Indian J. pure appl. Math., 33(1) : 97-109, January 2002
© Printed in India.

THERMOSOLUTAL INSTABILITY OF WALTERS (MODEL B′) VISCO-ELASTIC ROTATING FLUID PERMEATED WITH SUSPENDED PARTICLES AND VARIABLE GRAVITY FIELD IN POROUS MEDIUM

VEENA SHARMA

Department of Mathematics, Himachal Pradesh University, Summer Hill, Shimla 171 005, *India*

GIAN CHAND RANA

Department of Mathematics, Government Post Graduate College, Sarkaghat 175 024, *Himachal Pradesh, India*

(*Received* 16 *August* 2000; *after revision* 12 *January* 2001; *accepted* 23 *April* 2001)

The thermosolutal instability of Walters' (model B′) elastico-viscous fluid in porous medium is considered in the presence of uniform rotation, suspended particles and variable gravity field. The stable solute gradient, rotation, gravity field, suspended particles and viscoelaticity introduce oscillatory modes which were non-existent in their absence. For the stationary convection, the stable solute gradient and rotation have stabilizing effects and suspended particles are found to have destabilizing effect on the system, whereas the medium permeability has destabilizing or stabilizing effect on the system under certain conditions. The effects of stable solute gradient, rotation, suspended particles, medium permeability have also been shown graphically.

Key Words : Thermosolutal Instability Walters' (model B') Elastico-Viscous Fluid; Suspended particles; Variable Gravity; Rotation

1. INTRODUCTION

A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar[1]. Veronis[2] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The buoyancy forces can arise not only from density differences due to variations in temperature, but also from those due to variations in solute concentration. Thermosolutal convection problems arise in oceanography, limnology and engineering. Examples of particular interest are provided by ponds built to trap solar heat [Taber and Matz[3]] and some Antarctic lakes [Shirtcliffe[4]]. The physics is quite similar in the stellar case, in that helium acts like salt in raising the density and in diffusing more slowly than heat.

Stommel and Fedorov[5] and Linden[6] have remarked that the length scales characteristic of double-diffusive convecting layers in the ocean may be sufficiently large that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke[7] explained a double-diffusive

instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Nason *et al.*[8] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotationin the ultracentrifuge.

The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground-water hydrology and astrophysics. The development of geothermal power resources have increased general interest in the study of the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust [Lister[9]]. Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice-versa. The physical properties of comets, meteorites and inter-planetary dust strongly suggest the importance of porosity in the astophysical context [McDonnel[10]].

Thermal stability of a fluid layer under variable gravitational field heated from below or above is investigated analytically by Pradhan and Samal[11]. Although the gravity field of the Earth is varying with height from its surface, we usually neglect this variation for laboratory purposes and treat the field as constant. However, this may not be the case for large-scale flows in the ocean, the atmosphere or the mantle. It can become imperative to consider gravity as a quantity varying with distance from the centre. Recently, Sharma and Kumar[12] have studied the stability of two superposed Walters' (model B') viscoelastic fluids.

The present paper deals with the effects of suspended particles, rotation and variable gravity field on thermosolutal convection in Walters' (model B') elastico-viscous fluid in porous medium.

2. Formulation of the Problem and Perturbation Equations

Consider an infinite, horizontal, incompressible Walters' (model B') elastico-viscous fluid of thickness d, bounded by the planes $z = 0$ and $z = d$ in an isotropic and homogeneous porous medium of porosity ε and permeability k_1, which is acted upon by a uniform rotation $\boldsymbol{\Omega}(0, 0, \Omega)$ and variable gravity $\boldsymbol{g}(0, 0, -g)$, $g = \lambda g_0$, $g_0 (> 0)$ is the value of g at $z = 0$ and λ can be positive or negative as gravity increases or decreases upwards from its value g_0. This layer is heated and soluted from below such that a uniform temperature gradient $\beta\left(= \left| \frac{dT}{dz} \right| \right)$ and uniform solute gradient $\beta'\left(= \left| \frac{dc}{dz} \right| \right)$ are maintained.

