac{l}{\tau}(w_2-w_1) \quad \text{.. (1.8)}$$

$$\tau\frac{\partial w_2}{\partial t}=w_1-w_2 \quad \text{...(1.9)}$$

where $\tau = m/k$ is the relaxation time of dust particles and $l = mN_0/\rho$ is the mass concentration of the dust particles.

Eliminating w_2 from (1.8) and (1.9), we get

$$\frac{\partial^2 w_1}{\partial t^2} = \frac{\partial}{\partial t}\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}\right) + \nu\frac{\partial}{\partial t}\left(\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r}\right) - \left(\frac{l+1}{\tau}\right)\frac{\partial w_1}{\partial t} - \frac{1}{\tau}\left[\frac{1}{\rho}\frac{\partial p}{\partial z} - \nu\left(\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r}\right)\right]. \quad ...(1.10)$$

PART A

2. Pressure Gradient Varies Linearly with Time

We now assume that

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = a_0 + at. \quad ...(2.1)$$

Equation (1.10) then becomes

$$\frac{\partial^2 w_1}{\partial t^2} = a + \nu\frac{\partial}{\partial t}\left[\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r}\right] - \left(\frac{l+1}{\tau}\right)\frac{\partial w_1}{\partial t} + \frac{1}{\tau}\left[(a_0 + at) + \nu\left(\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r}\right)\right]. \quad ...(2.2)$$

Let $\bar{w}_1 = \int_0^\infty w_1 e^{-st} dt, \quad \bar{w}_2 = \int_0^\infty w_2 e^{-st} dt$

be the Laplace transforms of w_1 and w_2 and let w_1^0 and w_2^0 be the initial values of w_1 and w_2 respectively.

Multiplying eqns. (1.9) by e^{-st} and then integrating between the limits 0 to ∞, we get

$$\frac{\partial^2 \bar{w}_1}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{w}_1}{\partial r} - k^2 \bar{w}_1 = -\frac{1}{\nu}\left[\left(\frac{1+l+s\tau}{1+s\tau}\right) w_1^0 + \frac{a_0}{s} + \frac{a}{s^2}\right] \quad ...(2.3)$$

and

$$\bar{w}_2 = \left(\frac{1}{1+s\tau}\right)\bar{w}_1 + \left(\frac{\tau}{1+s\tau}\right) w_2^0 \quad ...(2.4)$$

where

$$k^2 = \frac{s(1+l+s\tau)}{\nu(1+s\tau)}.$$

We shall now find w_1^0 and w_2^0.

Initially the pressure gradient is a_0 and the motion is steady in the pipe. From eqns. (1.8) and (1.9), we obtain $w_1^0 = w_2^0$ and

$$\frac{d^2 w_1^0}{dr^2} + \frac{1}{r}\frac{dw_1^0}{dr} = -\frac{a_0}{\nu}. \qquad \ldots(2.5)$$

The boundary conditions are

$w_1^0 =$ finite, when $r = 0$

$w_1^0 = 0$, when $r = R$, where R is the radius of the pipe.

The solution of eqn. (2.5) under these boundary conditions is

$$w_1^0 = \frac{a_0}{4\nu}(R^2 - r^2).$$

Therefore $\quad w_1^0 = w_2^0 = \dfrac{a_0}{4\nu}(R^2 - r^2).$

Substituting the values of w_1^0 and w_2^0 in (2.3) and (2.4), we get

$$\frac{\partial^2 \bar{w}_1}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{w}_1}{\partial r} - k^2 \bar{w}_1 = -\frac{1}{\nu}\left[\left(\frac{1+l+s\tau}{1+s\tau}\right)(R^2 - r^2)\frac{a_0}{4\nu} + \frac{a_0}{s} + \frac{a}{s^2}\right] \qquad \ldots(2.6)$$

and

$$\bar{w}_2 = \left(\frac{1}{1+s\tau}\right)\bar{w}_1 + \left(\frac{\tau}{1+s\tau}\right)(R^2 - r^2)\frac{a_0}{4\nu}. \qquad \ldots(2.7)$$

The boundary conditions for $\bar{w}_1$ and $\bar{w}_2$ are

$$\left.\begin{array}{l} \bar{w}_1 = 0,\ \bar{w}_2 = 0;\ \text{when } r = R \\ \bar{w}_1 = \bar{w}_2 = \text{finite; when } r = 0. \end{array}\right\} \qquad \ldots(2.8)$$

