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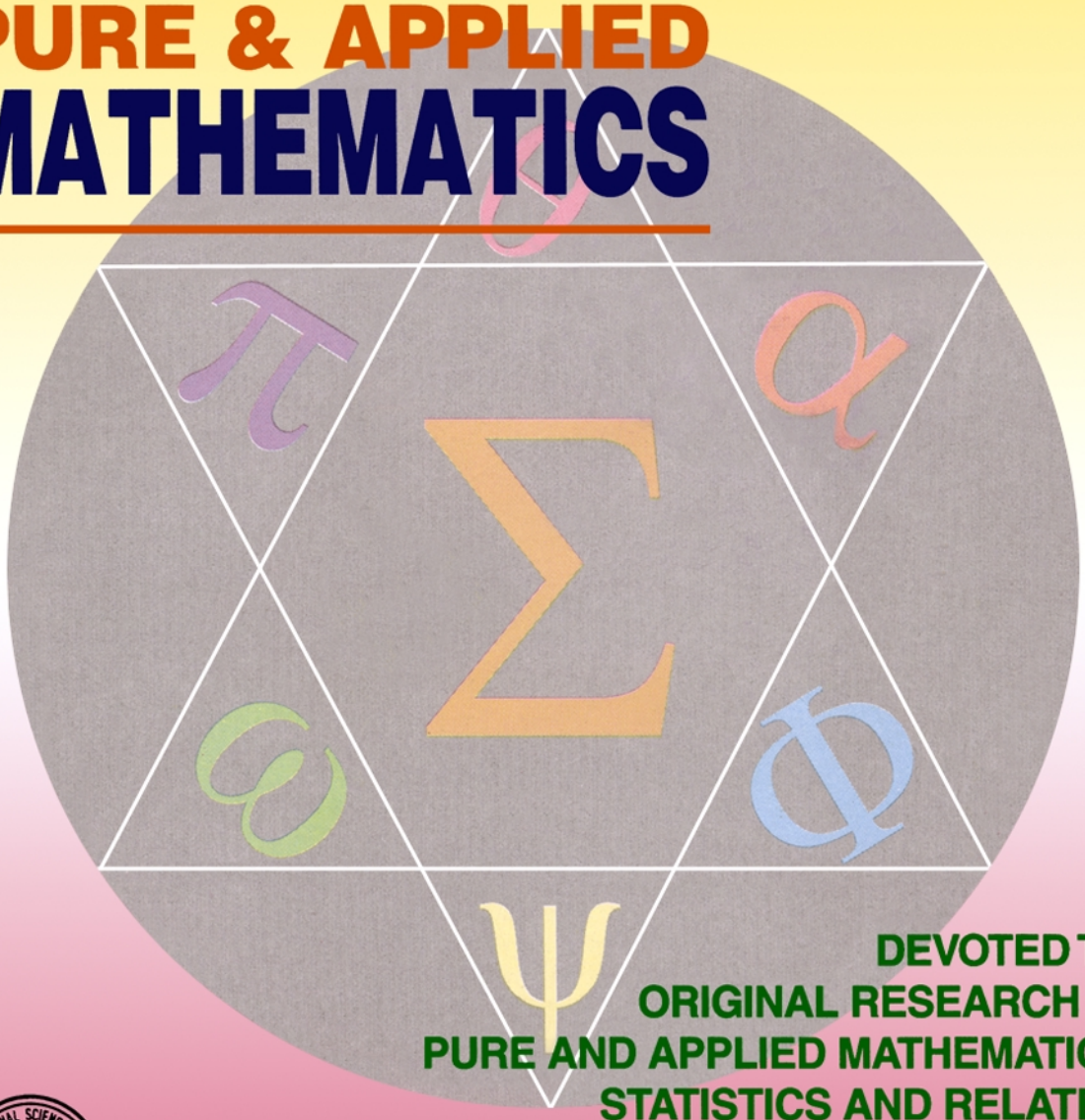

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A STUDY OF LOAD CAACITY OF FINITE SLIDER BEARINGS LUBRICATED WITH COUPLE STRESS FLUIDS CONSIDERING THERMAL EFFECTS

