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q-ANALOGUES OF SOME BILINEAR TRANSFORMATIONS

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q-analogues of the bilinear transformations of Carlitz (1970), Verma (1972) and Meixner (1942) are obtained. Some of their interesting special cases are also discussed, for instance, q-analogues of the well-known Hardy-Hille formula for Laguerre polynomials.

§1. Meixner (1942) proved the following bilinear generating functions, as generalizations of some of the results due to Erdélyi (1939) :

$$\sum_{n=0}^{\infty} \binom{a}{n} s^n\, {}_2F_1\left[\begin{matrix}-n, b; z\\ c\end{matrix}\right] {}_2F_1\left[\begin{matrix}-n, d; t\\ e\end{matrix}\right]$$

$$= (1+s)^a \sum_{n=0}^{\infty} \binom{a}{n} \frac{(szt)^n (b)_n (d)_n}{(1+s)^{2n} (c)_n (e)_n}$$

$$\times\, {}_2F_1\left[\begin{matrix}n-a, b+n; \dfrac{sz}{1+s}\\ c+n\end{matrix}\right] {}_2F_1\left[\begin{matrix}n-a, d+n; \dfrac{st}{1+s}\\ e+n\end{matrix}\right] \qquad \ldots(1.1)$$

$$\sum_{n=0}^{\infty} \binom{a}{n} s^n\, {}_2F_1\left[\begin{matrix}-n, b; z\\ c\end{matrix}\right] {}_2F_1\left[\begin{matrix}n-a, d; t\\ e\end{matrix}\right]$$

$$= (1+s)^a \sum_{n=0}^{\infty} \binom{a}{n} \frac{(-szt)^n (b)_n (d)_n}{(1+s)^{2n} (c)_n (e)_n}$$

$$\times\, {}_2F_1\left[\begin{matrix}n-a, b+n; \dfrac{sz}{1+s}\\ c+n\end{matrix}\right] {}_2F_1\left[\begin{matrix}n-a, d+n; \dfrac{t}{1+s}\\ e+n\end{matrix}\right] \qquad \ldots(1.2)$$

$$\sum_{n=0}^{\infty} \binom{a}{n} s^n\, {}_2F_1\left[\begin{matrix}n-a, b; z\\ c\end{matrix}\right] {}_2F_1\left[\begin{matrix}n-a, d; t\\ e\end{matrix}\right] =$$

(*equation continued on p.* 1644)

$$= (1+s)^a \sum_{n=0}^{\infty} \binom{a}{n} \frac{(szt)^n (b)_n (d)_n}{(1+s)^{2n} (c)_n (e)_n}$$

$$\times \, {}_2F_1\left[\begin{matrix} n-a, b+n; \dfrac{z}{1+s} \\ c+n \end{matrix}\right] {}_2F_1\left[\begin{matrix} n-a, d+n; \dfrac{t}{1+s} \\ e+n \end{matrix}\right] \quad \ldots(1.3)$$

and

$$\sum_{n=0}^{\infty} \binom{a}{n} s^n \frac{(c-b)_n (e-d)_n}{(c)_n (e)_n} \, {}_2F_1\left[\begin{matrix} n-a, b; z \\ c+n \end{matrix}\right] {}_2F_1\left[\begin{matrix} n-a, d; t \\ e+n \end{matrix}\right]$$

$$= (1+s)^a \sum_{n=0}^{\infty} \binom{a}{n} \frac{(b)_n (d)_n}{(c)_n (e)_n} \frac{[s(1-z)(1-t)]^n}{(1+s)^{2n}}$$

$$\times \, {}_2F_1\left[\begin{matrix} n-a, b+n; \dfrac{s+z}{1+s} \\ c+n \end{matrix}\right] {}_2F_1\left[\begin{matrix} n-a, d+n; \dfrac{s+t}{1+s} \\ e+n \end{matrix}\right]. \quad \ldots(1.4)$$

Later on, Carlitz (1970) generalized (1.1) and Verma (1972) generalized (1.2) and (1.3) to arbitrary series. In this note we obtain q-analogues of the results of Carlitz (1970), Verma (1972) and the q-analogue of the terminating version of (1.4). As applications of these formulae we deduce bilinear generating functions for q-Laguerre polynomials.

§2. *Definitions and Notations* — If we let

$$|q| < 1, \quad [a; q]_n = (1-a)(1-aq) \ldots (1-aq^{n-1}), \quad [a; q]_0 = 1$$

and
$$[a; q]_\infty = \prod_{r=0}^{\infty} (1 - aq^r)$$

then we may define the basic hypergeometric series as :

$${}_{p+1}\phi_{p+r}\left[\begin{matrix} a_1, a_2, \ldots, a_{p+1}; q; x \\ b_1, b_2, \ldots, b_{p+r} \end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} \frac{[a_1; q]_n \ldots [a_{p+1}; q]_n x^n (-)^{nr} q^{rn(n-1)/2}}{[q; q]_n [b_1; q]_n \ldots [b_{p+r}; q]_n}$$

where the series ${}_{p+1}\phi_{p+r}$ converges for all positive integral values of r and for all x, except, when $r = 0$, it converges only for $|x| < 1$. Following Jackson (1944) we define

$$l_n^a(x; q) = \frac{[aq; q]_n}{[q; q]_n}\, {}_2\phi_1\left[\begin{matrix} q^{-n}, 0; q; x \\ aq \end{matrix}\right]$$

and

$$L_n^a(x; q) = \frac{[aq; q]_n}{[q; q]_n}\, {}_1\phi_1\left[\begin{matrix} q^{-n}; q; -x \\ aq \end{matrix}\right]$$

as q-analogues of Laguerre polynomials. In fact these two q-analogues of Laguerre polynomials are connected by the relation

$$l_n^a(xq^n; q) = a^n L_n^{1/a}\left(-\frac{xp^2}{a}; p\right)$$

where $pq = 1$. The q-analogue of Jacobi polynomials is defined as (see Andrews and Askey 1977)

$$p_n(x; a, b/q) = {}_2\phi_1\left[\begin{matrix} q^{-n}, abq^{1+n}; q; xq \\ aq \end{matrix}\right].$$