The solution of (2.6) under the boundary conditions (2.8) is

$$\bar{w}_1 = \frac{a_0}{4\nu}\left(\frac{R^2 - r^2}{s}\right) + \frac{a(1+s\tau)}{s^3(1+l+s\tau)}\left[1 - \frac{I_0(kr)}{I_0(kR)}\right]. \qquad \ldots(2.9)$$

Therefore,

$$\bar{w}_2 = \frac{a_0}{4\nu}\left(\frac{R^2 - r^2}{s}\right) + \frac{a}{s^3(1+l+s\tau)}\left[1 - \frac{I_0(kr)}{I_0(kR)}\right]. \qquad \ldots(2.10)$$

Now applying Laplace inversion theorem, we get

$$w_1 = \frac{a_0}{4\nu}(R^2-r^2)+\frac{at}{4\nu}(R^2-r^2)-\frac{a(1+l)(R^2-r^2)(3R^2-r^2)}{64\nu^2}$$

$$+2a\sum_{m=1}^{\infty}\frac{1}{\alpha_m}\left(\frac{1+H_1\tau}{H_1}\right)^2\left\{\frac{1}{(1+H_1\tau)^2+l}\right\}\frac{J_0\left(\frac{r}{R}\alpha_m\right)}{J_1(\alpha_m)}e^{H_1 t}$$

$$+2a\sum_{m=1}^{\infty}\frac{1}{\alpha_m}\left(\frac{1+H_2\tau}{H_2}\right)^2\left\{\frac{1}{(1+H_2\tau)^2+l}\right\}\frac{J_0\left(\frac{r}{R}\alpha_m\right)}{J_1(\alpha_m)}e^{H_2 t} \quad \ldots(21.1)$$

$$w_2 = \frac{a_0}{4\nu}(R^9-r^2)+\frac{at}{4\nu}(R^2-r^2)-\frac{a\tau}{4\nu}(R^2-r^2)-\frac{a(1+l)}{64\nu^6}(R^2-r^2)(3R^2-r^2)$$

$$+2a\sum_{m=1}^{\infty}\frac{1}{\alpha_m}\left(\frac{1+H_1\tau}{H_1^2}\right)\left\{\frac{1}{(1+H_1\tau)^2+l}\right\}\frac{J_0\left[\frac{r}{R}\alpha_m\right]}{J_1(\alpha_m)}e^{H_1 t}$$

$$+2a\sum_{m=1}^{\infty}\frac{1}{\alpha_m}\left(\frac{1+H_2\tau}{H_2^2}\right)\left\{\frac{1}{(1+H_2\tau)^2+l}\right\}\frac{J_0\left[\frac{r}{R}\alpha_m\right]}{J_1(\alpha_m)}e^{H_2 t} \quad \ldots(2.12)$$

where $(m = 1, 2, 3, \ldots)$ are the positive zeros of $J_0(\alpha)=0$ and

$$H_1 = \frac{-\left(1+l+\nu\tau\frac{(\alpha_m^2)}{R^2}\right)+\sqrt{\left(1+l+\nu\frac{(\alpha_m^2)}{R^2}\tau\right)^2-4\nu\tau\frac{(\alpha_m^2)}{R^2}}}{2\tau}$$

$$H_2 = \frac{-\left(1+l+\nu\tau\frac{(\alpha_m^2)}{R^2}\right)-\sqrt{\left(1+l+\nu\frac{(\alpha_m^2)}{R^2}\tau\right)^2-4\nu\tau\frac{(\alpha_m^2)}{R^2}}}{2\tau}$$

Now we make eqns. (2.11) and (2.12) dimensionless by introducing

$$W_1=\frac{w_1}{U_0},\ \frac{r}{R}=\eta,\ T=\frac{\nu t}{R^2},\ \sigma=\frac{\nu\tau}{R^2},$$

where U_0 is a characteristic velocity. We then get

$$W_1=b_0(1-\eta^2)+bT(1-\eta^2)-\frac{b}{16}(3-\eta^2)(1-\eta^2)(1+l)$$

$$+32\,b\sigma^2\sum_{m=1}^{\infty}\frac{1}{\alpha_m}\left(\frac{2+M_1}{M_1}\right)^2\frac{1}{(2+M_1)^2+4l}\frac{J_0(\eta\alpha_m)}{J_1(\alpha_m)}\exp\left(\frac{M_1T}{2\sigma}\right)$$