Let p, ρ, T, C, α, α', υ, υ' and $\boldsymbol{q}(u, v, w)$ denote, respectively, pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, kinematic viscosity, kinematic viscoelasticity and velocity of the fluid, $K' = 6\pi\rho\upsilon\eta'$, η' being particle radius, is the Stokes' drag coefficient, $\bar{x} = (x, y, z)$, $E = \varepsilon + (1 - \varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_f}\right)$ is constant and E' is a constant analogous to E but corresponding to solute rather than heat; κ, κ' are the thermal diffusivity and solute diffusivity respectively; ρ_s, c_s ρ_0, c_f denote the density and heat capacity of solid (porous) matrix and fluid, respectively; $\boldsymbol{q}_d(\bar{x}, t)$ and $N(\bar{x}, t)$ denote filter velocity and number density of the suspended particles, respectively, the suffix zero refers to the values at reference level $z = 0$.

The equations expressing the conservation of momentum, mass, temperature, solute mass concentration and equation of state of Walters' (model B') elastico-viscous fluid are

$$\frac{1}{\varepsilon}\left[\frac{\partial \boldsymbol{q}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}\cdot\nabla)\boldsymbol{q}\right]=-\frac{1}{\rho_0}\nabla p+\boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\boldsymbol{q}$$

$$+\frac{K'N}{\varepsilon\rho_0}(\boldsymbol{q}_d-\boldsymbol{q})+\frac{2}{\varepsilon}(\boldsymbol{q}\times\boldsymbol{\Omega}), \qquad \ldots (1)$$

$$\nabla\cdot\boldsymbol{q}=0, \qquad \ldots (2)$$

$$E\frac{\partial}{\partial t}T+(\boldsymbol{q}\cdot\nabla)T+\frac{mNc_{pt}}{\rho_0 c_f}\left[\varepsilon\frac{\partial}{\partial t}+\boldsymbol{q}_d\cdot\nabla\right]T=\kappa\nabla^2 T, \qquad \ldots (3)$$

$$E'\frac{\partial}{\partial t}C+(\boldsymbol{q}\cdot\nabla)C+\frac{mNc'_{pt}}{\rho_0 c_f}\left[\varepsilon\frac{\partial}{\partial t}+\boldsymbol{q}_d\cdot\nabla\right]C=\kappa'\nabla^2 C \qquad \ldots (4)$$

and $$\rho=\rho_0[1-\alpha(T-T_0)+\alpha'(C-C_0]. \qquad \ldots (5)$$

Assuming uniform particles size, spherical shape and small relative velocities between the fluid and particles, the presence of particles add an extra force term in the equations of motion (1), proportional to the velocity difference between particles and fluid. The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. The distances between particles are assumed quite large compared with their diameters, so that interparticle reactions are ignored. The buoyancy force on the particles is neglected. If mN is the mass of particles per unit volume, then the equations of motion and continuity for particles, under the above assumptions, are

$$mN\left[\frac{\partial \boldsymbol{q}_d}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}_d\cdot\nabla)\boldsymbol{q}_d\right]=K'N(\boldsymbol{q}-\boldsymbol{q}_d), \qquad \ldots (6)$$

$$\varepsilon\frac{\partial N}{\partial t}=\nabla\cdot(N\boldsymbol{q}_d)=0. \qquad \ldots (7)$$

The initial state of the system is taken to be a quiescent layer (no settling) with a uniform particle distribution N_0. The initial state is

$$\boldsymbol{q}=(0,0,0),\ \boldsymbol{q}_d=(0,0,0)$$

$$T=-\beta z+T_0,\ C=-\beta' z+C_0,$$

$$\rho=\rho_0(1+\alpha\beta z-\alpha'\beta' z),\ N_0 = \text{constant} \qquad \ldots (8)$$

is an exact solution to the governing equations.

Let $\delta\rho$, δp, θ, γ, $\boldsymbol{q}(u, v, w)$ and $\boldsymbol{q}_d(l, r, s)$ denote, respectively, the perturbations in density ρ, pressure p, temperature T, solute concentration C, fluid velocity (0, 0, 0) and particle velocity (0, 0, 0). The change in density $\delta\rho$ caused by perturbations θ and γ in temperature and solute concentration is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \quad \ldots (9)$$