§3. In this section we prove the following formulae :

$$\sum_{n=0}^{\infty} \frac{[\alpha; q]_n}{[q; q]_n} z^n g_n[A; xq^n; m; q]\, g_n[B; yq^n; p; q]$$

$$= \frac{[\alpha z; q]_\infty}{[z; q]_\infty} \sum_{n=0}^{\infty} \frac{[\alpha; q]_n (xyz)^n}{[q; q]_n [\alpha z; q]_{2n}}\, q^{(m+p+2)n(n-1)/2}$$

$$\times \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{[\alpha q^n; q]_r [\alpha q^n; q]_s (-xz)^r (-yz)^s q^{(m+1)r(r-1+2n)/2}}{[q; q]_r [q; q]_s [\alpha z q^{2n}; q]_{r+s}}$$

$$\times\ q^{((p+1)s(s-1+2n)+2rs)/2} A_{n+r} B_{n+s} \qquad \ldots(3.1)$$

$$\sum_{n=0}^{\infty} \frac{[\alpha; q]_n (-z)^n q^{n(n-1)/2}}{[q; q]_n [\alpha z; q]_n} g_n[A; xq; m+1; q]\, g_n[B; yq; p+1; q]$$

$$= \frac{[z; q]_\infty}{[\alpha z; q]_\infty} \sum_{n=0}^{\infty} \frac{[q/z; q]_n (xyz^2)^n q^{((m+p)n(n-1)-2n)/2}}{[q; q]_n [\alpha; q]_n}$$

$$\times\ G_{n,n}[\alpha; A; xzq^{mn}; m+1; q]\, G_{n,n}[\alpha; B; yzq^{pn}; p+1; q] \qquad \ldots(3.2)$$

and

$$\sum_{n=0}^{\infty} \frac{z^n}{[q;q]_n} g_n[A; -xq^n; - ; q]\, g_n[B; -yq^n; - ; q]$$

$$= \frac{1}{[z;q]_\infty} \sum_{n=0}^{\infty} \frac{[z;q]_n}{[q;q]_n} (xyz)^n q^{n(n-1)} G_{0,n}[0; A; xzq^n; 1; q]$$

$$\times\ G_{0,n}[0; B; yzq^n; 1; q] \qquad \ldots(3.3)$$

where

$$g_n[A; x; m; q] = \sum_{r=0}^{n} \frac{[q^{-n};q]_r}{[q;q]_r} x^r q^{mr(r-1)/2} A_r;\ A = \{A_r\}$$

$$G_{n,m}\,[\alpha; A; x; p; q] = \sum_{r=0}^{\infty} \frac{[\alpha;q]_{n+r}}{[q;q]_r} x^r q^{pr(r-1)/2} A_{m+r}.$$

Proof of (3.1) — In view of the q-analogue of Saalschütz summation theorem [Slater 1966, 3.3.2.2] we can write

$$\frac{[\alpha;q]_n\,[q^{-n};q]_r\,[q^{-n};q]_s q^{nr}}{[q;q]_n\,[q;q]_r\,[q;q]_s} = \frac{[\alpha;q]_n\,[\alpha;q]_r\,[\alpha;q]_s(-)^{r+s}q^{((s+r)(s+r+1)-2sn)/2}}{[q;q]_r\,[q;q]_s\,[\alpha;q]_{r+s}\,[q;q]_{n-r-s}}$$

$$\times\ {}_3\phi_2\left[\begin{matrix}\alpha q^n,\ q^{-r},\ q^{-s};\ q;\ q\\ \alpha,\ q^{1+n-r-s}\end{matrix}\right] \quad \ldots(3.4)$$

multiplying both of the sides of (3.4) by $z^n q^{ns}$ and summing with respect to n from 0 to ∞, we get

$$\sum_{n=0}^{\infty} \frac{[\alpha;q]_n\,[q^{-n};q]_r\,[q^{-n};q]_s\, z^n q^{(r+s)n}}{[q;q]_n\,[q;q]_r\,[q;q]_s}$$

$$= \frac{[\alpha z;q]_\infty\,[\alpha;q]_r\,[\alpha;q]_s\,(-z)^{r+s}\,q^{(s+r)(s+r-1)/2}}{[z;q]_\infty\,[q;q]_r\,[q;q]_s\,[\alpha z;q]_{r+s}}\ {}_2\phi_1\left[\begin{matrix}q^{-r},\ q^{-s};\ q;\ q/z\\ \alpha\end{matrix}\right].$$

$$\ldots(3.5)$$

Next, multiplying both of the sides of (3.5) by $x^r y^s A_r B_s q^{(mr(r-1)+ps(s-1))/2}$ and then summing with respect to r and s from 0 to ∞, we get (3.1) on some reductions.

Proof of (3.2) — Multiplying both the sides of (3.4) by $\dfrac{(-z)^n\, q^{(n-r)(n-r-1)/2}}{[\alpha z;q]_n}$ and summing with respect to n from 0 to ∞, we get:

$$\sum_{n=0}^{\infty} \frac{[\alpha; q]_n [q^{-n}; q]_r [q^{-n}; q]_s (-z)^n q^{(n(n-1)+r(r+1))/2}}{[q; q]_n [q; q]_r [q; q]_s [\alpha z; q]_n}$$

$$= \frac{[z; q]_\infty [\alpha; q]_r [\alpha; q]_s z^{r+s} q^{(r(r-1)-2s)/2}}{[\alpha z; q]_\infty [q; q]_r [q; q]_s} {}_3\phi_2 \left[\begin{matrix} q^{-r}, q^{-s}, q/z; q; q \\ \alpha, 0 \end{matrix} \right]. \quad \ldots(3.6)$$

Furthermore, multiplying (3.6) by $x^r y^s A_r B_s q^{(mr(r-1)+(1+p)s(s-1)+2s)/2}$ and summing with respect to r and s from 0 to ∞, we get (3.2). Both (3.1) and (3.2) are q-analogues of a result of Carlitz (1970, 1.6).

Proof of (3.3) — Rewriting the q-analogue of Gauss's summation theorem (Slater 1966, 3.3.2.5) as

$$\frac{[q; q]_n}{[q; q]_{n-r} [q; q]_{n-s}} = \frac{1}{[q; q]_{n-r-s}} {}_2\phi_1 \left[\begin{matrix} q^{-r}, q^{-s}; q; q^{1+n} \\ q^{1+n-r-s} \end{matrix} \right], \quad \ldots(3.7)$$

multiplying by $\dfrac{z^n x^r y^s q^{(r(r-1)+s(s-1))/2} A_r B_s}{[q; q]_r [q; q]_s}$ and summing with respect to n, r and s from 0 to ∞, we get (3.3), q-analogue of a formula due to Carlitz (1970, 5.1).

