Indian J. pure appl. Math., 25 (6) : 655-664, June 1994

MAGNETOHYDRODYNAMIC UNSTEADY FLOW OF A NON-NEWTONIAN FLUID PAST AN INFINITE POROUS PLATE

MAGDY A. EZZAT

Department of Mathematics, Faculty of Science, University of Alexandria
Alexandria, Egypt

In this work we study the flow of an unsteady incompressible electrically conducting viscous non-Newtonian power-law fluid past a hot infinite porous flat plate with a periodically alternating suction-injection of the fluid in the presence of a transverse uniform magnetic field.

Solution for the velocity and temperature distributions are obtained.

The effects of various parameters such as the flow index of the power-law fluid, the frequency of oscillations, the parameter suction-injection and the magnetic filed parameter on the flow are studied.

Numerical computations are carried out and represented graphically.

INTRODUCTION

Non-Newtonian fluids are of increasing importance in modern technology. This was probably caused by the growing use of non-Netonian fluids in many activities such as molten plastics, paints, drilling of petroleum, and polymer solutions.

The boundary layer concept in the theory of non-Newtonian fluids is relevant to a number of engineering activities, among which may be cited the possibility of reducing frictional drag on bearings and on immersed bodies such as ship hulls and submarines.

The boundary layer concept in the theory of non-Newtonian power-law fluids was introduced by Schwolter[1].

Acrivos[2] was apparently the first to investigate the natural convection behavior of non-Newtonian fluid from a body with an isothermal surface.

The steady flow for such fluids over a plate was studied by Acrivos, Shan and Petersen[3,4]. The steady laminar flow of a conducting power-law fluid in the presence of a magnetic field has been discussed in a number of papers[5-7]. The methods of similarity solutions have been used to obtain exact solutions of the boundary layer equations[8].

The problem of unsteady displacement flow of a conducting power-law fluid in the presence of a magnetic field has been studied by Martinson[9]. He also studied the behaviour of magnetohydrodynamic unsteady flow of a power-law fluid over a moving plate with variable injection[10].

Our aim in the present work to study the flow of a power-law fluid past an infinite porous plate whose temperature and the suction-injection velocity at the plate fluctuates harmonically with time about a constant mean. The free stream velocity oscillates in magnitude but not in direction. All the fluctuations have the same frequency. A magnetic field of uniform strength is applied transversely to the plate. The solution is obtained using a method suggested by Lighthill[11] and Stuart[12].

FORMULATION OF THE PROBLEM

Let us consider the flow of a power-law fluid of density ρ, consistency k, flow index n and electrical conductivity σ, occupying a semi-infinite region of space bounded by a porous, infinite plate. The velocity components of the fluid are u and v in the x and y directions respectively, taken parallel and perpendicular to the plate. Since the plate is of infinite extent, the variables of the problem depend upon y and the time t only. A constant magnetic field of strength H_0 is applied in the positive y-direction. Assuming the magnetic Reynolds number ($R_m = VL\,\sigma\,\mu$) to be very small, which is the case in most applications, the induced magnetic field may be neglected as in Rao[13] and El-Dabe[14]. In the above expression for R_m, V is a characteristic velocity of the fluid, L the characteristic velocity length and μ the magnetic permeability of the fluid. Under the above assumptions the magnetic induction has one non-vanishing component $B_y = \mu H_0 = B_0$ (constant), while the pendermotive force has one non-vanishing component in x-direction given by $F_x = -\sigma B_0^2 u/\rho$. Also in the equation of energy we include viscous dissipation but neglect Joule's dissipation. With these assumptions the boundary-layer equations describing the two-dimensional flow of an electrically conducting power-law fluid past an infinite porous plate in the presence of a transverse magnetic field are given by (Saponkoff[8])

$$\frac{\partial v}{\partial y} = 0 \qquad \text{... (1)}$$

$$\rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + k\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) - \sigma B_0^2 u \qquad \text{... (2)}$$

$$\rho\, C\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = \lambda\frac{\partial^2 T}{\partial y^2} + k\left|\frac{\partial u}{\partial y}\right|^{n+1}. \qquad \text{... (3)}$$

In the above equations p is the pressure, C the specific heat of the fluid, λ the thermal conductivity, B_0 the component of the electromagnetic induction and T the temperature in the boundary layer. The boundary conditions of the problem are taken as (Raptis[15])

$$u = 0,\ T = T_m(1 + \varepsilon\, e^{i\omega t}) - T_\infty\, \varepsilon\, e^{i\omega t} \text{ at } y = 0$$

$$u = U(t),\ T \to T_\infty \quad \text{as} \quad y \to \infty \qquad \text{... (4)}$$

where T_m is the mean value about which the temeprature fluctuates, T_∞ the temperature of the free stream, and ω is the frequency of the fluctuation. We shall assume that $\varepsilon \ll 1$.