Then the linearized perturbation equations relevant to the problem are

$$\frac{1}{\varepsilon}\frac{c\boldsymbol{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p + \boldsymbol{g}\left(\frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)\boldsymbol{q} + \frac{K'N}{\varepsilon\rho_0}\boldsymbol{q}_d - \boldsymbol{q}) + \frac{2}{\varepsilon}(\boldsymbol{q}\times\boldsymbol{\Omega}), \quad \ldots (10)$$

$$\nabla\cdot\boldsymbol{q} = 0, \quad \ldots (11)$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\boldsymbol{q}_d = \boldsymbol{q}, \quad \ldots (12)$$

$$(E + b\,\varepsilon)\frac{\partial\theta}{\partial t} = \beta(w + bs) + \kappa\nabla^2\theta, \quad \ldots (13)$$

and

$$(E' + b'\,\varepsilon)\frac{\partial\gamma}{\partial t} = \beta'(w + b's) + \kappa'\nabla^2\gamma, \quad \ldots (14)$$

where

$$b = \frac{mNc_{pt}}{\rho_0 c_f},\ b' = \frac{mNc'_{pt}}{\rho_0 c_f}$$

and w, s are the vertical fluid and particle velocities.

In the cartesian form, eqs. (10)-(14) with the help of eq. (9) can be expressed as

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial x}\delta p - \frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)u$$

$$\frac{mN}{\varepsilon\rho_0}\frac{\partial u}{\partial t} + \frac{2}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\Omega v, \quad \ldots (15)$$

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial y}\delta p - \frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)v$$

$$-\frac{mN}{\varepsilon\rho_0}\frac{\partial v}{\partial t} - \frac{2}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\Omega u, \quad \ldots (16)$$

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\frac{\partial}{\partial z}\delta p - \frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)\left(\upsilon - \upsilon'\frac{\partial}{\partial t}\right)w$$

$$+ g\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)(\alpha\theta - \alpha'\gamma) - \frac{mN}{\varepsilon\rho_0}\frac{\partial w}{\partial t} \quad \ldots (17)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad \ldots (18)$$

$$(E + b\,\varepsilon)\frac{\partial\theta}{\partial t} = \beta(w + bs) + \kappa\nabla^2\theta \quad \ldots (19)$$

and $$(E' b' + \varepsilon)\frac{\partial \gamma}{\partial t} = \beta' (w + b' s) + \kappa' \nabla^2 \gamma, \qquad \ldots (20)$$

where $$b = \frac{mNc_{pt}}{\rho_0 c_f} \text{ and } b' = \frac{mNc'_{pt}}{\rho_0 c'_f}.$$

Operating eqs. (15) and (16) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$, respectively, adding and using eq. (18), we get

$$\frac{1}{\varepsilon}\left(\frac{m\partial}{K\partial t}+2\right)\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)=\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\nabla^2-\frac{\partial^2}{\partial z^2}\right)\delta p-\frac{1}{k_1}$$
$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial z}$$
$$-\frac{mN}{\varepsilon\rho_0}\frac{\partial}{\partial t}\frac{\partial w}{\partial z}-\frac{2}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega\zeta, \qquad \ldots (21)$$

where $$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

is the z-component of vorticity.

Operating eqs. (17) and (21) by $\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$ respectively and adding, to eliminate δp between eqs. (17) and (21), we get

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\nabla^2 w = g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial}{\partial y^2}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)(\alpha\theta-\alpha'\gamma)$$
$$-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\nabla^2 w-\frac{mN}{\varepsilon\rho_0}\frac{\partial}{\partial t}\nabla^2 w-\frac{2}{\varepsilon}\Omega\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial z}, \qquad \ldots (22)$$

where $$\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

Operating eqs. (15) and (16) by $-\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively and adding, we get

$$\frac{1}{\varepsilon}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial t}=-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\zeta-\frac{mN}{\varepsilon\rho_0}\frac{\partial\zeta}{\partial t}$$
$$+\frac{2}{\varepsilon}\Omega\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial w}{\partial z}. \qquad \ldots (23)$$