Next, we discuss some of the interesting special cases of (3.1) - (3.3). (3.1) for $m = 0, p = 1, A_r = \dfrac{1}{[\alpha; q]_r}, B_s = \dfrac{1}{[\alpha; q]_s}$, yields (on replacing α by αq]

$$\sum_{n=0}^{\infty} \frac{[q; q]_n z^n}{[\alpha q; q]_n} l_n^\alpha (xq^n; q) L_n^\alpha (yq^n; q)$$

$$= \frac{[\alpha zq; q]_\infty}{[z; q]_\infty} \sum_{n=0}^{\infty} \frac{(xyz)^n q^{3n(n-1)/2}}{[q; q]_n [\alpha q; q]_n [\alpha zq; q]_{2n}}$$

$$\times \sum_{r=0}^{\infty} \frac{(-xz)^r q^{(r(r-1)+2nr)/2}}{[q; q]_r [\alpha zq^{1+2n}; q]_r} {}_1\phi_0 \left[\begin{matrix} q^{-r}; q; -\dfrac{y}{x} q^{n+r} \\ - \end{matrix} \right]$$

$$= \frac{[\alpha zq; q]_\infty}{[z; q]_\infty} \sum_{n=0}^{\infty} \frac{(xyz)^n q^{3n(n-1)/2}}{[q; q]_n [\alpha q; q]_n [\alpha zq; q]_{2n}} {}_1\phi_1 \left[\begin{matrix} -\dfrac{y}{x} q^n; q; xzq^n \\ \alpha zq^{1+2n} \end{matrix} \right]. \quad \ldots(3.8)$$

whereas (3.2) for $m = p = 0$, $A_r = \dfrac{1}{[\alpha; q]_r} = B_r$, gives (on replacing α by αq)

$$\sum_{n=0}^{\infty} \frac{[q;q]_n (-z)^n q^{n(n-1)/2}}{[\alpha q;q]_n [\alpha z q;q]_n} L_n^{\alpha}(xq;q) L_n^{\alpha}(yq;q)$$

$$= \frac{[z;q]_\infty}{[\alpha z q;q]_\infty} \sum_{n=0}^{\infty} \frac{\left[\frac{q}{z};q\right]_n (xyz^2 q^{-1})^n}{[q;q]_n [\alpha q;q]_n}$$

$$\times {}_0\phi_0\left[\begin{matrix} - ; q; -xz \\ - \end{matrix}\right] {}_0\phi_0\left[\begin{matrix} - ; q; -yz \\ - \end{matrix}\right] \qquad \text{...(3.9)}$$

$$= \frac{[z;q]_\infty [-xz;q]_\infty [-yz;q]_\infty}{[\alpha z q;q]_\infty} {}_2\phi_1\left[\begin{matrix} \frac{q}{z}, 0; q; \frac{xyz^2}{q} \\ \alpha q \end{matrix}\right]. \qquad \text{...(3.10)}$$

Formulae (3.8) and (3.10) are q-analogues of the well known Hardy-Hille formula (Carlitz 1970, 1.1) for the Laguerre polynomials.

Next, (3.1) and (3.2), for $\alpha \to 0$ yield

$$\sum_{n=0}^{\infty} \frac{z^n}{[q;q]_n} g_n[A; xq^n; m; q]\, g_n[B; yq^n; p; q]$$

$$= \frac{1}{[z;q]_\infty} \sum_{n=0}^{\infty} \frac{(xyz)^n q^{(m+p+2)n(n-1)/2}}{[q;q]_n}$$

$$\times \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{(-xz)^r (-yz)^s A_{n+r} B_{n+s} q^{(m(m+1)r(r-1+2n)+(p+1)s(s-1+2n))/2}}{[q;q]_r [q;q]_s q^{-rs}}$$

$$\text{...(3.11)}$$

and

$$\sum_{n=0}^{\infty} \frac{(-z)^n q^{n(n-1)/2}}{[q;q]_n} g_n[A; xq; m+1; q]\, g_n[B; yq; p+1; q]$$

$$= [z;q]_\infty \sum_{n=0}^{\infty} \frac{\left[\frac{q}{z};q\right]_n (xyz^2)^n q^{((m+p)n(n-1)-2n)/2}}{[q;q]_n}$$

$$\times G_{0,n}[0; A; xzq^{mn}; m+1; q]\, G_{0,n}[0; B; yzq^{pn}; p+1; q]$$

$$\text{...(3.12)}$$

(3.3), (3.11) and (3.12) are all q-analogues of a result of Carlitz (1970, 5.1). In (3.3), replacing x, y by $-x$, $-y$ respectively and then setting $A_r = \frac{1}{[cq;\, q]_r}$, $B_s = \frac{1}{[dq;\, q]_s}$, we get

$$\sum_{n=0}^{\infty} \frac{[q;\, q]_n\, z^n}{[cq;\, q]_n\, [dq;\, q]_n}\, l_n^c(xq^n;\, q)\, l_n^d(yq^n;\, q)$$

$$= \frac{1}{[z;\, q]_\infty} \sum_{n=0}^{\infty} \frac{[z;\, q]_n\, (xyz)^n\, q^{n(n-1)}}{[q;\, q]_n\, [cq;\, q]_n\, [dq;\, q]_n}$$

$$\times\ {}_1\phi_1\begin{bmatrix} 0;\, q;\, xzq^n \\ cq^{1+n} \end{bmatrix} {}_1\phi_1\begin{bmatrix} 0;\, q;\, yzq^n \\ dq^{1+n} \end{bmatrix} \qquad \ldots(3.13)$$

whereas (3.11) for $m = 1, p = 0$, $A_r = \frac{1}{[cq;\, q]_r}$, $B_s = \frac{1}{[dq;\, q]_s}$ and (3.12) for $m = p = 0$, $A_r = \frac{1}{[cq;\, q]_r}$, $B_s = \frac{1}{[dq;\, q]_s}$ yield

$$\sum_{n=0}^{\infty} \frac{[q;\, q]_n\, z^n}{[cq;\, q]_n\, [dq;\, q]_n}\, L_n^c(xq^n;\, q]\, l_n^d(yq^n;\, q)$$

$$= \frac{1}{[z;\, q]_\infty} \sum_{n=0}^{\infty} \frac{(xyz)^n\, q^{3n(n-1)/2}}{[q;\, q]_n\, [cq;\, q]_n\, [dq;\, q]_n}$$