Applying eqn. (2) to the main stream, we get

$$\rho \frac{dU}{dt} = -\frac{\partial p}{\partial x} - \sigma B_0^2 U. \quad \text{... (5)}$$

In view of (5), we get from (2)

$$\rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) = -\frac{dU}{dt} + k\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) + \sigma B_0^2 (U-u). \quad \text{... (6)}$$

Using the non-dimensional variables

$$x^* = \frac{x}{L},\quad y^* = \frac{Y}{L}\mathrm{Re}^{1/(n+1)},\quad u^* = \frac{u}{U_0}$$

$$t^* = \frac{U_0 t}{L},\quad U^* = \frac{U}{U_0},\quad V^* = \frac{V}{U_0}\mathrm{Re}^{1/(n+1)}$$

$$\omega^* = \frac{L\omega}{U_0},\quad M^* = \frac{\sigma B_0^2 L}{\rho U_0},\quad P_r = \frac{U_0 L}{\lambda}\mathrm{Re}^{-2/(n+1)} \quad \text{... (7)}$$

$$E = \frac{U_0^2}{(T_m - T_\infty)C},\quad \theta = \frac{T - T_\infty}{T_m - T_\infty}$$

where $\mathrm{Re} = \rho U_0^{2-n} L^n / k$ is Reynolds number, P_r the Prandtl number, E the Eckert number, M the magnetic parameter and U_0 is the mean velocity of the free stream.

Equations (1), (6) and (3) take the following from (dropping the asterisks for convenience)

$$\frac{\partial v}{\partial y} = 0 \quad \text{... (8)}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{dU}{dt} + \frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) + M(U-u) \quad \text{... (9)}$$

$$\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + E\left|\frac{\partial u}{\partial y}\right|^{n+1} \quad \text{... (10)}$$

Boundary conidtions (4), now reduce to

$$u = 0,\quad \theta = (1 + \varepsilon e^{i\omega t}) \quad \text{at} \quad y = 0$$

$$u = U(t),\quad \theta \to 0 \quad \text{as} \quad y \to \infty. \quad \text{... (11)}$$

SOLUTION OF THE PROBLEM

Integrating (8), we get

$$v = -\varepsilon A e^{i\omega t} \qquad \text{... (12)}$$

where A is the suction-injection parameter and from now on we shall consider a free stream of the form

$$U(t) = 1 + \varepsilon e^{i\omega t}. \qquad \text{... (13)}$$

In order to solve (9) and (10) subject to boundary conditions (11) in the neighbourhood of the plate, we take (Rao[13])

$$u(y, t) = u_1(y) + \varepsilon\, e^{i\omega t} u_2(y) \qquad \text{... (14)}$$

$$\theta(y, t) = \theta_1(y) + \varepsilon\, e^{i\omega t} \theta_2(y). \qquad \text{... (15)}$$

Substituting (12), (13), (14) and (15) into (9) and (10) and comparing the harmonic terms, we get

$$n\, u_1'' \, |u_1'|^{n-1} + M(1 - u_1) = 0 \qquad \text{... (16)}$$

$$n\, |u_1'|^{n-1} u_2'' + n(n-1)\, |u_1'|^{n-2} u_1'' u_2' + (M + iw)(1 - u_2) = -A u_1', \qquad \text{... (17)}$$

$$\theta_1'' = -P_r E\, |u_1'|^{n+1} \qquad \text{... (18)}$$

and

$$\theta_2'' - iwP_r\, \theta_2 = -A\theta_1' - P_r E\,(n+1)\, |u_1'|^{n+1} u_2' \qquad \text{... (19)}$$

where primes now denote differentiation with respect to y.