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This paper considers an iterative technique for a theoretical study of thermal effects in finite slider bearings lubricated with couple stress fluid. polymeric lubricants are fluids with couple stresses. The variation of lubricant-viscosity of polymeric lubricant with temperature, raised by the frictionalheat developed by the flow of fluid, is assumed. A three-dimensional model has been considered in order to include side flow along with heat-effect on polymeric lubricants. An expression for a modified Reynolds equation is derived in order to obtain pressure gradient. Analytical expressions for mean temperature, lubricant parameters such as temperature dependent viscosity etc., and load capacity are derived. An iterative technique in a finite difference method of partial differential equations is developed. The variation of lubricant-viscosity and load bearing capacity are calculated numerically by using the iterative technique. Values of load bearing capacity are calculated numerically by using the iterative technique. Values of load bearing capacity for various values of couple stress and thermal parameters, and maximum load capacities for fixed values of couple stress parameter and the length-breath ratio along with fixed thermal parameter, and the corresponding inlet-outlet film ratio have been presented. Assuming Newtonian (in absence of couple stress parameter) and non-Newtonian (in presence of couple stress parameter) behaviour of lubricants, a comparative study of load capacities and maximum load capacities for finite and infinite slider bearings for these two cases have been analyzed. It is observed that the maximum load capacity depends jointly on couple stress parameter, the corresponding inlet-outlet film height ratio, the length-breath ratio and other temperature dependent parameters. The analysis also indicates that the effects of temperature, where there is a side-flow of lubricants, are significant.

Key Words : Couple Stress Fluid; Temperature Effects; Mean Temperature; Side-flow; Load Capacity; Maximum Load Capacity; Inlet-outlet Film Height Ratio; Length-Breath Ratio

Nomenclature

a, A	width of the plate, inlet-outlet film height ratio
A_{wmax}	inlet-outlet film ratio corresponding to maximum load capacity
B	non-dimensional coefficient of temperature
c_p	specific heat
d	length of the plate
E	Eckert number
h, H, h_0	dimensional, non-dimensional film height, least film height
$H_{i,j}$	values of H at ijth node on X-Z plane

k	thermal conductivity
L	non-dimensional couple stress parameter
M	non-dimensional viscosity coefficient
p, P	dimensional, non-dimensional fluid pressure
$P_{i,j}$	values of P at ijth node on X-Z plane
Pr	Prandtl number
(r), $(r+1)$	previous,current iteration levels
s, S	dimensional, non-dimensional defined values
S_{ij}	values of S at ijth node on X-Z plane
t, T	dimensional, non-dimensional mean temperature
t_0	temperature at $p = 0$ (i.e., ambient temperature)
u,v,w	dimensional fluid velocity components along x-, y-, z-directions
U,V,W	non-dimensional velocity components along x-, y-, z-directions
u_0	uniform sliding velocity of lower plate
v_0, V_0	dimensional, non-dimensional, squeezing velocity of upper plate
W,W_{max}	non-dimensionl load bearing capacity, maximum value of W
x,y,z	cartesian coordinates in the flow region
X,Y,Z	non-dimensional cartesian coordinates
$\Delta X, \Delta Z$	increments on X-, Z-directions in the fluid region
β	coefficient of temperature
η	couple stress parameter
λ	length-breadth ratio of the plate
μ, μ_0	coefficient of viscosity,μ at $p = 0$

1. INTRODUCTION

The lubrication behaviours of different Newtonian and non-Newtonian fluids have been examined and analyzed by many researchers. It is observed that, in many appliances of modern industrial technology, the use of non-Newtonian fluids as lubricants is more important than that of Newtonian fluids. The lubricants with stable suspensions of fine particles of insoluble solids, which have different material characters and can be used as the viscosity-index improver in order to prevent the viscosity variation with temperature, exhibit non-Newtonian behaviour. The interactions of such suspended particles in the flow of fluid have been referred to as couple stresses. A number of theories of micro-continuum have been developed to explain the behaviours of these non-Newtonian polymeric fluids[1, 2]. A special theory of fluids proposed by Stokes[3] has been applied to explain the effect of couple stresses in fluids, known as couple stress theory of fluids. Many papers have been published considering this couple stress theory to explain theoretically the effects of couple stresses in various journals and slider bearings. Bujurke and Jayaraman[4] used the couple stress theory to explain the effects of couple stresses in a squeeze film synovial-joints. Ramanaiah and Sarkar[5] and Ramanaiah[6] have also considered the couple stress fluid model to explain the effect of couple stresses on squeeze film thrust bearing. A similar analysis dealing with load capacity, frictional force etc. has been

considered by Sinha and Singh[7], Bujurke and Naduvinami[8] for rolling bearing and Lin[9] for finite journal bearings. A comparative study of various lubricating factors for different forms of slider bearings lubricated with couple stress fluid has been considered by Das and Bhattacharjee[10]. All such analyses have been considered under isothermal consideration.

For a good understanding of the frictional behaviour on different lubricating factors such as viscosity, load capacity etc., the effect of temperature produced by the flow of viscous fluids as well as bearing motion is to be considered. So, the lubricating factors in lubrication technology are assumed to be functions of temperature. Some papers presenting the effect of temperature in some special cases have been published. Sadeghi *et al.*[11] considered a study of thermal effects in two-dimensional rolling/sliding contacts lubricated with incompressible Newtonian fluids. Johnson and Greenwood[12] have considered the Eyring fluid model for analyzing the thermal effects in EHD traction. Considering power law fluid model the thermal effects have been analyzed by Prasad *et al.* for rollers without cavitation[13] and with cavitation[14] and by Shukla and Isa[15] for externally pressurized bearing with squeeze films.