$$\times \sum_{r=0}^{\infty} \frac{(-xz)^r\, q^{r(r-1+2n)}}{[q;\, q]_r\, [cq^{1+n};\, q]_r}\ {}_1\phi_1\begin{bmatrix} 0;\, q;\, yzq^{n+r} \\ dq^{1+n} \end{bmatrix} \qquad \ldots(3.14)$$

and

$$\sum_{n=0}^{\infty} \frac{[q;\, q]_n\, (-z)^n\, q^{n(n-1)/2}}{[cq;\, q]_n\, [dq;\, q]_n}\, L_n^c(xq;\, q)\, L_n^d(yq;q)$$

$$= [z;\, q]_\infty \sum_{n=0}^{\infty} \frac{\left[\frac{q}{z};\, q\right]_n (xyz^2)^n\, q^{-n}}{[q;\, q]_n\, [cq;\, q]_n\, [dq;\, q]_n}$$

$$\times\ {}_1\phi_1\begin{bmatrix} 0;\, q;\, -xz \\ cq^{1+n} \end{bmatrix} {}_1\phi_1\begin{bmatrix} 0;\, q;\, -yz \\ dq^{1+n} \end{bmatrix} \qquad \ldots(3.15)$$

respectively. (3.13) – (3.15) are q-analogues of a formula due to Carlitz (1970, 5.10). (3.3) for $A_r = \frac{[a;\, q]_r}{[c;\, q]_r}$, $B_s = \frac{[b;\, q]_s}{[d;\, q]_s}$ yields

$$\sum_{n=0}^{\infty} \frac{z^n}{[q;q]_n} p_n\left(-xq^{n-1}; \frac{c}{q}, \frac{a}{c} q^{-n}/q\right) p_n\left(-yq^{n-1}; \frac{d}{q}, \frac{b}{d} q^{-n}/q\right)$$

$$= \frac{1}{[z;q]_\infty} \sum_{n=0}^{\infty} \frac{[z;q]_n\, [a;q]_n}{[q;q]_n\, [c;q]_n} \frac{[b;q]_n\, (xyz)^n\, q^{n(n-1)}}{[d;q]_n}$$

$$\times\, {}_1\phi_1\left[\begin{matrix} aq^n; q; -xzq^n \\ cq^n \end{matrix}\right] {}_1\phi_1\left[\begin{matrix} bq^n; q; -yzq^n \\ dq^n \end{matrix}\right] \qquad \ldots(3.16)$$

Next, in (3.16) replacing x and y by x/z and y/z respectively and then replacing z by xy/z, we get

$$\sum_{n=0}^{\infty} \frac{x^n y^n}{[q;q]_n\, z^n}\, {}_2\phi_1\left[\begin{matrix} q^{-n}, a; q; \frac{-z}{y} q^n \\ c \end{matrix}\right] {}_2\phi_1\left[\begin{matrix} q^{-n}, b; q; \frac{-z}{x} q^n \\ d \end{matrix}\right]$$

$$= \frac{1}{\left[\frac{xy}{z}; q\right]_\infty} \sum_{n=0}^{\infty} \frac{\left[\frac{xy}{z}; q\right]_n [a;q]_n}{[q;q]_n\, [c;q]_n} \frac{[b;q]_n\, z^n q^{n(n-1)}}{[d;q]_n}$$

$$\times\, {}_1\phi_1\left[\begin{matrix} aq^n; q; -xq^n \\ cq^n \end{matrix}\right] {}_1\phi_1\left[\begin{matrix} bq^n; q; -yq^n \\ dq^n \end{matrix}\right] \qquad \ldots(3.17)$$

which is a q-analogue of a result due to Erdelyi (1940). It may be worth-while to remark that similar bilinear transformations may be obtained from (3.11) and (3.12) by suitably choosing the sequences $\{A_r\}$ and $\{B_s\}$.

Lastly (3.2) for $A_r = \dfrac{[a;q]_r}{[\alpha;q]_r\, [-axz;q]_r}$, $B_s = \dfrac{[b;q]_s}{[\alpha;q]_s\, [-byz;q]_s}$, $m = p = 0$

$$\sum_{n=0}^{\infty} \frac{[\alpha;q]_n (-z)^n q^{n(n-1)/2}}{[q;q]_n\, [\alpha z;q]_n}\, {}_2\phi_2\left[\begin{matrix} q^{-n}, a; q; -xq \\ \alpha, -axz \end{matrix}\right] {}_2\phi_2\left[\begin{matrix} q^{-n}, b; q; -yq \\ \alpha, -byz \end{matrix}\right]$$

$$= \frac{[z;q]_\infty\, [-xz;q]_\infty\, [-yz;q]_\infty}{[\alpha z;q]_\infty\, [-axz;q]_\infty\, [-byz;q]_\infty}\, {}_3\phi_2\left[\begin{matrix} a, b, q/z; q; xyz^2q^{-1} \\ \alpha, 0 \end{matrix}\right] \qquad \ldots(3.18)$$

which is a q-analogue of a result of Weisner (Carlitz 1961, 1.1).

Before we close this section, we would prove the following q-analogue of a result of Carlitz (1961, 3.11) :

$$\frac{[-axt;q]_\infty\,[-byt;q]_\infty}{[-xt;q]_\infty\,[-yt;q]_\infty}\ {}_2\phi_3\left[\begin{matrix} a,\,b;\,q;\,cxyt^2 \\ c,\,-axt,\,-byt\end{matrix}\right]$$

$$= \sum_{s=0}^{\infty}\frac{[b;q]_s\,(-yt)^s}{[q;q]_s}\ {}_4\phi_2\left[\begin{matrix} a,\,\dfrac{q^{1-s}}{c},\,q^{-s},\,0;\,q;\,\dfrac{cxq^s}{by} \\ c,\,\dfrac{q^{1-s}}{b}\end{matrix}\right]. \qquad \ldots(3.19)$$

Proof of (3.18) — Denoting the left hand side of (3.19) by S, we have (using q-binomial theorem (Slater 1966, 3.2.2.11)

$$S = \sum_{n=0}^{\infty}\frac{[a;q]_n\,[b;q]_n\,(cxyt^2)^n\,q^{n(n-1)}}{[q;q]_n\,[c;q]_n}$$

$$\times\ {}_1\phi_0\left[\begin{matrix} aq^n;\,q;\,-xt \\ -\end{matrix}\right]{}_1\phi_0\left[\begin{matrix} bq^n;\,q;\,-yt \\ -\end{matrix}\right]$$

$$= \sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{[a;q]_r\,[b;q]_s\,(-)^{r+s}\,x^r y^s t^{r+s}}{[q;q]_r\,[q;q]_s}\ {}_2\phi_1\left[\begin{matrix} q^{-r},\,q^{-s};\,q;\,cq^{r+s} \\ c\end{matrix}\right]$$

$$= \sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{[a;q]_r\,[b;q]_s\,[c;q]_{r+s}\,(-t)^{r+s}\,x^r y^s}{[q;q]_s\,[q;q]_r\,[c;q]_r\,[c;q]_s}$$

setting $r + s = j$ and simplifying, we get (3.19).