The boundary conditions on u_1, u_2, θ_1 and θ_2 become

$$u_1 = u_2 = 0, \quad \theta_1 = \theta_2 = 1 \quad \text{at} \quad y = 0 \qquad \text{... (20)}$$

$$u_1 \to 1, \quad u_2 \to 1, \quad \theta_1 \to 0, \quad \theta_2 \to 0 \quad \text{as } y \to \infty.$$

We shall now consider the following cases :

(I) *Pseudoplastic Fluids* ($n < 1$)

Solving eqns. (16) and (17) subject to conditions (2), we get

$$u_1 = 1 - (1 + sy)^{\frac{n+1}{n-1}} \qquad \text{... (21)}$$

$$u_2 = 1 - \left[1 + \frac{As(n+1)}{i\omega(1-n)}\right](1 + sy)^{-h} + \frac{As(n+1)}{i\omega(1-n)}(1 + sy)^{2/(n-1)} \qquad \text{... (22)}$$

where

$$s = \frac{1-n}{1+n}\left[\frac{M(n+1)}{2n}\right]^{1/(n+1)}$$

$$h = \alpha + i\beta = \frac{1}{2}\left[1 + \left(1 + m\left(1 + \frac{i\omega}{M}\right)\right)^{1/2}\right]$$

$$\alpha = \frac{1}{2}\left(1 + \left[\frac{1}{2}\left(1 + m\left((1+m)^2 + \frac{\omega^2 m^2}{M^2}\right)^{1/2}\right)\right]^{1/2}\right)$$

$$\beta = \frac{1}{2}\left[\frac{1}{2}\left(\left((1+m)^2 + \frac{\omega^2 m^2}{M^2}\right)^{1/2} - 1 - m\right)\right]^{1/2}$$

and

$$m = \frac{8(1+n)}{(1-n)^2}.$$

We find from (22) that the real and imaginary parts of $u_2 = M_r + iM_i$ are given by

$$M_r = 1 - (1+sy)^{-\alpha}\left[\cos\log(1+sy)^{\beta} - \frac{As\,(n+1)}{w\,(1-n)}\sin\log(1+sy)^{\beta}\right] \quad \text{... (23)}$$

$$M_i = \frac{As\,(n+1)}{w\,(1-n)}(1+sy)^{-\alpha}\cos\log(1+sy)^{\beta} - \sin\log(1+sy)^{\beta}$$

$$- \frac{As\,(n+1)}{w\,(1-n)}(1+sy)^{2/(n-1)}, \quad \text{... (2.4)}$$

We thus obtain from (14), for the real part of $u(y,\ t)$

$$u(y,\ t) = u_1(y) + \varepsilon\,(M_r\cos\omega t - M_i\sin\omega t). \quad \text{... (25)}$$

Substituting for u_1 from (21) into (18) and solving the resulting equation with conditions (20), we get

$$\theta_1 = (1+sy)^{4n/(n-1)}, \quad \text{... (26)}$$

provided that

$$P_r = \frac{4n\,(3n+1)}{E\,(n+1)^2}S^{1-n}.$$

Substituting for u_1, u_2 and θ_1 from (21), (22) and (26) into (19) and solving the resulting equation subject to conditions (20) we get

$$\theta_2 = e^{-\sqrt{i\omega P_r}\,y} + \frac{\alpha^*}{i\,\omega\,P_r}\left[(1+sy)^{\frac{n+1}{n-1}-h} - e^{-\sqrt{i\omega P_r}\,y}\right]$$

$$- \frac{\beta^*}{i\omega P_r}\left[(1+sy)^{\frac{3n+1}{n-1}} - e^{-\sqrt{i\omega P_r}\,y}\right] - \frac{\gamma^*}{i\omega P_r}\left[(1+sy)^{\frac{n+3}{n-1}} - e^{-\sqrt{i\omega P_r}\,y}\right] \quad \dots (27)$$

$$+ \sum_{m=1}^{\infty} \frac{D^{2m}}{m!\,(i\omega P_r)^m}\left[\alpha^*(1+sy)^{\frac{n+1}{n-1}-h} - \beta^*(1+sy)^{\frac{3n+1}{n-1}} - \gamma^*(1+sy)^{\frac{n+1}{n-1}}\right]$$

where

$$\alpha^* = P_r E h s^{n+1}\left(\frac{n+1}{1-n}\right)\left[1 + \frac{As}{i\omega}\left(\frac{n+1}{1-n}\right)\right]$$

$$\beta^* = \frac{4n}{1-n} As \quad \dots (28)$$

$$\gamma^* = \frac{2AP_r E}{i\omega}\left(\frac{s}{1-n}\right)$$

and D stands for d/dy.

Equations (21) and (16) give the mean of the velocity and the temperature while the fluctuating parts are given by eqns. (22) and (27).