An analysis of the maximum load capacity has been made by Das[16], considering power law fluids as lubricants in the absence of any body force. A similar analysis in presence of magnetic field has been presented by Das[17], considering couple stress fluid model and side-flow. But we have found no theoretical analysis, considering thermal effects, for couple stress fluid used as lubricant. The present analysis deals with thermal effects in finite slider bearings lubricated with couple stress fluid model proposed by Stokes'. In order to present the joint effects of temperature and side-flow of polymeric lubricant, variation of viscosity with temperature raised by the frictional heat produced by the flow of fluids with couple stresses has also been considered. For numerical calculations, an iterative scheme in a finite difference method of partial differential equations is developed. The variations of lubricating factors such as couple stress parameter, inlet-outlet film-height ratio and length-breadth ratio are analysed numerically by using the iterative technique developed. A comparative study of load capacities for various values of couple stress and bearing parameters, and maximum load capacities of a finite and an infinite slider bearings has been presented, considering the present non-Newtonian fluid and Newtonian fluid models as lubricants in both thermal and isothermal conditions. A comparison of present results with existing available results is also made. The comparison reveals the validity of the numerical technique developed for thermal analysis.

2. EQUATION OF MOTION

The physical configuration in our problem is a three-dimensional finite slider bearing shown in Fig. 1. It has two spaced surfaces, the lower one is a straight rigid surface moving with uniform sliding velocity u_0 and the upper one is rigid in general form with finite length d and width a, squeezing down with velocity v_0. A steady and incompressible flow of couple stress fluid is considered between the surfaces in the form of film of thickness $h(x)$. We assume the thermal flow of the lubricants and non-thermal property of the surfaces. The basic equations under above assumptions in the simplified form are :

Continuity equation :

$$(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z) = 0 \qquad \ldots (1)$$

Momentum equation :

$$\partial p/\partial x = \mu (\partial^2 u/\partial y^2) - \eta (\partial^4 u/\partial y^4) \qquad \ldots (2)$$

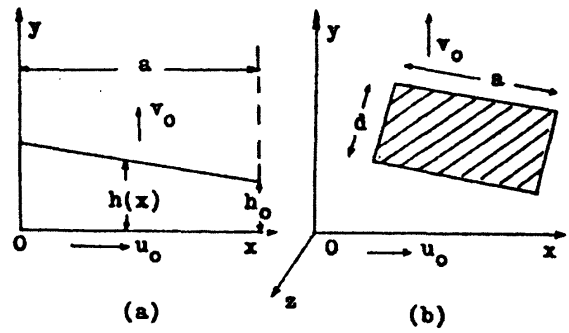

FIG. 1. Physical configuration of finite slider bearings : (a) Normal section, (b) Three dimensional

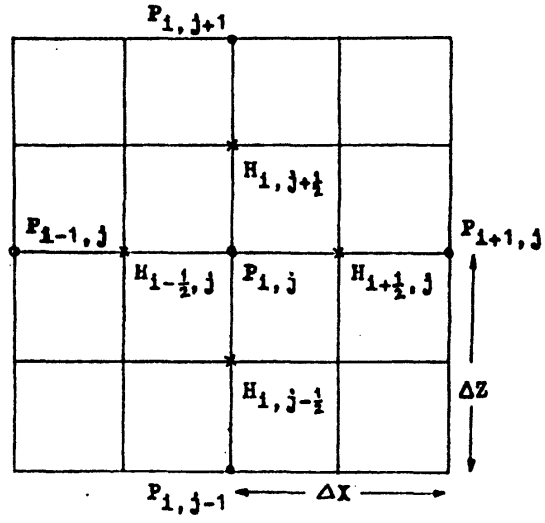

FIG. 2. Grid point notation for the film region in the X-Z plane

$$\partial p/\partial y = 0,\ \partial p/\partial z = \mu\,(\partial^2 w/\partial y^2) - \eta\,(\partial^4 w/\partial y^4) \qquad \ldots (3\ \&\ 4)$$

Energy equation :

$$k\,(\partial^2 t/\partial y^2) = -\,\mu\,[(\partial u/\partial y)^2 + (\partial w/\partial y)^2]$$

$$+\ \eta\,[(\partial u/\partial y)\,(\partial^3 u/\partial y^3) + (\partial w/\partial y)\,(\partial^3 w/\partial y^3)] \qquad \ldots (5)$$

where u, v, w are the velocity components of the fluid, μ, η, $p\,(x, z)$, k and t are respectively viscosity coefficient, material constant of couple stress, pressure, thermal conductivity and temperature of the fluid.