3. DISPERSION RELATION

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, \theta, \gamma, \zeta] = [W(z), \Theta(z), \Gamma(z), Z(z)] \exp [ik_x x + ik_y y + nt], \quad \ldots (24)$$

where k_x, k_y are the wave numbers along the x and y directions respectively, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using expression (24), eqs. (22), (23), (19) and (20) become

$$\frac{n}{\varepsilon}\left(\frac{d^2}{dz^2} - k^2\right) W = -gk^2(\alpha\Theta - \alpha'\Gamma) - \frac{1}{k_1}(\upsilon - \upsilon' n)\left(\frac{d^2}{dz^2} - k^2\right) W$$

$$- \frac{mNn}{\varepsilon\rho_0\left(\frac{mn}{K'} + 1\right)}\left(\frac{d^2}{dz^2} - k^2\right) W - \frac{2}{\varepsilon}\Omega\frac{dZ}{dz}, \quad \ldots (25)$$

$$\frac{n}{\varepsilon} Z = -\frac{1}{k_1}(\upsilon - \upsilon' n) Z - \frac{mNn}{\varepsilon\rho_0\left(\frac{mn}{K'} + 1\right)} Z + \frac{2}{\varepsilon}\Omega\frac{dW}{dz}, \quad \ldots (26)$$

$$(E + b\varepsilon) n\Theta = \beta(W + bs) + \kappa\left(\frac{d^2}{dz^2} - k^2\right)\Theta \quad \ldots (27)$$

and

$$(E' + b'\varepsilon) n\Gamma = \beta'(W + b's) + \kappa'\left(\frac{d^2}{dz^2} - k^2\right)\Gamma. \quad \ldots (28)$$

Eqs. (25)-(28), in non-dimensional form, become

$$\left[\frac{\sigma}{\varepsilon}\left(1 + \frac{M}{1 + \tau_1\sigma}\right) + \frac{1 - F\sigma}{P_l}\right](D^2 - a^2) W + \frac{ga^2 d^2}{\upsilon}$$

$$(\alpha\Theta - \alpha'\Gamma) + \frac{2\Omega d^3}{\varepsilon\upsilon} DZ = 0, \quad \ldots (29)$$

$$\left[\frac{\sigma}{\varepsilon}\left(1 + \frac{M}{1 + \tau_1\sigma}\right) + \frac{1 - F\sigma}{P_l}\right] Z = \left(\frac{2\Omega d}{\varepsilon\upsilon}\right) DW, \quad \ldots (30)$$

$$(D^2 - a^2 - E_1 p_1 \sigma)\Theta = -\frac{\beta d^2}{\kappa}\left(\frac{B + \tau_1\sigma}{1 + \tau_1\sigma}\right) W \quad \ldots (31)$$

and

$$(D^2 - a^2 - E_1' q_1' \sigma)\Gamma = -\left(\frac{\beta d^2}{\kappa'}\right)\left(\frac{B' + \tau_1\sigma}{1 + \tau_1\sigma}\right) W, \quad \ldots (32)$$

where we have put $a = kd$, $\sigma = \frac{n d^2}{\upsilon}$, $\tau = \frac{m}{K'}$, $\tau_1 = \frac{\tau \upsilon}{d^2}$, $M = \frac{mN}{\rho_0}$, $P_l = \frac{k_1}{d^2}$, $E_1 = E + b\,\varepsilon$, $E_1' = E' + b'\varepsilon$, $B = b + 1$, $B' = b' + 1$, $F = \frac{\upsilon'}{d^2}$ and $D^* = d\frac{d}{dz} = dD$ and superscript * is suppressed, $p_1 = \frac{\upsilon}{\kappa}$, is the thermal Prandtl number and $q = \frac{\upsilon}{\kappa'}$, is the Schmidt number.

Eliminating Θ, Γ and Z between eqs. (29)-(32), we obtain

$$\left[\frac{\sigma}{\varepsilon}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1 - F\sigma}{P_l}\right]$$

$$(D^2 - a^2)(D^2 - a^2 - E_1 p_1 \sigma)(D^2 - a^2 - E_1' q_1' \sigma)$$

$$- Ra^2 \lambda\left(\frac{B + \tau_1\sigma}{1+\tau_1\sigma}\right)(D^2 - a^2 - E_1' q_1' \sigma) W + Sa^2 \lambda\left(\frac{B' + \tau_1\sigma}{1+\tau_1\sigma}\right)$$

$$(D^2 - a^2 - E_{1pq_1}\sigma) W$$

$$+ \frac{\frac{T_A}{\varepsilon^2}(D^2 - a^2 - E_1 p_1 \sigma)(D^2 - a^2 - E_1' q_1' \sigma)}{\left[\frac{\sigma}{\varepsilon}\left(1 + \frac{M}{1+\tau_1\sigma}\right) + \frac{1-F\sigma}{P_l}\right]} D^2 W = 0, \qquad \ldots (33)$$

where $R = \frac{g_0 \alpha \beta d^4}{\upsilon \kappa}$, is the thermal Rayleigh number,

$S = \frac{g_0 \alpha' \beta' d^4}{\upsilon \kappa'}$, is the analogous solute Rayleigh number,

$T_A = \left(\frac{2\Omega d^2}{\upsilon}\right)^2$, is the Taylor number.