§4. In this section we prove the q-analogues of a result of Verma (1972), which is a generalization of Meixner (1942):

$$\sum_{n=0}^{\infty}\frac{z^n}{[q;q]_n}\,g_n\,[A;\,xq^n;\,m;\,q]\,G_{n,0}\,[\alpha;\,B;\,y;\,p;\,q]$$

$$= \frac{[\alpha z;q]_\infty}{[z;q]_\infty}\sum_{n=0}^{\infty}\frac{[\alpha;q]_n\,(-\alpha xyz)^n\,q^{(m+p+3)n(n-1)/2}}{[q;q]_n\,[\alpha z;q]_{2n}}$$

$$\times\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{[\alpha q^n;q]_r\,[\alpha q^n;q]_s\,(-xz)^r\,y^s A_{n+r} B_{n+s}\,q^{((m+1)r(r-1+2n)+ps(s-1+2n))/2}}{[q;q]_r\,[q;q]_s\,[\alpha z q^{2n};q]_{r+s}}$$

$$\ldots(4.1)$$

and

$$\sum_{n=0}^{N} \frac{[\alpha; q]_n [q^{-N}; q]_n q^n}{[q; q]_n \left[\frac{q^{1-N}}{z}; q\right]_n} \left\{ \sum_{r=0}^{n} \frac{[q^{-n}; q]_r \left[\frac{q^{1-N}}{z}; q\right]_r A_r (xz)^r q^{Nr}}{[q; q]_r [q^{1+N-n}; q]_r} \right\}$$

$$\times \left\{ \sum_{s=0}^{\infty} \frac{[\alpha q^n; q]_s y^s B_s}{[q; q]_s} \right\}$$

$$= \frac{[\alpha z; q]_N}{[z; q]_N} \sum_{n=0}^{\infty} \frac{[\alpha; q]_n [\alpha z q^N; q]_n \left[\frac{q^{1-N}}{z}; q\right]_n (\alpha xyz)^n q^{n(n-1+N)}}{[q; q]_n [\alpha z; q]_n \left[\frac{q}{z}; q\right]_n}$$

$$\times \left\{ \sum_{r=0}^{\infty} \frac{[\alpha q^n; q]_r \left[\frac{q^{1-N+n}}{z}; q\right]_r (xz)^r q^{Nr} A_{n+r}}{[q; q]_r \left[\frac{q^{1+n}}{z}; q\right]_r} \right\}$$

$$\times \left\{ \sum_{s=0}^{\infty} \frac{[\alpha q^n; q]_s [\alpha z q^{n+N}; q]_s y^s B_{n+s}}{[q; q]_s [\alpha z q^n; q]_s} \right\}. \qquad \ldots(4.2)$$

Proof of (4.1) — Rewriting the q-analogue of Gauss' summation theorem in the form :

$$[\alpha; q]_r [\alpha; q]_s \, {}_2\phi_1 \left[\begin{matrix} q^{-r}, q^{-s}; q; \alpha q^{r+s} \\ \alpha \end{matrix} \right] = [\alpha; q]_{r+s}. \qquad \ldots(4.3)$$

Multiplying both the sides by

$$\frac{[\alpha z q^{r+s}; q]_\infty (-xz)^r y^s A_r B_s q^{((m+1)r(r-1)+ps(s-1))/2}}{[z; q]_\infty [q; q]_r [q; q]_s}$$

and summing with respect to r and s from 0 to ∞, we get

$$\frac{[\alpha z; q]_\infty}{[z; q]_\infty} \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \sum_{n=0}^{\min(r,s)} \frac{[\alpha; q]_r [\alpha; q]_s [q^{-r}; q]_n [q^{-s}; q]_n (-xz)^r y^s \alpha^n A_r B_s q^{((m+1)r(r-1)+ps(s-1))/2}}{[q; q]_r [q; q]_s [\alpha z; q]_{r+s} [q; q]_n [\alpha; q]_n q^{-n(r+s)}}$$

$$= \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{[\alpha; q]_{r+s} (-xz)^r y^s A_r B_s q^{((m+1)r(r-1)+ps(s-1))/2} [\alpha z q^{r+s}; q]_\infty}{[q; q]_r [q; q]_s [z; q]_\infty}$$

or

$$\frac{[\alpha z; q]_\infty}{[z; q]_\infty} \sum_{n=0}^{\infty} \frac{[\alpha; q]_n (-\alpha xyz)^n q^{(m+p+3)n(n-1)/2}}{[q; q]_n [\alpha z; q]_{2n}} \times$$

(*equation continued on p.* 1653)

$$\times \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{[\alpha q^n; q]_r [\alpha q^n; q]_s (-xz)^r y_s}{[q; q]_r [q; q]_s [\alpha z q^{2n}; q]_{r+s}}$$

$$\times A_{n+r} B_{n+s} q^{((m+1)r(r-1+2n)+ps(s-1+2n))/2}$$

$$= \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{[\alpha; q]_{r+s} (-xz)^r y^s A_r B_s q^{((m+1)r(r-1)+ps(s-1))/2}}{[q; q]_r [q; q]_s}$$

$$\times \sum_{n=0}^{\infty} \frac{[\alpha q^{r+s}; q]_n z^n}{[q; q]_n}$$

$$= \sum_{n=0}^{\infty} \frac{z^n}{[q; q]_n} g_n [A; xq^n; m; q]\, G_{n,0} [\alpha; B; y; p; q].$$