We have considered the variation of viscosity μ with temperature due to the frictional heat generated by the flow of fluid. Further, the relation for temperature-dependent μ (pressure independent) is defined as follows[13]:

$$\mu = \mu_0 \exp\,[-\beta\,(t_m - t_0)] \qquad \ldots (6)$$

where μ_0 and t_0 are respectively the viscosity coefficient and temperature when $p = 0$ (ambient pressure). β is the temperature viscosity coefficient and t_m, the mean temperature across the film thickness h, is defined as

$$t_m = (1/h) \int_0^h t \, dy \qquad \ldots (7)$$

The boundary conditions for velocity components, pressure and temperature in the lubricant region are :

$$u = u_0, \; v = 0, \; w = 0 \text{ at } y = 0; \; u = 0, \; v = v_0, \; w = 0 \text{ at } y = h \qquad \ldots (8)$$

$$p = 0 \text{ at } x = 0, \; a \text{ and at } z = 0, \; d \qquad \ldots (9)$$

$$t = t_0 \text{ at } y = 0 \text{ and } y = h \qquad \ldots (10)$$

No-couple stress conditions:

$$\partial^2 u/\partial y^2 = \partial^2 w/\partial y^2 = 0 \text{ at } y = 0 \text{ and } y = h(x) \qquad \ldots (11)$$

3. SOLUTIONS OF EQUATION

Since μ defined in eq. (6) is function of x and y only, the solution of u and w can be obtained from eqns. (2) and (4) with the boundary conditions (8) and (9) and no-couple stress condition (11) in the forms :

$$u = u_0 (1 - y/h) + [(\partial p/\partial x)/(2 \mu)] \cdot [y (y - h) + (2 \eta/\mu) \{1 - \cosh \{(2y - h)/ 2 (\eta/\mu)^{1/2}\} / \cosh \{h/2 (\eta/\mu)^{1/2}\}\}] \qquad \ldots (12)$$

and

$$w = [(\partial p/\partial z)/(2 \mu)] \cdot [y (y - h) + (2 \eta/\mu) \{1 - \cosh \{(2y - h) / 2 (\eta/\mu)^{1/2}\}/ \cosh \{h/2 (\eta/\mu)^{1/2}\}\}] \qquad \ldots (13)$$

We integrate the continuity eq. (1) with respect to y with the boundary condition of v in (8) and with the help of the principle of integration under differentiation. The modified Reynolds equation is obtained :

$$(\partial/\partial x) [s (\partial p/\partial x)] + (\partial/\partial z) [s (\partial p/\partial z)] = 6 \mu u_0 (dh/dx) \qquad \ldots (14)$$

where

$$s = h^3 - 12 \eta [h - 2 (\eta/\mu)^{1/2} \tanh \{h/2 (\eta/\mu)^{1/2}\}]/\mu \qquad \ldots (15)$$

We introduce the following dimensionless quantities :

$$X = x/a, \; Y = y/h_0, \; Z = z/d, \; H = h/h_0, \; M = \mu/\mu_0, \; L = (\eta/\mu_0)^{1/2}/h_0, \; U = u/u_0$$

$$V_0 = v_0\, a/(h_0\, u_0),\ W = wa/u_0\, d,\ P = ph_0^2/(\mu_0\, u_0\, a),\ T = t/t_0,$$

$$T_m = t_m/t_0,\ Pr = \mu_0\, c_p/k,$$

$$E = u_0^2/(c_p\, t_0),\ B = \beta\, t_0,\ S = Ss/h_0^3,\ \lambda = d/a,$$

where h_0 is the minimum film thickness. The non-dimensional form of eqs. (5) to (7) and boundary conditions (9) and (10) can respectively be rewritten as

$$(\partial^2 T/\partial Y^2) = Pr.\, E\,[\{-M\,(\partial U/\partial Y)^2 + L^2\,(\partial U/\partial Y)\,(\partial^3 U/\partial Y^3)\}$$

$$+\ \lambda^2\{-M\,(\partial W/\partial Y)^2 + L^2\,(\partial W/\partial Y)\,(\partial^3 W/\partial Y^3)\}] \qquad \ldots (16)$$

$$M = \exp\ \{-B\,(T_m - 1)\} \qquad \ldots (17)$$

$$T_m = (1/H)\int_0^H T\,dY \qquad \ldots (18)$$

$$P = 0 \text{ at } x = 0,\ 1 \text{ and } Z = 0,\ 1 \qquad \ldots (19)$$

$$T = 1 \text{ at } Y = 0 \text{ and } Y = H. \qquad \ldots (20)$$

Eqs. (12)-(15) also reduce to

$$U = (\partial P/\partial X)\,[Y^2 - HY + (2L^2/M)\ \{1 - \cosh$$

$$(2Y - H)\,M^{1/2}/2L\ \}\ /\ \cosh\ \{HM^{1/2}/2L\}\}]/\ 2M + (1 - Y/H) \qquad \ldots (21)$$