We consider the case where both the boundaries are free and perfect conductors of heat, while the adjoining medium is assumed to be electrically non-conducting. The appropriate boundary conditions for this case are[1 & 2]

$$W = D^2 W = \Gamma = \Theta = DZ = 0 \text{ at } z = 0 \text{ and } 1. \qquad \ldots (34)$$

The case of two free boundaries, though little artificial is the most appropriate for stellar atmospheres. Using the boundary conditions (34), we can show that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence the proper solution of W characterizing the lowest mde is

$$W = W_0 \sin \pi z, \qquad \ldots (35)$$

where W_0 is constant.

Substituting (35) in equation (33), we obtain the dispersion relation

$$R_1 x\lambda = \left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+\tau_1 i\sigma_1\pi^2}\right)+\frac{1-Fi\sigma_1\pi^2}{P}\right]$$

$$\frac{(1+x)(1+x+E_1 p_1 i\sigma_1)(1+i\sigma_1\tau_1\pi^2)}{(B+i\sigma_1\tau_1\pi^2)}$$

$$+\frac{S_1 x\lambda(1+x+E_1 p_1 i\sigma_1)(B'+i\sigma_1\tau_1\pi^2)}{(B+i\sigma_1\tau_1\pi^2)(1+x+E_1' p_1 i\sigma_1)}$$

$$+\frac{\frac{T_{A_1}}{\varepsilon^2}(1+x+E_1 p_1 i\sigma_1)(1+i\sigma_1\tau_1\pi^2)}{\left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+i\sigma_1\tau_1\pi^2}\right)+\frac{1-Fi\sigma_1\pi^2}{P}\right](B+i\sigma_1\tau_1\pi^2)}, \qquad \text{... (36)}$$

where $R_1=\frac{R}{\pi^4}, T_{A_1}=\frac{T_A}{\pi^4}, x=\frac{a^2}{\pi^2}, S_1=\frac{S}{\pi^4}, i\sigma_1=\frac{\sigma}{\pi^2}$ and $P=\pi^2 P_l$.

Eq. (36) is the required dispersion relation studying the effect of suspended particles, stable solute gradient, medium permeability, rotation and variable gravity field of the system.

4. Stability of the System and Oscillatory Modes

Here we examine the possibility of oscillatory modes, if any, on the Walter's (model B′) elastico-viscous fluid due to the presence of suspended particles, stable solute gradient, rotation, viscoelasticity and variable gravity field. Multiplying eq. (29) by W^*, the complex conjugate of W, integrating over the range of z and making use of (30)-(32) with the help of boundary conditions (34), we obtain

$$\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\frac{1-F\sigma}{P_l}\right]I_1-\frac{\lambda g_0\alpha\kappa a^2}{\upsilon\beta}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_2+\sigma^* p_1 E_1 I_3)$$

$$+\frac{\lambda g_0\alpha'\kappa' a^2}{\upsilon\beta'}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)(I_4+\sigma^* q' E_1' I_5)$$

$$+d^2\left[\frac{\sigma^*}{\varepsilon}\left(1+\frac{M}{1+\tau_1\sigma^*}\right)+\frac{1-F\sigma^*}{P_l}\right]I_6=0, \qquad \text{... (37)}$$

where $I_1=\int_0^1 (|DW|^2+a^2|W|^2)\,dz,$

$$I_2 = \int_0^1 (|D\Theta|^2 + a^2 |\Theta|^2)\, dz,$$

$$I_3 = \int_0^1 |\Theta|^2\, dz,$$

$$I_4 = \int_0^1 (|D\Gamma|^2 + a^2 |\Gamma|^2)\, dz,$$

$$I_5 = \int_0^1 (|\Gamma|^2)\, dz$$

and

$$I_6 = \int_0^1 |Z|^2\, dz.$$

The integrals I_1-I_6 are all positive definite. Putting $\sigma = i\sigma_1$ in eq. (37), where σ_1 is real and equating the imaginary parts, we obtain