$$+32b\,\sigma^2\sum_{m=1}^{\infty}\frac{1}{\alpha_m}\left(\frac{2+M_2}{M_2}\right)^2\frac{1}{(2+M_2)^2+4l}\frac{J_0(\eta\alpha_m)}{J_1(\alpha_m)}\exp\left(\frac{M_3T}{2\sigma}\right) \quad \ldots(2.13)$$

and

$$W_2=b_0(1-\eta^2)+bT(1-\eta^2)-b\sigma(1-\eta^2)-\frac{b}{16}(3-\eta^2)(1-\eta^2)(1+l)$$

$$+64b\,\sigma^2\sum_{m=1}^{\infty}\frac{1}{\alpha_m}\left(\frac{2+M_1}{M_1^2}\right)\frac{1}{(2+M_1)^2+4l}\,\frac{J_0(\eta\alpha_m)}{J_1(\alpha_m)}\exp\left(\frac{M_1T}{2\sigma}\right)$$

$$+64b\,\sigma^2\sum_{m=1}^{\infty}\frac{1}{\alpha_m}\left(\frac{2+M_2}{M_2^2}\right)\frac{1}{(2+M_2)^2+4l}\,\frac{J_0(\eta\alpha_m)}{J_1(\alpha_m)}\exp\left(\frac{M_2T}{2\sigma}\right) \qquad ...(2.14)$$

where $b_0=\frac{a_0}{4\nu}\frac{R^2}{U_0}$, $b=\frac{a\,R^4}{4\nu^2U_0}$ are all dimensionless numbers and

$$M_1=-(1+l+\sigma(\alpha_m)^2)+\sqrt{(1+l+\sigma(\alpha_m)^2)^2-4\sigma(\alpha_m)^2}$$

$$M_2=-(1+l+\sigma(\alpha_m)^2-\sqrt{(1+l+\sigma(\alpha_m)^2)^2-4\sigma\alpha^2} \qquad ...(2.15)$$

The non-dimensional total flux for fluid particles is given by

$$Q_1=\frac{b_0}{2}+\frac{bT}{2}-\left(\frac{1+l}{12}\right)b$$

$$+64b\,\sigma^2\sum_{m=1}^{\infty}\frac{1}{\alpha_m^2}\left(\frac{2+M_1}{M_1}\right)^2\frac{1}{(2+M_1)^2+4l}\exp\left(\frac{M_1T}{2\sigma}\right)$$

$$+64b\,\sigma^2\sum_{m=1}^{\infty}\frac{1}{(\alpha_m)^2}\left(\frac{2+M_2}{M_2}\right)^2\frac{1}{(2+M_2)^2+4l}\exp\left(\frac{M_2T}{2\sigma}\right) \qquad ...(2.16)$$

where

$$Q_1=\frac{q_1}{\pi R^2U_0}.$$

The corresponding expression for dust particles is

$$Q_2=\frac{b_0}{2}+\frac{bT}{2}-\frac{b\sigma}{2}-\frac{1}{12}(1+l)b+$$

$$+128b\sigma^2\sum_{m=1}^{\infty}\frac{1}{(\alpha_m)^2}\left(\frac{2+M_1}{M^2_1}\right)\frac{1}{(2+M_1)^2+4l}\exp\left(\frac{M_1T}{2\sigma}\right)$$

$$+128b\sigma^2\sum_{m=1}^{\infty}\frac{1}{(\alpha_m)^2}\left(\frac{2+M_2}{M^2_2}\right)\frac{1}{(2+M_2)^2+4l}\exp\left(\frac{M_2T}{2\sigma}\right) \qquad (\ldots 2.17)$$

where $Q_2=\frac{q_2}{\pi R^2U_0}$ and M_1, M_2 are same as in (2.15).

Figures 1 and 2 have been drawn taking $b_0=1$, $b=1$ and $l=1$. From Fig. 1, it is observed that as σ increases the velocity of the fluid particles increases, but that of the dust particles decreases. The velocities of the fluid and dust particles become the same when the relaxation time, τ tends to zero, i.e. when the dust particles