$$W = (1/\lambda^2)\,(\partial P\,.\,\partial Z)\,[Y^2 - HY + (2L^2/M)$$

$$\{1 - \cosh\ \{(2Y - H)\,M^{1/2}/2L\}\ /\ \cosh\ \{HM^{1/2}/2L\}\}\ /\ 2M \qquad \ldots (22)$$

$$(\partial/\partial X)\,(S\,\partial P/\partial X) + (1/\lambda^2)\,(\partial/\partial Z)\,(S\,\partial P/\partial Z) = 6M\,(dH/dX) \qquad \ldots (23)$$

$$S = H^3 - 12\,L^2\,\{H - (2L/M^{1/2})\tanh\,(HM^{1/2}/2L)\}/M \qquad \ldots (24)$$

Since the temperature-dependent viscosity coefficient M is a function of T, so the solution for T can be obtained from eq. (16) by using eqs. (21), (22) and boundary condition (20) for T. Using this value of T in eq. (18), we obtain the value of T_m in the form :

$$T_m = 1 + Pr.\, E\ [\{(\partial p/\partial x)^2 + (1/\lambda^2)\,(\partial p/\partial z)^2\}\ \{(H^4/240\,M) + L^3/2M^3\ \Phi$$

$$\times \tanh\,(HM^{1/2}/2L\} + M/12] \text{ where } \Phi = 6L - (12\,L^2 + MH^2)/(M^{1/2}\,H) \qquad \ldots (25)$$

As a particular case, we have taken film thickness $H(X)$ of inclined slider bearing i.e.,

$$H(X) = A - (A-1)X, 0 \leqq X \leqq 1 \quad \ldots (26)$$

4. NUMERICAL CALCULATIONS

The dimensionless modified Reynolds equation is solved for pressure P numerically using a finite difference method in partial differential equations.[18]. The film region is divided by the grid spacing as shown in Figure 2, and eq. (23) is discretized as :

$$[\{S_{i+1/2,j}(P_{i+1,j}-P_{i,j})/\Delta X\} - \{S_{i-1/2,j}(P_{i,j}-P_{i-1,j})/\Delta X\}] / \Delta X$$

$$+ (1/\lambda^2)[\{S_{i,j+1/2}(P_{i,j+1}-P_{i,j})/\Delta Z\} - \{S_{i,j-1/2}(P_{i,j}-P_{i,j-1})/\Delta Z\}] / \Delta Z$$

$$= 6M_{i+1/2,j}(H_{i+1/2,j}-H_{i-1/2,j})/\Delta X. \quad \ldots (27)$$

Eq. (27) results in $m \times n$ simultaneous equations for $m \times n$ points in the film region. These equations are solved numerically assuming $P = 0$ at the boundaries. In this paper, 50 divisions (i.e., $\Delta X = \Delta Z = 0.02$) have been taken in each unit length in both X- and Z-directions. The film pressures have been solved numerically using the conjugate gradient method of iteration.

The initial values for first iteration is assumed as $T_m = 1$, which suggests $M = 1$ [from eq. (17)]. Using these values of T_m and M in eq. (24) we obtain S. Using S in eq. (27), we obtained P at each nodal point. These values of P are referred as values at zero iteration i.e., $(r) = 0$. We have discretized the relations for T_m, M, S in eqs. (25), (17), (24) and expression in eq. (27) in terms of iteration level in the following superscriptial notations :

$$T_m^{r+1} = 1 + Pr.E\ [\{\{(P_{i+1,j}^{(r)} - P_{i,j}^{(r)})/\Delta X\}^2]\ (1/\lambda^2)\ \{P_{i,j+1}^{(r)} - P_{i,j}^{(r)}/\Delta Z\}^2\}$$

$$\times \{(H^4/240\,M^{(r)}) + \Phi\ \{L^3/2(M^{(r)})^3\}\ \tanh\ \{H(M^{(r)})^{1/2}/2L\}\} + M^{(r)}/12] \quad \ldots (28)$$