$$\sigma_i \left[\left\{ \frac{1}{\varepsilon}\left(1 + \frac{M}{\tau_1^2 \sigma_1^2}\right) - \frac{F}{P_l} \right\} (I_1 - d^2 I_6) \right.$$

$$+ \frac{\lambda g_0 \alpha \kappa a^2}{\upsilon \beta} \left\{ \frac{\tau_1 (B-1)}{B^2 + \tau_1^2 \sigma_1^2} \frac{I_2 + B + \tau_1^2 \sigma_1^2 E_1 p_1 I_3}{B^2 + \tau_1^2 \sigma_1^2} \right\}$$

$$\left. - \frac{\lambda g_0 \alpha' \kappa' a^2}{\upsilon \beta'} \left\{ \frac{\tau_1 I_4 (B-1)}{B^2 + \tau_1^2 \sigma_1^2} + \frac{B + \tau_1^2 \sigma_1^2}{B^2 + \tau_1^2 \sigma_1^2} E_1' p_1 I_5 \right\} \right] \quad \ldots (38)$$

Eq. (38) implies that $\sigma_i = 0$ or $\sigma_i \neq 0$ which means that modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of rotation, stable solute gradient, gravity field and viscoelasticity, which were non-existence in their absence.

5. STATIONARY CONVECTION

For stationary convection, putting $\sigma = 0$ in eq. (36) reduces it to

$$R_1 = \left[\frac{1+x}{\lambda x}\left(\frac{1+x}{P} + \frac{T_{A_1} P}{\varepsilon^2} \right) + S_1 B' \right] \frac{1}{B'}, \quad \ldots (39)$$

and Walters' (model B′) elastico-viscous fluid behaves like an ordinary Newtonian fluid since visco-elastic parameter F vanishes with σ.

To study the effects of suspended particles, rotation, stable solute gradient and medium permeability, we examine the behaviour of $\frac{dR_1}{dB}, \frac{dR_1}{dT_{A1}}, \frac{dR_1}{dS_1}$ and $\frac{dR_1}{dP}$ analytically.

Eq. (39) yields

$$\frac{dR_1}{dB} = -\left[\frac{1+x}{\lambda x}\left(\frac{1+x}{P} + \frac{T_{A_1} P}{\varepsilon^2}\right) + S_1 B'\right]\frac{1}{B^2}, \qquad \ldots (40)$$

which is negative implying thereby that the effect of suspended particles is to destabilize the system when gravity increases upwards from its value g_0 and stabilizes the system when gravity decreases upwards, if

$$\frac{1+x}{\lambda x}\left(\frac{1+x}{P} + \frac{T_{A_1} P}{\varepsilon^2}\right) > S_1 B'.$$

From eq. (39), we get

$$\frac{dR_1}{dT_{A_1}} = \frac{(1+x) P}{\varepsilon^2 \lambda x B}, \qquad \ldots (41)$$

which shows that rotation has stabilizing effect on the system in porous medium when gravity increases upwards from its value g_0 and destablizes the system when gravity decreases upwards which is in contrast to the Newtonian fluids.

It is evident from eq. (39) that

$$\frac{dR_1}{dS_1} = \frac{B'}{B}, \qquad \ldots (42)$$

which is positive implying thereby stable solute gradient has stabilizing effect.

Also from eq. (39), we get

$$\frac{dR_1}{dP} = -\left(\frac{1+x}{\lambda x B}\right)\left[\frac{1+x}{P^2} - \frac{T_{A_1}}{\varepsilon^2}\right]. \qquad \ldots (43)$$

From (42), we see that medium permeability has destabilizing effect when $\frac{1+x}{p^2} > \frac{T_{A_1}}{\varepsilon^2}$ and medium permeability has stabilizing effect when $\frac{1+x}{p^2} < \frac{T_{A_1}}{\varepsilon^2}$, when gravity increases upwards from its value g_0.

