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THERMAL-DIFFUSION AS WELL AS TRANSPIRATION EFFECTS ON MHD FREE CONVECTION AND MASS TRANSFER FLOW PAST AN ACCELERATED VERTICAL POROUS PLATE

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An unsteady free convection and mass transfer flow of a viscous, incompressible and electrically conducting fluid past a moving infinite vertical porous plate is studied taken into account the thermal diffusion effect. Similarity equations of the momentum, energy and concentration equations are derived by introducing a time dependent length scale. The suction velocity is taken to be inversely proportional to the above length scale. The energy and concentration equations are solved analytically. But due to the complexity of the momentum equation, it is solved numerically by the method of superposition. The effects of the magnetic parameter, the Grashof number, the modified Grashof number, the Soret number and the transpiration parameter on the velocity are shown graphically and thereafter discussed. Finally, the corresponding skin-friction is shown in tabular form.

1. INTRODUCTION

Many natural phenomena and engineering problems are susceptible to MHD analysis. Geophysicists encounter MHD phenomena in the interactions of conducting fluids and magnetic fields that are present in and around heavenly bodies. Engineers employ MHD principles in the design of heat exchangers, pumps and flowmeters, in space vehicle propulsion, thermal protection, braking, control and re-entry, in creating novel power generating systems, and in developing confinment schemes for controlled fusion. From the technological point of view, MHD free-convection flows have also great significance for the applications in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics (cf. Alfven[1], Cramer and Pai[5], Lui[9]). Model studies of the above phenomena of MHD free convection flows have been made by many. Some of them are Georgantopoulos[3], Nanousis *et al.*[11] and Raptis and Singh[13]. On the other hand, along with the free convection currents, caused by the temperature difference, the flow is also affected by the difference in concentrations on material constitution. Many investigators have studied the phenomena of MHD free convection and mass transfer flows of whom the names of Haldavnekar and Soundalgekar[6], Soundalgekar *et al.*[16], Nanousis and

Goudas[10] and Georgantopoulos and Nanousis[4] are worth mentioning. However, in the above studies the thermal diffusion effect was ignored. This assumption is true when the concentration level is very low. There are, however, exceptions. The thermal diffusion effect (commonly known as Soret effect) for instance, has been utilized for isotope separation and in mixtures between gases with very light molecular weight (H_2, H_e) and the medium molecular weight (N_2, air) the diffusion-thermoeffect was found to be of a magnitude such that it can not be neglected (Eckert and Drake[2]). In view of the importance of this diffusion thermo effect, recently Jha and Singh[7] studied the free convection and mass transfer flow in an infinite vertical plate moving impulsively in its own plane, taking into account the Soret effect. Very recently Kafoussias[8] studied the MHD free convection and mass transfer flow taking into account Soret effect. The analytical studies of Jha and Singh and Kafoussias were based on Laplace transform technique.

Hence, our aim is to introduce a similarity technique to study the thermal diffusion effects as well as the transpiration effects on MHD free convection and mass transfer flow past an accelerated vertical plate having a variable suction. The plate is assumed to be moving in its own plane with an arbitrary velocity $U_0F(t)$ where U_0 is the uniform velocity and $F(t)$ is a function of time.

2. Mathematical Analysis

Let us consider an unsteady free convection and mass transfer flow of a viscous, incompressible and electrically conducting fluid past a moving infinite vertical porous plate having a variable suction at the plate. The flow is assumed to be in the x-direction, which is taken along the plate in the upward direction and y-axis is normal to it. A uniform magnetic field B_0 is taken to be acting along the y-axis. At time $t > 0$, the plate is accelerated in its own plane with a velocity $U(t)$. The plate temperature and concentration are instantly raised to $T_w(> T_\infty)$ and $C_w(> C_\infty)$, which are thereafter maintained constant, where T_∞ and C_∞ are the temperature and concentration of the uniform flow. The induced magnetic field is assumed to be negligible, so that $\mathbf{B} = (0, B_0, 0)$. The equation of conservation of electric charge $\nabla \cdot \mathbf{J} = 0$ gives J_y = constant, where $\mathbf{J} = (J_x, J_y, J_z)$. Since the plate is electrically nonconducting, this constant is zero and hence $J_y = 0$ every where in the flow. With in the frame work of such assumptions and under Boussinesq's approximation, the equations relevant to the problem are