Proof of (4.2) — Multiplying both the sides of (4.3) by $\dfrac{[\alpha z q^s; q]_N (xz)^r y^s A_r B_s}{[q; q]_r [q; q]_s [zq^{-r}; q]_N}$ and summing with respect to r and s from 0 to ∞, we get

$$\sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \sum_{n=0}^{min(r,s)} \frac{[\alpha; q]_r [\alpha; q]_s [\alpha z q^s; q]_N [q^{-r}; q]_n [q^{-s}; q]_n (xz)^r y^s \alpha^n q^{n(r+s)} A_r B_s}{[q; q]_r [q; q]_s [q; q]_n [\alpha; q]_n [zq^{-r}; q]_N}$$

$$= \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{[\alpha; q]_{r+s} (xz)^r y^s A_r B_s [\alpha z q^s; q]_N}{[q; q]_r [q; q]_s [zq^{-r}; q]_N}$$

or

$$\frac{[\alpha z; q]_N}{[z; q]_N} \sum_{n=0}^{\infty} \frac{[\alpha; q]_n [\alpha z q^N; q]_n \left[\dfrac{q^{1-N}}{z}; q\right]_n (\alpha x y z)^n q^{n(n-1+N)}}{[q; q]_n [\alpha z; q]_n \left[\dfrac{q}{z}; q\right]_n}$$

$$\times \left\{ \sum_{r=0}^{\infty} \frac{[\alpha q^n; q]_r \left[\dfrac{q^{1-N+n}}{z}; q\right]_r (xz)^r q^{Nr} A_{n+r}}{[q; q]_r \left[\dfrac{q^{1+n}}{z}; q\right]_r} \right\}$$

$$\times \left\{ \sum_{s=0}^{\infty} \frac{[\alpha q^n; q]_s [\alpha z q^{n+N}; q]_s}{[q; q]_s [\alpha z q^n; q]_s} y^s B_{n+s} \right\} =$$

(*equation continued on p.* 1654)

$$= \sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{[\alpha;q]_{r+s}(xz)^r y^s A_r B_s}{[q;q]_r[q;q]_s}\sum_{n=0}^{N}\frac{[\alpha q^{r+s};q]_n[q^{-N};q]_n q^n}{[q;q]_n\left[\frac{q^{1-N+r}}{z};q\right]_n}$$

$$= \sum_{n=0}^{N}\frac{[\alpha;q]_n[q^{-N};q]_n q^n}{[q;q]_n\left[\frac{q^{1-N}}{z};q\right]_n}\left\{\sum_{r=0}^{n}\frac{[q^{-n};q]_r\left[\frac{q^{1-N}}{z};q\right]_r(xz)^r q^{Nr}A_r}{[q;q]_r[q^{1+N-n};q]_r}\right\}$$

$$\times\left\{\sum_{s=0}^{\infty}\frac{[\alpha q^n;q]_s y^s B_s}{[q;q]_s}\right\}.$$

If in (4.2) we set $\alpha = q^{-M}$ (M a non-negative integer) and then let $N\to\infty$, we get

$$\sum_{n=0}^{M}\frac{z^n}{[q;q]_n}g_n[A;-xq;1;q]\,G_{n,0}[q^{-M};B;y;-;q]$$

$$= \frac{[zq^{-M};q]_\infty}{[z;q]_\infty}\sum_{n=0}^{M}\frac{[q^{-M};q]_n(-xy)^n q^{n(3n-1-2M)/2}}{[q;q]_n\left[\frac{q}{z};q\right]_n[zq^{-M};q]_n}$$

$$\times\left\{\sum_{r=0}^{M-n}\frac{[q^{-M+n};q]_r(-x)^2 A_{n+r}q^{r(r+1+2n)/2}}{[q;q]_r\left[\frac{q^{1+n}}{z};q\right]_r}\right\}$$

$$\times\left\{\sum_{s=0}^{M-n}\frac{[q^{-M+n};q]_s y^s B_{n+s}}{[q;q]_s[zq^{-M+n};q]_s}\right\}. \qquad \ldots(4.4)$$

(4.4) is a q-analogue of the terminating version of (1.2).

On the otherhand, q-analogues of yet another result of Verma (1972) which is a generalization of (1.3) are :

$$\sum_{n=0}^{\infty}\frac{z^n}{[q;q]_n[\alpha;q]_n}G_{n,0}[\alpha;A;x;-;q]\,G_{n,0}[\alpha;B;y;-;q]$$

$$= \frac{[\alpha z;q]_\infty}{[z;q]_\infty}\sum_{n=0}^{\infty}\frac{[\alpha;q]_n(\alpha xyz)^n q^{n(n-1)}}{[q;q]_n[\alpha z;q]_{2n}}$$

$$\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\frac{[\alpha q^n;q]_r\,]\alpha q^n;q]_s\,x^r y^s A_{n+r}B_{n+s}}{[q;q]_r[q;q]_s[\alpha zq^{2n};q]_{r+s}} \qquad \ldots(4.5)$$

and

$$\sum_{n=0}^{N} \frac{[\alpha;q]_n\,[c;q]_n\,[q^{-N};q]_n\,q^n}{[q;q]_n\,[\alpha z;q]_n \left[\frac{c}{z}\,q^{1-N};q\right]_n} \left\{\sum_{r=0}^{\infty} \frac{[\alpha q^n;q]_r \left[\frac{c}{z}\,q^{1-N};q\right]_r x^r A_r}{[q;q]_r \left[\frac{c}{z}\,q^{1-N+n};q\right]_r}\right\}$$

$$\times \left\{\sum_{s=0}^{\infty} \frac{[\alpha q^n;q]_s\,[\alpha z;q]_s\,y^s B_s}{[q;q]_s\,[\alpha z q^n;q]_s}\right\}$$

$$= \frac{[z;q]_N \left[\frac{\alpha z}{c};q\right]_N}{\left[\frac{z}{c};q\right]_N [\alpha z;q]_N}$$

$$\times \sum_{n=0}^{N} \frac{[q^{-N};q]_n\,[\alpha;q]_n\,[c;q]_n\,[\alpha z;q]_n \left[\frac{c}{z}\,q^{1-N};q\right]_n \left(\frac{\alpha x y}{c}\right)^n q^{nN}}{[q;q]_n\,[\alpha z q^N;q]_n \left[\frac{\alpha z}{c};q\right]_n \left[\frac{cq}{z};q\right]_n \left[\frac{q^{1-N}}{z};q\right]_n}$$

$$\times \left\{\sum_{r=0}^{\infty} \frac{[\alpha q^n;q]_r \left[\frac{q}{z};q\right]_r \left[\frac{cq^{1-N+n}}{z};q\right]_r x^r A_{n+r}}{[q;q]_r \left[\frac{c}{z}\,q^{1+n};q\right]_r \left[\frac{q^{1+n-N}}{z};q\right]_r}\right\}$$

$$\times \left\{\sum_{s=0}^{\infty} \frac{[\alpha q^n;q]_s\,[\alpha z q^n;q]_s \left[\frac{\alpha z}{c}\,q^N;q\right]_s y^s B_{n+s}}{[q;q]_s\,[\alpha z q^{n+N};q]_s \left[\frac{\alpha z}{c}\,q^n;q\right]_s}\right\}. \quad \ldots(4.6)$$