where $\Phi = 6L - (12L^2 + M^{(r)}H^2)/\{M^{(r)})^{1/2}H\}$

$$M^{(r+1)} = \exp\ \{-B(T_m^{(r+1)} - 1)\} \quad \ldots (29)$$

$$S^{(r+1)} = H^3 - 12L^2/[H - \{2L/(M^{(r+1)})^{1/2}\}$$

$$\tanh\ \{H(M^{(r+1)\,1/2})/2L\}] / M^{(r+1)} \quad \ldots (30)$$

and $[\{S_{i+1/2,j}^{(r+1)}(P_{i+1,j}^{(r+1)} - P_{i,j}^{(r)})/\Delta X\} - \{S_{i-1/2,j}^{(r+1)}(P_{i,j}^{(r)} - P_{i-1,j}^{(r)})/\Delta X\}]/\ \Delta X$

$$+ (1/\lambda^2)\ [\{S_{i,j+1/2}^{(r+1)}(P_{i,j+1}^{(r} + 1) - P_{i,j}^{(r)})/\Delta Z\}$$

$$- \{S_{i,j-1/2}^{(r+1)}(P_{i,j}^{(r)} - P_{i,j-1}^{(r)})/\Delta Z\}]/Dz$$

$$6\, M_{i+1/2j}^{(r+1)} (H_{i+1/2,j} - H_{i-1/2,j})/\Delta X \qquad \ldots (31)$$

where (r) and $(r + 1)$ are respectively previous and current iteration levels. Using the initial values (i.e., when $(r) = 0$) of M, T_m, S and P in eqs. (28) - (31), we have obtained P at different nodal points at various iteration levels. Reaching to the iteration levels $(r) = 4$ and $(r) = 5$, we have obtained that the values in both levels are almost identical. Thus $(r) = 5$ is the last iteration level in our consideration.

For numerical calculation, the following values or wide ranges of values of dimensionless quantities have been used :

$$B = 0.7,\ V_0 = 0,\ 0 \leqq Pr.E \leqq 1,\ 1 \leqq A \leqq 2,\ 0.5 \leqq \lambda < \infty \text{ and } 0 \leqq L \leqq 10.$$

Dimensionless load capacity W is calculated from the equation as follows :

$$W = \int_0^1 \int_0^1 P\, dX\, dZ = \Delta X\, \Delta Z\, \Sigma_i\, \Sigma_j\, P_{i,j} \qquad \ldots (32)$$

5. RESULTS AND DISCUSSION

Dimensionless numerical values of load capacity W for various values of inlet-outlet film height ratio A, dimensionless maximum load capacity W_{max} and inlet-outlet film height ratio corresponding to maximum load A_{wmax} for dimensionless couple stress parameter L, length-breath ratio λ and thermal parameter Pr.E have been presented graphically and numerically.

Fig. 3 presents W vs. A for two values of Pr.E and three values of λ (corresponding to three forms of finite slider bearings). The figure shows that for each pair of λ and Pr.E, W first increases rapidly as A increases from unity and then reduces slowly, attaining a maximum value

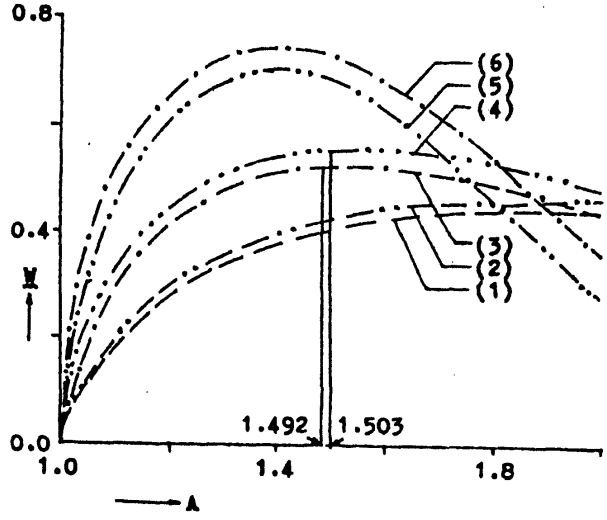

FIG. 3. W against A for $L = 3$ and for Pr.E = 0.3, [(1) $\lambda = 0.8$, (3) $\lambda = 1.0$, (5) $\lambda = 1.2$; Pr.E = 0.0, [(2) $\lambda = 0.8$, (4) $\lambda = 1.0$, (6) $\lambda = 1.2$]

(W_{max}) at a particular value of A (A_{wmax}). The figure also reveals that W increases with the increase of λ but decreases with the increase of Pr.E i.e., the effect of temperature has noticeable influence on load capacity of a longer slider bearing. The figure also shows that the position of A_{wmax} shifts towards the inlet region with the increase of Pr.E. A_{wmax} = 1.503 for curve (4) with Pr.E = 0 and A_{wmax} = 1.492 for curve (3) with Pr.E = 0.3, both for fixed λ = 1.