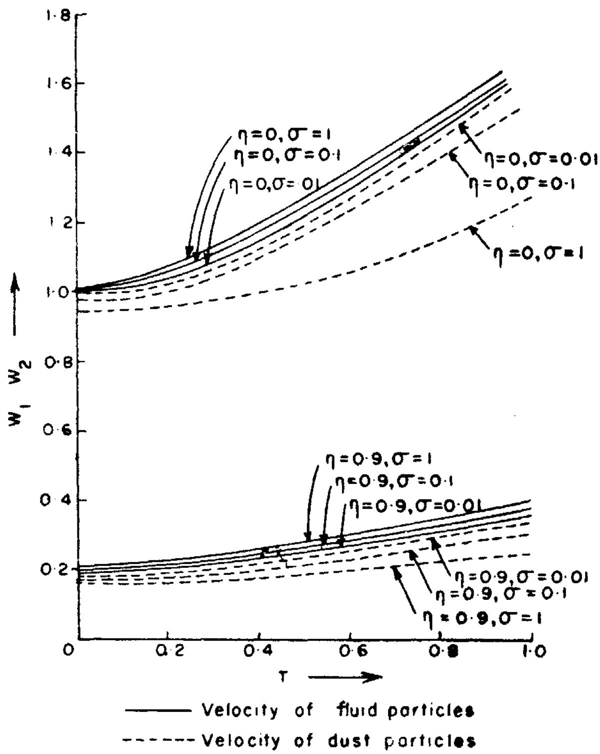

FIG. 1.

become very fine. It is also seen that the transient parts are insignificant for very large $T(\gg 1)$. It means that the dusty fluid velocity will take more time to vary linearly with T, than the clean fluid velocity. It is also clear from Fig. 1 that

W_1 and W_2 increase with T for fixed η and for any T, W_1 and W_2 decrease with the increase of η and are maximum when $\eta=0$. It means that both the fluid and dust particles which are nearer to the axis of the pipe move with greater velocity.

From Fig. 2, it is seen that as σ increases the total flux for fluid particles increases, but that for the dust particles decreases.

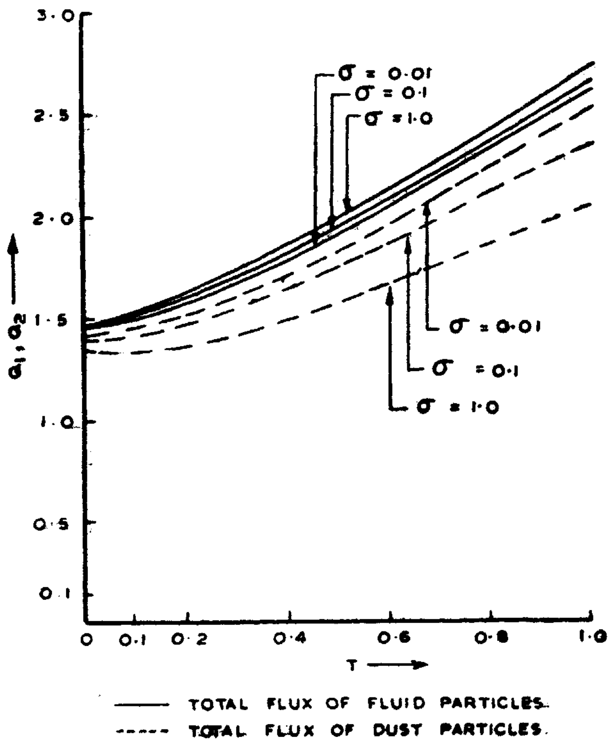

FIG. 2.

PART B

3. PRESSURE GRADIENT DECREASES EXPONENTIALLY WITH TIME

We take

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}=a_0+\sum_{n=1}^{\infty}a_n e^{-nt}. \qquad \ldots (3.1)$$

Equation (1.10) then becomes

$$\frac{\partial^2 w_1}{\partial t^2} = -\sum_{n=1}^{\infty} n\, a_n\, e^{-nt} + \nu \frac{\partial}{\partial t}\left[\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r}\right] - \left(\frac{l+1}{\tau}\right)\frac{\partial w_1}{\partial t}$$

$$+ \frac{1}{\tau}\left[a_0 + \sum_{n=1}^{\infty} a_n\, e^{-nt} + \nu\left(\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r}\right)\right]. \qquad \ldots (3.2)$$

Let $\bar{w}_1 = \int_0^{\infty} w_1\, e^{-st}\, dt$, $\bar{w}_2 = \int_0^{\infty} w_2\, e^{-st}\, dt$ be the Laplace transforms of w_1 and w_2 and let w_1^0 and w_2^0 be the initial values of w_1 and w_2 respectively. Multiplying (1.9) and (3.2) by e^{-st} and integrating between the limts 0 to ∞, we have

$$\frac{\partial^2 \bar{w}_1}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{w}_1}{\partial r} - k^2\, \bar{w}_1$$

$$= -\frac{1}{\nu}\left[\left(\frac{1+l+s\tau}{1+s\tau}\right) w_1^0 + \frac{a_0}{s} + \sum_{n=1}^{\infty} \frac{(1-n\tau)\, a_n}{(s+n)\,(1+s\tau)}\right] \qquad \ldots (3.3)$$

and

$$\bar{w}_2 = \left(\frac{1}{1+s\tau}\right)\bar{w}_1 + \left(\frac{\tau}{1+s\tau}\right) w_1^0. \qquad \ldots (3.4)$$