The dispersion relation (39) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

In Fig. 1, R_1 is plotted against B for $\lambda = 2$, $P = 0.2$, $S_1 = 10$, $T_{A_1} = 5$, $\varepsilon = 0.5$, $B' = 2$ for fixed wave numbers $x = 0.5$ and $x = 0.8$. For the wave numbers $x = 0.5$ and $x = 0.8$, suspended particles have a destabilizing effect. In figures 2, R_1 is plotted against T_{A_1} for $\lambda = 2$, $P = 0.2$, $S_1 = 10$, $B = 3$, $\varepsilon = 0.5$, $B' = 2$ for fixed wave numbers $x = 0.5$ and $x = 0.8$. This shows that rotation has a stabilizing effect. In figure 3, R_1 is plotted against S_1 for $\lambda = 2$, $P = 0.2$, $T_{A_1} = 5$, $\varepsilon = 0.5$, $B = 3$, $B' = 2$ for fixed wave numbers $x = 0.5$ and $x = 0.8$. This shows that stable solute gradient has a stabilizing effect.

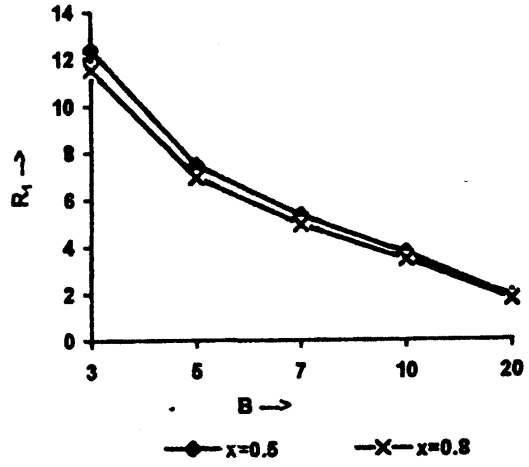

FIG. 1. Variation of Rayleigh number (R_1) with suspended particles (B) for $\lambda = 2$, $S_1 = 10$, $T_{A_1} = 5$, $\epsilon = 0.5$, $P = 0.2$, $B' = 2$ for fixed wave numbers $x = 0.5$ and $x = 0.8$

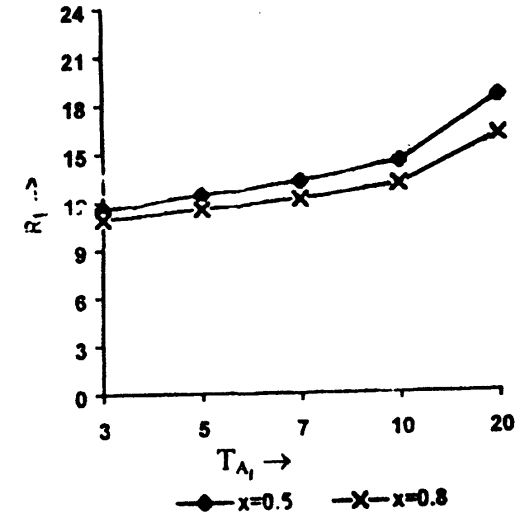

FIG. 2. Variation of Rayleigh number (R_1) with rotation (T_{A_1}) for $\lambda = 2$, $S_1 = 10$, $P = 0.2$, $\epsilon = 0.5$, $B = 3$, $B' = 2$ for fixed wave numbers $x = 0.5$ and $x = 0.8$

In figures 4, R_1 is plotted against P for $\lambda = 2$, $S_1 = 10$, $T_{A_1} = 5$, $\varepsilon = 0.5$, $B = 3$, $B' = 2$ for fixed wave numbers $x = 0.5$ and $x = 0.8$. This shows that medium permeability has a destabilizing effect for $P = 0.1$ to 0.3 and has a stabilizing effect for $P = 0.3$ to 1.0.

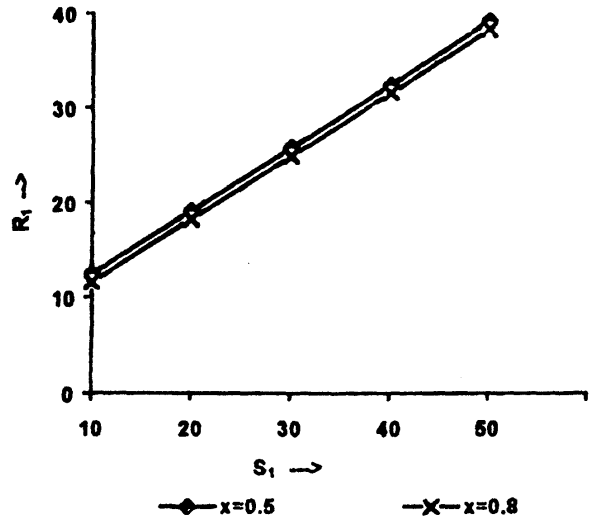

FIG. 3. Variation of Rayleigh number (R_1) with stable solute gradient (S_1) for $\lambda = 2$, $P = 0.2$, $T_{A_1} = 5$, $\epsilon = 0.5$, $B = 3$, $B' = 2$ for fixed wave numbers $x = 0.5$ and $x = 0.8$