Fig. 4 represents a comparative study of load capacity W against L for two values of Pr.E and three values of λ (where $\lambda \to \infty$ corresponds to infinitely long slider bearing and λ = 0.8, 1.2

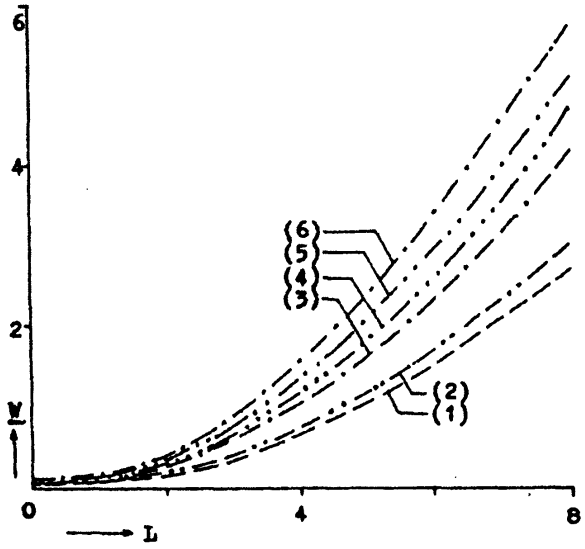

FIG. 4. W against L for A = 1.6 and for Pr.E = 0.3, [(1) λ = 0.8, (3) λ =1.2, (5) $\lambda \to \infty$]; Pr.E = 0.0, [(2) λ = 0.8, (4) λ = 1.2, (6) $\lambda \to \infty$]

correspond to finite slider bearings). The figure shows that W increases with the increase of both L and/or λ. The figure also reveals a realistic case that the decrease of load capacity W due to thermal effect is more noticeable when the plate is longer and/or L assumes larger values. The figure further

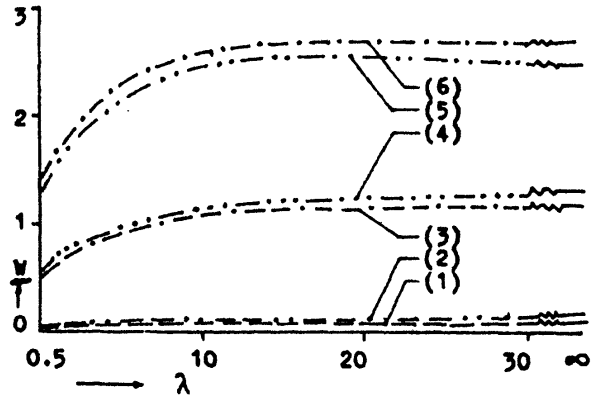

FIG. 5. W against λ for A = 1.4 and for Pr.E = 0.3, [(1) L = 0, (3) L = 3, (5) L = 6]; Pr.E = 0.0, [(2) L = 0, (4) L = 3, (6) L = 6]

infers that couple stress effect enhances the load capacity even when the thermal effects have been considered.

In Fig. 5, we have presented W vs. λ for two values of Pr.E and three values of L. The figure indicates that W increases with the increase of λ for given values of Pr.E and L. But this increment is higher with larger value of L and lower with that of Pr.E. The figure also reveals that the increasing rate of W for fixed values of L and Pr.E decreases even for larger values of λ. In this case, $\lambda = 500$ is the consideration of $\lambda \to \infty$. The values of W change significantly with the lower values of λ. The values in the Newtonian case, in the absence of thermal parameter (L = Pr.E = 0) are found to be identical with those given by Gross et al.[19], under similar conditions for slider bearing with the variation of λ. These results confirm the validity of the numerical technique in consideration.

Fig. 6 presents A_{wmax} vs. L for two values of Pr.E and three values of λ. The figure displays that A_{wmax} decreases with the increase of L and the decreasing rate is larger for lower values of L. The figure also reveals that A_{wmax} is diminished with the increase of Pr.E or λ. This variation is higher for higher values of L. The figure also indicates that the value of A_{wmax} for finite inclined slider bearing (λ = 1) in the isothermal Newtonian case (L = Pr.E = 0) is 2.512. Whereas, under the similar circumstances, this value even for step slider bearing, as presented in the work of Das[17], is 2.431. This comparison suggests that values of A_{wmax} are reliable.