Here again

$$w_1^0 = w_2^0 = \frac{a_0}{4\nu}(R^2 - r^2).$$

The solution of eqn. (3.3) under the boundary conditions (2.8) is

$$\bar{w}_1 = \frac{a_0\,(R^2 - r^2)}{4\nu s} + \sum_{n=1}^{\infty} \frac{(1-n\tau)\, a_n}{s\,(s+n)\,(1+l+s\tau)}\left[1 - \frac{I_0\,(kr)}{I_0(kR)}\right]. \qquad \ldots (3.5)$$

Therefore

$$\bar{w}_2 = \frac{a_0}{4\nu}\left(\frac{R^2 - r^2}{s}\right) + \sum_{n=1}^{\infty} \frac{(1-n\tau)a_n}{s\,(s+n)\,(1+s\tau)\,(1+l+s\tau)}\left[1 - \frac{I_0(kr)}{I_0(kR)}\right]. \qquad \ldots (3.6)$$

Now applying Laplace inversion theorem, we get

$$w_1 = \frac{a_0}{4\nu}(R^2-r^2) - \sum_{n=1}^{\infty} \frac{a_n(1-n\tau)e^{-nt}}{n(1+l-n\tau)}\left[1-\frac{J_0(\phi r)}{J_0(\phi R)}\right]$$

$$+2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{a_n(1-n\tau)}{\alpha_m}\times\left[\frac{(1+H_1\tau)}{(H_1+n)\{(1+H_1\tau)^2+l\}}e^{H_1 t}\right.$$

$$\left.+\frac{(1+H_2\tau)}{(H_2+n)}\frac{1}{(1+H_2\tau)^2+l}e^{H_2 t}\right]\frac{J_0\left(\frac{r}{R}\alpha_m\right)}{J_1(\alpha_m)}$$

$$= w_1' + w_1'' + w_1''' . \qquad \ldots(3.7)$$

where

$$\phi = \sqrt{\frac{n(1+l-n\tau)}{\nu(1-n\tau)}}$$

and

$$w_2 = \frac{a_0(R^2-r^2)}{4\nu} + \sum_{n=1}^{\infty} a_n\left[\frac{\tau^2}{l}e^{-t/\tau} - \frac{e^{-nt}}{n(1+l-n\tau)}\left\{1-\frac{J_0(\phi r)}{J_0(\phi R)}\right\}\right]$$

$$+2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{a_n(1-n\tau)}{\alpha_m}\left[\frac{1}{(H_1+n)\{(1+H_1\tau)^2+l\}}e^{H_1 t}\right.$$

$$\left.+\frac{1}{(H_2+n)(1+H_2\tau)^2+l}e^{H_2 t}\right]\frac{J_0\left(\frac{r}{R}\alpha_m\right)}{J_1(\alpha_m)}$$

$$= w_2' + w_2'' + w_2''' . \qquad \ldots(3.8)$$

These expressions for w_1 and w_2 do not agree with Rao's result (1969). His expressions for w_1 and w_2 do not contain w_1''' and w_2''' respectively. These differences are due to the fact, that Rao has assumed the form of w_1 and w_2 as

$$w_1 = w_{10} + \sum_{n=1}^{\infty} w_{1n}e^{-nt}, \quad w_2 = w_{20} + \sum_{n=1}^{\infty} w_{2n}e^{-nt}$$

where w_{10}, w_{1n}, w_{20} and w_{2n} are functions of r only. Naturally then the parts w_1' and w_2''' will be absent in his expressions for w_1 and w_2 respectively. But $(w_1'+w_1''+w_1''')$ and $(w_2'+w_2''+w_2''')$ are more general solutions of (3.2) and (1.9) respectively and they are confirmed by Laplace transform method used in the present paper.

When the dust is very fine, the relaxation time of dust particles decreases and ultimately as $\tau \rightarrow 0$ the velocity of dusty fluid becomes that of the clean fluid in both the cases.

If the masses of the dust particles are small, their influence on the fluid flow is reduced and in the limit as $m \rightarrow 0$, the fluid becomes ordinary viscous and we get the problem studied by Srivastava (1963).

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