Proof of (4.5) — In the Euler's transformation (Sears 1951)

$${}_2\phi_1\left[\begin{matrix}\alpha q^r,\ \alpha q^s;\ q;\ z\\ \alpha\end{matrix}\right] = \frac{[\alpha z q^{r+s};q]_\infty}{[z;q]_\infty}\ {}_2\phi_1\left[\begin{matrix}q^{-r},\ q^{-s};\ q;\ \alpha z q^{r+s}\\ \alpha\end{matrix}\right]$$

multiplying both the sides by $\frac{[\alpha;q]_r\ [\alpha;q]_s\ x^r y^s A_r B_s}{[q;q]_r\ [q;q]_s}$ and summing with respect to r and s from 0 to ∞, we get (4.5) on simplification.

Proof of (4.6) — In the Sears transformation (1951, 8.3)

$${}_4\phi_3\left[\begin{matrix}\alpha q^r,\ \alpha q^s,\ c,\ q^{-N};\ q;\ q\\ \alpha,\ \frac{c}{z}\,q^{1-N+r},\ \alpha z q^s\end{matrix}\right]$$

$$= \frac{[zq^{-r};q]_N \left[\frac{\alpha z}{c}\,q^s;q\right]_N}{\left[\frac{z}{c}\,q^{-r};q\right]_N [\alpha z q^s;q]_N}\ {}_4\phi_3\left[\begin{matrix}q^{-r}\ \ q^{-s},\ c,\ q^{-N};\ q;\ q\\ \alpha,\ zq^{-r}.\ \frac{c}{\alpha z}\,q^{1-N-s}\end{matrix}\right]$$

multiplying both the sides by $\frac{[\alpha; q]_r [\alpha; q]_s x^r y^s A_r B_s}{[q; q]_r [q; q]_s}$ and summing with respect to r and s from 0 to ∞, we get (4.6), on some reductions. Lastly, we derive the q-analogue of the terminating version of (1.4) in the form :

$$\sum_{n=0}^{N} \frac{[q^{-N}; q]_n \left[\frac{c}{b}; q\right]_n \left[\frac{e}{d}; q\right]_n [s; q]_n (bdq)^n}{[q; q]_n [c; q]_n [e; q]_n}$$

$$\times {}_3\phi_2\left[\begin{matrix} q^{-N+n}, d, -tq^n; q; q \\ eq^n, 0 \end{matrix}\right] {}_3\phi_2\left[\begin{matrix} q^{-N+n}, b, -zq^n; q; q \\ cq^n, 0 \end{matrix}\right]$$

$$= \sum_{n=0}^{N} \frac{[q^{-N}; q]_n [b; q]_n [d; q]_n [s; q]_n (ztq)^n}{[q; q]_n [c; q]_n [e; q]_n s^{2n-N}}$$

$$\times {}_3\phi_2\left[\begin{matrix} q^{-N+n}, bq^n, -\frac{z}{s}; q; q \\ cq^n, 0 \end{matrix}\right] {}_3\phi_2\left[\begin{matrix} q^{-N+n}, dq^n, -\frac{t}{s}; q; q \\ eq^n, 0 \end{matrix}\right] \quad \ldots(4.7)$$

Proof of (4.7) — The q-analogue of Saalschütz summation theorem :

$$\frac{[q^{-N+p}; q]_n [q^{-N+k}; q]_n}{[q^{-N}; q]_n [q^{-N+k+p}; q]_n} = {}_3\phi_2\left[\begin{matrix} q^{-n}, q^{-k}, q^{-p}; q; q \\ q^{-N}, q^{1+N-p-k-n} \end{matrix}\right]$$

may be rewritten as (using the q-analogue of Vandermonde's summation theorem)

$$(bd)^n q^{n(k+p)} \frac{[q^{-N}; q]_n \left[\frac{c}{b}; q\right]_n \left[\frac{e}{d}; q\right]_n}{[q; q]_n [c; q]_n [e; q]_n} \sum_{r \geqslant p} \frac{[q^{-N+n}; q]_r [d; q]_r q^r}{[q; q]_p [q; q]_{r-p} [eq^n; q]_r}$$

$$\times \sum_{s \geqslant k} \frac{[q^{-N+n}; q]_s [b; q]_s q^s}{[q; q]_k [q; q]_{s-k} [cq^n; q]_s}$$

$$= \sum_{m \geqslant o} \frac{[q^{-N+k+p}; q]_{n-m} [q^{-N}; q]_m [b; q]_m [d; q]_m q^{m(m+1-N)}}{[q; q]_{n-m} [q; q]_m [c; q]_m [e; q]_m}$$

$$\times \sum_{u \geqslant p-m} \frac{[q^{-N+m}; q]_u [dq^m; q]_u q^u}{[q; q]_{p-m} [q; q]_{u-p+m} [eq^m; q]_u}$$

$$\times \sum_{j \geqslant k-m} \frac{[q^{-N+m}; q]_j [bq^m; q]_j q^j}{[q; q]_{k-m} [q; q]_{j-k+m} [cq^m; q]_j}.$$

Now multiplying both the sides of the above expression by $[s; q]_n\, z^k t^p q^{k(k-1)/2} \times q^{(p(p-1)+2n)/2}$ and summing with respect to n, k, p the left-hand side may be written in the form :

$$= \sum_{n=0}^{N} \sum_{p\geqslant 0} \sum_{k\geqslant 0} \frac{[q^{-N}; q]_n [s; q]_n \left[\frac{c}{b}; q\right]_n \left[\frac{e}{d}; q\right]_n}{[q; q]_n [c; q]_n [e; q]_n} (bd)^n q^{n(k+p+1)}$$

$$\times \sum_{r\geqslant p} \frac{[q^{-N+n}; q]_r [d; q]_r q^r}{[q; q]_p [q; q]_{r-p} [eq^n; q]_r}$$

$$\times \sum_{s\geqslant k} \frac{[q^{-N+n}; q]_s [b; q]_s q^s z^k t^p q^{(k(k-1)+p(p-1))/2}}{[q; q]_k [q; q]_{s-k} [cq^n; q]_s}$$