When the transverse magnetic field is large h behaves like $2/1 - n$ and hence we have in this case the real and imaginary parts of $\theta_2 = N_r + iN_i$ are given by

$$N_r = X(1+sy)^{\frac{n+3}{n-1}} + (1-X)\,e^{\frac{\sqrt{-\omega P_r y}}{2}} \cos\frac{\sqrt{\omega P_r}\,y}{2}$$

$$+ \left(Y - \frac{\beta^*}{\omega P_r}\right) e^{-\frac{\sqrt{\omega P_r}\,y}{2}} \sin\frac{\sqrt{\omega P_r}\,y}{2}$$

$$- \sum_{z=1}^{\infty} (-1)^z \frac{D^{4z}}{2z!\,(wP_r)^{2z-1}}\left[Y(1+sy)^{\frac{n+3}{n-1}} - \frac{\beta^*}{\omega P_r}(1+sy)^{\frac{3n+1}{n-1}}\right]$$

$$+ \sum_{z=1}^{\infty} (-1)^z \frac{D^{2(2z-1)}}{(2z-1)!\,(\omega P_r)^{2(z-1)}} X(1+sy)^{\frac{n+3}{n-1}} \quad \dots (29)$$

and

$$N_i = (X-1) + e^{-\frac{\sqrt{\omega P_r}\,y}{2}} \cos\frac{\sqrt{\omega P_r}\,y}{2}$$

$$+ \left(\frac{\beta^*}{\omega P_r} - Y\right)\left[(1+sy)^{\frac{n+3}{n-1}} - e^{-\frac{\sqrt{\omega P_r}\,y}{2}} \cos\frac{\sqrt{\omega P_r}\,y}{2}\right]$$

$$- \sum_{z=1}^{\infty} (-1)^z D \frac{4z}{2z!\,(\omega P_r)^{2z-1}} X(1+sy)^{\frac{n+3}{n-1}}$$

$$+ \sum_{z=1}^{\infty} (-1)^{z+1} \frac{D^{2(2z-1)}}{(2z-1)!\,(\omega P_r)^{2(z-1)}}$$

$$\left[Y(1+sy)^{\frac{n+3}{n-1}} - \frac{\beta^*}{\omega P_r} (1+sy)^{\frac{3n+1}{n-1}} \right] \quad \dots (30)$$

where

$$X = \frac{2AEs}{\omega^2(1-n)} \left[1 - \left(\frac{n+1}{1-n} \right)^2 s^{n+1} \right]$$

$$Y = \frac{2E(n+1)}{\omega(1-n)^2} s^{n+1}.$$

We thus obtain from (15), for the real part of $\theta(y, t)$

$$\theta(y,t) = \theta_1(y) + \varepsilon (N_r \cos \omega t - N_i \sin \omega t). \quad \dots (31)$$

(II) *Dilatant Fluids (n > 1)*

For this case the thickness of the boundary layer is given by $y_0 = -1/s$

For $0 > y > y_0$ the expressions for u_1, u_2, θ_1 and θ_2 are the same as in the previous case. However, for $y > y_0$ we have

$$u_1 = u_2 = 1, \quad \dots (32)$$

$$\theta_1 = \theta_2 = 0. \quad \dots (33)$$

For the shearing stress at the plate. The skin friction is given by

$$\tau = k \left| \frac{\partial u}{\partial y} \right|^n_{y=0} \quad \dots (34)$$

Using the transformation (7), we obtain

$$\tau = k \left| \frac{\partial u}{\partial y} \right|^n_{y=0} = \rho\, U_0^n \, \mathrm{Re}^{n/(n+1)} \left| \frac{\partial u^*}{\partial y^*} \right|^n_{y=0} \quad \dots (35)$$

From (14) and (32), we get

$$\tau = \left| \frac{\partial u_1}{\partial y} + \varepsilon \frac{\partial u_2}{\partial y} e^{i\omega t} \right|^n_{y=0} \quad \dots (36)$$

Substituting from (21) and (22) into (33), we get

$$\tau = \left[\frac{M(n+1)}{2n}\right]\left(1 + n\varepsilon\left[\frac{2As}{i\omega(n-1)} + h\left(\frac{As}{i\omega} - \frac{n-1}{n+1}\right)\right]e^{i\omega t}\right)$$

$$= \frac{M(n+1)}{2n} + \varepsilon\,|F|\,e^{i(\omega t + \phi^*)} \qquad \text{... (37)}$$

where

$$F = \sqrt{F_r^2 + F_i^2}, \qquad \tan\phi^* = \frac{F_i}{F_r},$$

$$F_r = \frac{M}{2}\left[\alpha(1-n) + \frac{As\,\beta(n+1)}{\omega}\right]$$

$$F_i = \frac{M}{2}\left[\beta(1-n) + \frac{As(n+1)(2-\alpha-\alpha n)}{\omega(1-n)}\right].$$