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g_0\,\beta(T - T_\infty) + g_0\,\beta^*\,(C - C_\infty) - \frac{p\,B_0^2\,u}{\rho} \quad \text{... (1)}$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{... (2)}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \quad \text{... (3)}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D_1\frac{\partial^2 T}{\partial y^2} \qquad \text{... (4)}$$

where u is the velocity components along x directions, v is the suction velocity, T and C are respectively the temperature and concentration of the fluid, ν is the kinematic viscosity of the fluid, ρ the density of the fluid, k the thermal conductivity, C_p the specific heat at constant pressure, β the volumetric co-efficient of thermal expansion, β^* the volumetric co-efficient of expansion with concentration, g_0 the acceleration due to gravity, p is the electric conductivity, D the molecular diffusivity and D_1 the thermal diffusivity.

The boundary conditions for the problem are

$$u = U(t), \quad v = v(t), \quad T = T_w, \quad C = C_w \text{ at } y = 0$$

$$u = 0, \; v = 0, \; T = T_\infty, \; C = C_\infty \text{ as } y \to \infty. \qquad \text{... (5)}$$

We now introduce a similarity parameter σ as

$$\sigma = \sigma(t) \qquad \text{... (6)}$$

such that σ is the length scale. In terms of this length scale, a convenient solution of equation (2) is considered to be

$$v = -v_0\frac{\nu}{\sigma} \qquad \text{... (7)}$$

where v_0 is the suction parameter.

Now for reasons of similarity, the plate velocity is taken to be

$$U(t) = U_0 F(t) \qquad \text{... (8)}$$

where U_0 is a constant uniform velocity and $F(t)$ is taken to be equal to σ_*^2, where $\sigma_* = \frac{\sigma}{\sigma_0}$, σ_0 being the value of σ at $t = t_0$.

We now introduce the following dimensionless variables

$$\eta = \frac{y}{\sigma}$$

$$u = U(t)\, f(\eta) \qquad \text{... (9)}$$

$$T = T_\infty + (T_w - T_\infty)\,\theta(\eta)$$

$$C = C_\infty + (C_w - C_\infty)\,\phi(\eta).$$

Then introducing the relations (6)-(9) in equations (1), (3) and (4), respectively, we obtain the following ordinary differential equations

$$\frac{\sigma}{\nu}\frac{\partial\sigma}{\partial t}(-\eta f' + 2f) - v_0 f' = f'' + G_r\theta + G_m\phi - Mf \qquad \text{... (10)}$$

$$-\frac{\sigma}{\nu}\frac{\partial\sigma}{\partial t}\eta\,\theta' - v_0\,\theta' = \frac{1}{P_r}\theta'' \qquad \text{... (11)}$$

$$-\frac{\sigma}{\nu}\frac{\partial\sigma}{\partial t}\eta\,\phi' - v_0\,\phi' = \phi''\frac{1}{S_c} + S_0\,\theta'' \qquad \text{... (12)}$$

where $G_r\left(=\frac{g_0\,\beta\sigma_0^2\,(T_w - T_\infty)}{U_0\nu}\right)$ is the Grashof number, $G_m\left(=\frac{g_0\,\beta^*\,\sigma_0^2\,(C_w - C_\infty)}{U_0\nu}\right)$ is the modified Grashof number, $M\left(=\frac{p\,B_0^2\,\sigma^2}{\nu\rho}\right)$ is the magnetic parameter, $P_r\left(=\frac{\rho\,C_p\,\nu}{k}\right)$ is the Prandtl number, $S_c\left(=\frac{\nu}{D}\right)$ is the Schmidt number and $S_0\left(=\frac{D_1\,(T_w - T_\infty)}{\nu\,(C_w - C_\infty)}\right)$ is the Soret number. In eqns. (10)-(12) primes denote the differentiation with respect to $\eta\left[(\,)' = \frac{d(\,)}{d\eta}\right]$.