$$= \sum_{n=0}^{N} \frac{[q^{-N}; q]_n \left[\frac{c}{b}; q\right]_n \left[\frac{e}{d}, q\right]_n [s; q]_n (bdq)^n}{[q; q]_n [c; q]_n [e; q]_n}$$

$$\times \sum_{r=0}^{N-n} \frac{[q^{-N+n}; q]_r [d; q]_r q^r}{[q; q]_r [eq^n; q]_r} \sum_{s=0}^{N-n} \frac{[q^{-N+n}; q]_s [b; q]_s q^s}{[q; q]_s [cq^n; q]_s}$$

$$\times {}_1\phi_0 \left[\begin{matrix} q^{-s}; q; -zq^{n+s} \\ - \end{matrix} \right] {}_1\phi_0 \left[\begin{matrix} q^{-r}; q; -tq^{n+r} \\ - \end{matrix} \right] \qquad \ldots(4.8)$$

whereas the right-hand side becomes

$$S = \sum_{n\geqslant 0} \sum_{k\geqslant 0} \sum_{p\geqslant 0} \sum_{m\geqslant 0} \frac{[q^{-N+k+p}; q]_{n-m} [q^{-N}; q]_m [b; q]_m [d; q]_m [s; q]_n z^k t^p q^{k(k-1)/2}}{[q; q]_{n-m} [q; q]_m [c; q]_m [e; q]_m}$$

$$\times q^{(p(p-1)+2m(m+1-N)+2n)/2} \sum_{u\geqslant p-m} \frac{[q^{-N+m}; q]_u [dq^m; q]_u q^u}{[q; q]_{p-m} [q; q]_{u-p+m} [eq^m; q]_u}$$

$$\times \sum_{j\geqslant k-m} \frac{[q^{-N+m}; q]_j [bq^m; q]_j q^j}{[q; q]_{k-m} [q; q]_{j-k+m} [cq^m; q]_j}$$

$$= \sum_{k\geqslant 0} \sum_{p\geqslant 0} \sum_{m\geqslant 0} \frac{[q^{-N}; q]_m [b; q]_m [d; q]_m [s; q]_m z^k t^p q^{(k(k-1)+p(p-1)+2m(m+2-N))/2}}{[q; q]_m [c; q]_m [e; q]_m}$$

$$\times {}_2\phi_1 \left[\begin{matrix} q^{-N+k+p}, sq^m; q; q \\ 0 \end{matrix} \right] \sum_{u\geqslant p-m} \frac{[q^{-N+m}; q]_u [dq^m; q]_u q^u}{[q; q]_{p-m} [q; q]_{u-p+m} [eq^m; q]_u}$$

$$\times \sum_{j\geqslant k-m} \frac{[q^{-N+m}; q]_j [bq^m; q]_j q^j}{[q; q]_{k-m} [q; q]_{j-k+m} [cq^m; q]_j}.$$

Summing the ${}_2\phi_1$ and then replacing k by $k + m$, we get

$$S = \sum_{p\geqslant 0}\sum_{m\geqslant 0} \frac{[q^{-N}; q]_m\,[b; q]_m\,[d; q]_m\,[s; q]_m\, z^m t^p q^{(p(p-1)+m(m+3-2p))/2}}{[q; q]_m\,[c; q]_m\,[e; q]_m\, s^{p-N+m}}$$

$$\times \sum_{u\geqslant p-m} \frac{[q^{-N+m}; q]_u\,[dq^m; q]_u\, q^u}{[q; q]_{p-m}\,[q; q]_{u-p+m}\,[eq^m; q]_u} \sum_{j\geqslant 0} \frac{[q^{-N+m}; q]_j\,[bq^m; q]_j\, q^j}{[q; q]_j\,[cq^m; q]_j}$$

$$\times\ {}_1\phi_0\left[\begin{matrix} q^{-j};\ q;\ -\dfrac{z}{s}\,q^j \\ - \end{matrix}\right]$$

$$= \sum_{m=0}^{N} \frac{[q^{-N}; q]_m\,[b; q]_m\,[d; q]_m\,[s; q]_m\,(ztq)^m}{[q; q]_m\,[c; q]_m\,[e; q]_m\, s^{2m-N}}$$

$$\times\ {}_3\phi_2\left[\begin{matrix} q^{-N+m},\ bq^m,\ -\dfrac{z}{s};\ q;\ q \\ cq^m,\ 0 \end{matrix}\right]$$

$$\times \sum_{u\geqslant 0} \frac{[q^{-N+m}; q]_u\,[dq^m; q]_u\, q^u}{[q; q]_u\,[eq^m; q]_u}\ {}_1\phi_0\left[\begin{matrix} q^{-u};\ q;\ -\dfrac{t}{s}\,q^u \\ - \end{matrix}\right] \qquad \ldots(4.9)$$

Summing the ${}_1\phi_0$ in (4.8) and (4.9) we get (4.7).

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REFERENCES

Andrews, G. E., and Askey, R. (1977). Enumeration of partition : The role of Eulerian series and q-orthogonal polynomials, in Higher Combinatories (ed. M. Aigner, D. Reidel). Dordrecht, Boston, pp. 3-26.

Carlitz, L. (1961). Some generating functions of Weisner. *Duke Math. J.*, **28**(4), 523-30.

———— (1970). Transformations of certain bilinear generating functions. *Ann. Math. Pura Applic.*, **86**, 155-68.

Erdélyi, A. (1939). Transformation einer genwssen nach produkten Konfluenten hypergeometrischer Functionen fortschreitenden Reihe. *Compositio Math.*, **6**, 336-47.

———— (1940). Transformation of a certain series of products of confluent hypergeometric functions, applications to Laguerre and Charlier polynomials. *Compositio Math.*, **7**, 340-52.

Jackson, F. H. (1944). Basic double Hypergeometric Functions (II). *Q. Jl Math. (Oxford)*, **15**, 49-61.

Meixner, J. (1942). Umformung gewisser Reihen deren Glieder Produkte hypergeometrischer Funktionen sind. *Deutsche Math.*, **6**, 341-49.

Sears, D. B. (1951). On the transformations of basic hypergeometric functions. *Proc. London math. Soc.* (2), **53**, 158-80.

Slater, L. J. (1966). Generalized Hypergeometric Functions. Cambridge University Press, London.

Verma, A. (1972). Some transformations of series with arbitrary terms. *Istituto Lambardo di Scienze e Lettere*, **106**, 342-53.