The skin friction τ is zero when $2n\varepsilon\,|F|/M(n+1) \geq 1$ and $\cos(\omega t + \phi^*) = -M(n+1)/2n\varepsilon\,|F|$. Then the velocity profile is of a separation type. The skin friction is negative when $2n\varepsilon\,|F|/M(n+1) \geq 1$ and $\cos(\omega t + \phi^*) < -M(n+1)/2n\varepsilon\,|F|$, and in this case the velocity profile is of a separated type with reverse flow near the wall. Some values of the absolute value of the skin friction obtained from (37) are illustrated graphically in Figures 3 and 4 for different values of parameters. In all calculations values of $\omega t = \pi/4$ and $\varepsilon = 0.01$ are taken.

CONCLUSIONS

In this paper we have studied a magnetohydrodynamic non-Newtonian power-law fluid moving past an infinite porous plate, when the plate temperature oscillates in magnitude about a constant mean, and the plate is subjected to alternating suction-injection whose magnitude oscillates with respect to time about a constant mean. The effects of the flow-index, the magnetic field, the frequency of the oscillations and the alternating suction-injection parameter on the velocity, the temperature and the skin friction have been studied. The results are illustrated graphically in Figs 1-4.

Figure 1 show the behaviour of the mean velocity for different values of the parameters M and n. It is seen from this figure that the mean velocity starts with a value of zero at the surface of the plate and increases until it reaches its maximum value U_0 at the end of the boundary layer. It was found that the effect of decreasing M is to decrease the values of the velocity. On the other hand an increase in the value of n results in a decrease of the velocity value. The thickness of the boundary layer also decreases with increasing n.

We observe from Fig. 2 that the mean temperature starts from its maximum value at the plate and decreases until it reaches its constant value T_∞ at the end of the boundary layer. This figure shows also that the mean temperature decreases with the increase of either M or n.

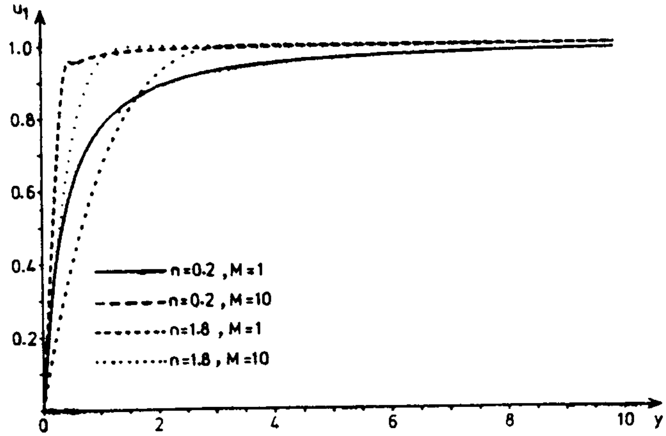

FIG. 1. Mean velocity distribution against n and M.

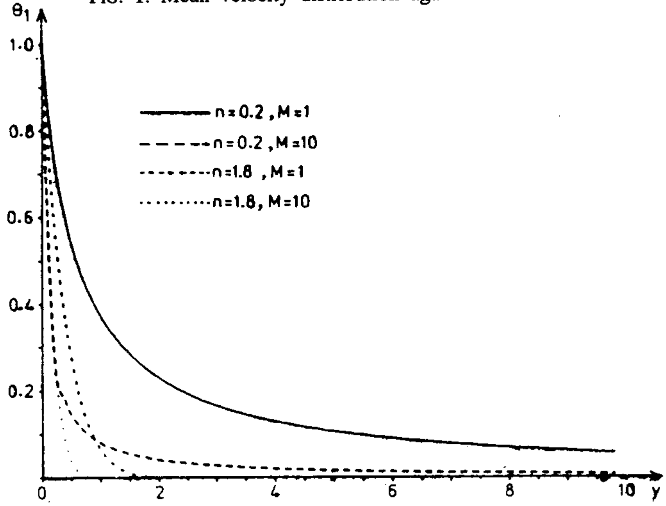

FIG. 2. Mean temperature distribution against n and M.