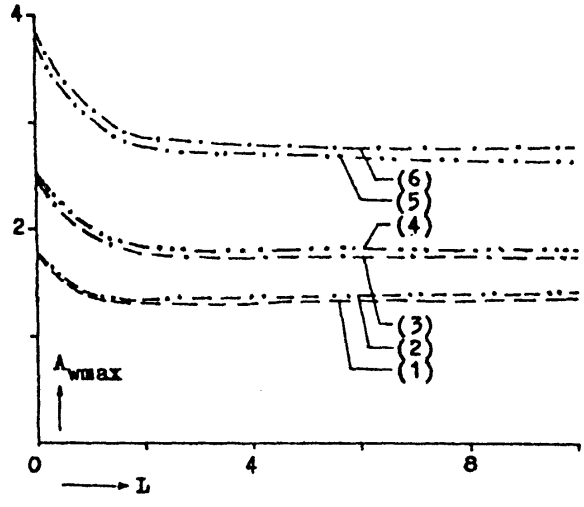

FIG. 6. A_{wmax} against L for Pr.E = 0.3, [(1) λ = 1.2, (3) λ = 1.0, (5) λ = 0.8]; Pr.E = 0.0, [(2) λ = 1.2, (4) λ = 1.0, (6) λ = 0.8]

Fig. 7 represents W_{max} vs. L for two values of Pr.E and three values of λ. In this figure, it is observed that W_{max} increases with the increase of L. But the increasing rate is intensified with the increase of λ but the increasing rate is diminished by the increase of Pr.E. In this figure, the value of W_{max} is 0.352, when L =1, Pr.E = 0 and $\lambda \to \infty$ and it is interesting to note that the value is almost identical with that given by Das[17], under similar conditions.

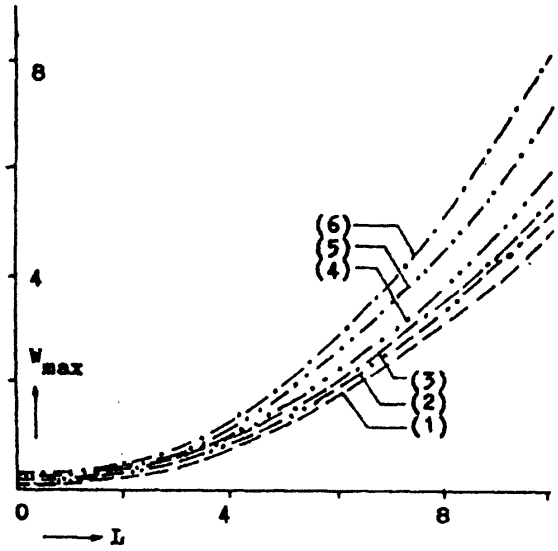

FIG. 7. W_{max} against L for Pr.E = 0.3, [(1) $\lambda = 0.8$, (3) $\lambda = 1.0$, (5) $\lambda \to \infty$]; Pr.E = 0.0, [(2) $\lambda = 0.8$, (4) $\lambda = 1.0$, (6) $\lambda \to \infty$]

In Table 1, we have presented W_{max} vs. pr.E for two values of L and three values of λ. In this table we observe that W_{max} decreases with the increase of Pr.E, but increases with the increase of L and/or λ. Further, the increasing rate is comparatively larger for higher values of L and/or λ. The close scrutiny infers the beneficial effect of the presence of couple stress parameter even when the thermal effect on maximum load capacity has been considered.

TABLE 1 : W_{max} against Pr.E for two values of L and for three values of λ.

	$L = 2$			$L = 6$		
Pr.E	$\lambda = 0.8$			$\lambda = 1.0$	$\lambda = 1.2$	
0.0	0.2262	0.2517	0.3397	1.9434	2.1846	2.9339
0.2	0.2078	0.2257	0.3136	1.9045	2.1492	2.8202
0.4	0.1878	0.2012	0.2995	1.8621	2.1202	2.7315
0.6	0.1624	0.1815	0.2802	1.8102	1.9821	2.6233
0.8	0.1488	0.1656	0.2636	1.7723	1.9386	2.5325
1.0	0.1332	0.1539	0.2486	1.7332	1.9032	2.4133

6. CONCLUSIONS

Based on a special micro-continuum theory, the generalized Reynolds equation for thermal analysis is derived, and applied to investigate the performance of finite slider bearings lubricated with couple stress fluids. From this study the following conclusions have been drawn :

1. The numerical technique developed can be applied to examine thermal effects on polymeric lubricants.

2. Load capacity, maximum load capacity and inlet-outlet film height ratio corresponding to maximum load capacity all depend on couple stress parameter, length-breadth ratio of the slider plate and frictional temperature factor conjointly.

3. For both types of slider bearing (finite and infinite), maximum load capacity increase with the increase of couple stress parameter and decrease with the increase of temperature factor.

4. The value of inlet-outlet film ratio corresponding to maximum load capacity, decreases with the increase of couple stress parameter or temperature parameter or length-breath ratio of the slider plate.

5. The maximum load capacity increases with the increase of length-breadth ratio of the slider plate but decrease with the increase of frictional temperature factor. This decrement in load capacity due to temperature parameter is greater when the length-breath ratio of the slider plate is infinite.

The over all study of this analysis reveals that many important lubricating factors such as maximum load capacity etc. are temperature dependent.

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