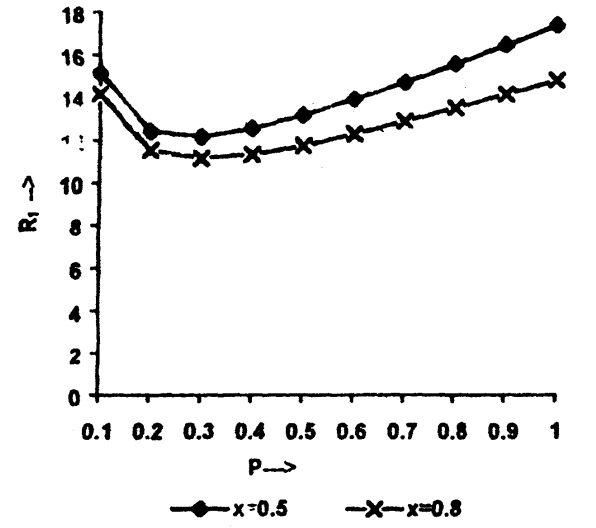

FIG. 4. Variation of Rayleigh number (R_1) with medium permeability (P) for $\lambda = 2$, $S_1 = 10$, $T_{A_1} = 5$, $\epsilon = 0.5$, $B = 3$, $B' = 2$ for fixed wave numbers $x = 0.5$ and $x = 0.8$

CONCLUSIONS

For the stationary convection, the stable solute gradient is found to have stabilizing effect whereas effect of rotation is stabilizing for $\lambda > 0$ and destabilizing for $\lambda < 0$ which is in contrast to the Newtonian fluids. The suspended particles are found to have destabilizing effect on the system as gravity increases upwards from its value g_0 (i.e., for $\lambda > 0$) and stabilizing effect for $\frac{1+x}{\lambda x}\left(\frac{1+x}{P}+\frac{T_{A_1}P}{\epsilon^2}\right) > S_1 B'$ as gravity decreases upwards from its value g_0 (i.e. for $\lambda < 0$) whereas

the medium permeability has destabilizing/stabilizing effect on the system for $\frac{1+x}{P^2} > \frac{T_{A_1}}{\varepsilon^2}/\frac{1+x}{P^2} < \frac{T_{A_1}}{\varepsilon^2}$. The presence of stable solute gradient, rotation, gravity field, suspended particles and viscoelasticity introduce oscillatory modes which were non-existent in their absence. The effects of stable solute gradient, rotation, suspended particles and medium permeability on thermosolutal instability have also been shown graphically.

REFERENCES

1. S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*, Dover Publications, New York, 1981.
2. G. Veronis, *J. marine Res.* **23** (1965) 1.
3. H. Tabor and R. Matz, *Solar Energy* **9** (1965) 177.
4. T. G. L. Shirtcliffe, *J. Geophys. Res.* **69** (1964) 5257.
5. H. Stommel and K. N. Fedorov, *Tellus*, **19** (1967) 306.
6. P. F. Linden, Geophys. *Fluid Dyn.* **6** (1974) 1.
7. M. K. Brikke, *Arch. Biochem. Biophys.* **55** (1955) 175.
8. P. Nason, V. Schumaker, B. Halsall and J. Schweds, *Biopolymers*, **7** (1969) 241.
9. C. R. B. Lister Geophys. *J. Roy astr. Soc.* **261** (1972) 515.
10. J. A. M. McDonnel, John Wiley and Sons, Toronto, 1978, 330.
11. G. K. Pradhan and P. C. Samal, *J. math. Anal. Appl.* **122** (1987) 487.
12. R. C. Sharma and P. Kumar, *Czech. J. Phys.* **47** (1997) 197.