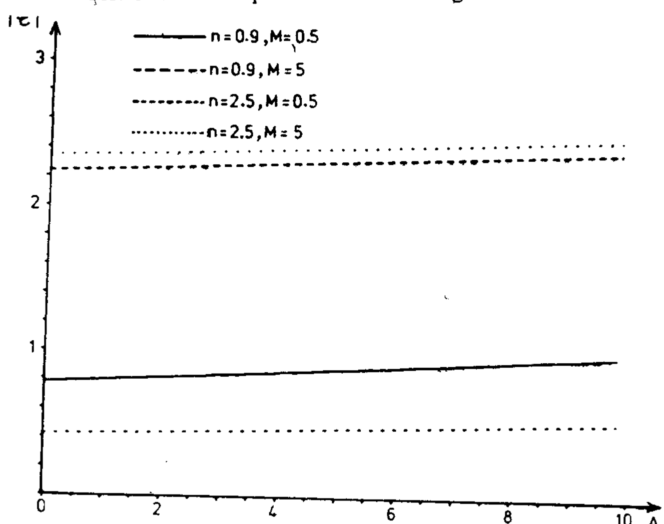

FIG. 3. Skin friction distribution against A for different values of n and M.

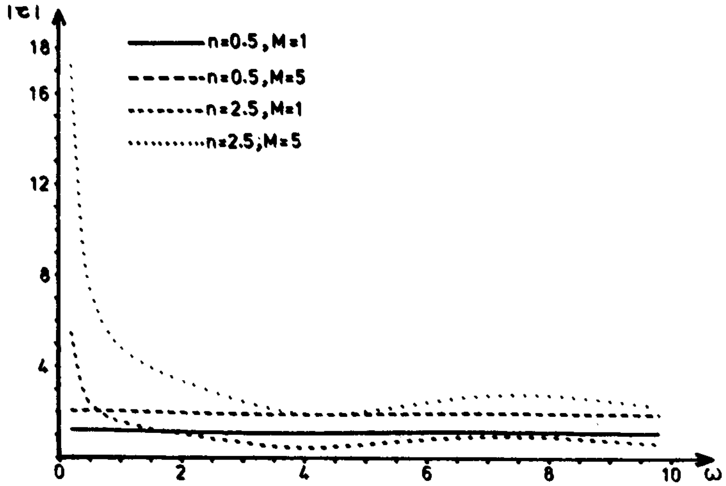

FIG. 4. Skin friction distribution against A for different values of n and M.

Figure 3 illustrates the relation between the absolute value of the skin friction against the variable parameter suction A. It was found that the magnitude of the skin friction increases with the increase of A. The effect of increasing either M or n is to increase the skin friction.

Figure 4 represent the dependence of the absolute value of the skin friction on the frequency ω for different values of the parameters M and n. It is found that its behaviour depends on the type of the fluid considered either pseudoplastic or dilatant. For pseudoplastic fluids the effect of increasing ω is to increase the skin friction slightly. The increase of M results in a increase of the skin friction.

REFERENCES

1. W. R. Schowlter, *A. I. Ch. E. J.* **6** (1960), 24.
2. A. Acrivos, *A. I. Ch. E. J.* **6** (1960), 584.
3. A. Acrivos, M. J. Shan and E. E. Petersen, *A. I. Ch. E. J.* **6** (1960), 312.
4. A. Acrivos, M. J. Shan and E. E. Petersen, *Ch. E. Sci. J.* **20** (1965), 101.
5. L. Martinson and K. Pavlof, *J. MGD*, **3** (1966), 690.
6. D. B. Sharikadza and M. A. Ezzat, *TGU. Tibilisi* **24** (1988), 246.
7. M. A. Ezzat, *TGU. Tibilisi* **25** (1989), 25.
8. U. Saponkoff, *J. MGD.* **6** (1967), 77.
9. L. Martinson and K. Pavlof, *J. MGD* **2** (1970), 50.
10. L. Martinson, *J. MGD* **1** (1974), 148.
11. M. J. Lighthill, *Proc. R. Soc. Lond.* **A224**, (1954), 1.
12. J. T. Stuart, *Proc. R. Soc. Lond.* **A231** (1955), 116.
13. U. Suryabrakasa Rao, *ZAMM* **42** (1962), 133.
14. N. El-Dabe, *Can. J. Phys.* **64** (1986), 84.
15. A. Raptis, *Int. J. Engng. Sci.* **21** (1983), 345.