The boundary conditions (5) now transform to

$$f = 1,\ \theta = 1,\ \phi = 1, \text{ at } \eta = 0$$
$$f = 0,\ \theta = 0,\ \phi = 0 \text{ as } \eta \to \infty. \qquad \text{... (13)}$$

Equation (10)-(12) are similar except for the term $\frac{\sigma}{\nu}\frac{\partial\sigma}{\partial t}$ where time t appears explicitly. Thus the similarity condition requires that $\frac{\sigma}{\nu}\frac{\partial\sigma}{\partial t}$ in eqns. (10)-(12) must be a constant quantity. Hence following the works of Sattar and Hossain[14] and Sattar[15] it is assumed that

$$\frac{\sigma}{\nu}\frac{\partial\sigma}{\partial t} = 2. \qquad \text{... (14)}$$

Integrating (14) it then follows that $\sigma = 2\sqrt{\nu t}$, which exactly corresponds to the usual scaling factor for the unsteady boundary layer flows Schlichting[17]. Finally introducing (14) in equations (10)-(12) we have respectively

$$f'' + 2f'\,\zeta - f(4 + M) + G_r\,\theta + G_m\,\phi = 0 \qquad \text{... (15)}$$

$$\theta'' + 2P_r\,\theta'\,\zeta = 0 \qquad \text{... (16)}$$

$$\phi'' + 2S_c\,\phi'\,\zeta + S_0\,S_c\,\theta'' = 0 \qquad \text{... (17)}$$

where $\zeta = \eta + \frac{1}{2}v_0$.

The above equations thus describe the basis of our problem, the solutions of which are now sought. In the preceding section these solutions are obtained. The solution of the eqns. (16) and (17) are, however, obtained analytically but due to the complexity of eqn. (15) it is solved numerically by the method of superposition Na[12].

3. Solutions

The solutions of eqns. (16) and (17) are straightforward and are obtained as

$$\theta(\eta) = \frac{\operatorname{erfc}(\sqrt{P_r}\,\zeta)}{\operatorname{erfc}\left(\sqrt{P_r}\,\frac{v_0}{2}\right)} \qquad \ldots (18)$$

and

$$\phi = \frac{K\sqrt{\pi}}{4(P_r - S_c)\sqrt{P_r}} \operatorname{erf}(\sqrt{P_r}\,\zeta) + C_1 \frac{\sqrt{\pi}}{2\sqrt{S_c}} \operatorname{erf}(\sqrt{S_c}\,\zeta) + C_2 \qquad \ldots (19)$$

where

$$K = -\frac{4P_r\sqrt{P_r}\,S_0 S_c}{\sqrt{\pi}\,[1 - \operatorname{erf}(\sqrt{P_r}\,v_0/2)]}$$

$$C_1 = \frac{K\sqrt{S_c}}{2(P_r - S_c)\sqrt{P_r}} \frac{\operatorname{erfc}(\sqrt{P_r}\,v_0/2)}{\operatorname{erfc}(\sqrt{S_c}\,v_0/2)} - \frac{2\sqrt{S_c}}{\sqrt{\pi}\operatorname{erfc}(\sqrt{S_c}\,v_0/2)}$$

$$C_2 = \frac{K\sqrt{\pi}}{4(P_r - S_c)\sqrt{P_r}} - \frac{C_1\sqrt{\pi}}{2\sqrt{S_c}}.$$

Now substituting (18) and (19) into eqn. (15) it's solution is obtained numerically as mentioned above. For the purpose of numerical integration, the Runge-Kutta Merson Integration Scheme has been used. The values of the parameters are taken as those of Kafoussias[8] for consistency. Now denoting τ is the skin-friction, the numerical values of τ is obtained from the process of numerical integrations. These values are stored in Tables I and II.

4. Results and Discussion

For the purpose of discussing the effects of various parameters on the flow behaviour near the plate, numerical calculations have been carried out for differential values of G_r, G_m, S_0, v_0 and M for fixed values of P_r and S_c. The values of G_r are taken to be both +ve and –ve, since these values represent respectively cooling and heating of the plate. The value of P_r is taken to be 0.71 which corresponds to air at 20°C. Since water vapour is used as a diffusing chemical species of most common interest in air, the value of S_c is taken to be 0.60 (water vapour). The values of G_m, S_0, v_0 and M are, however, taken from Kafoussias[8].

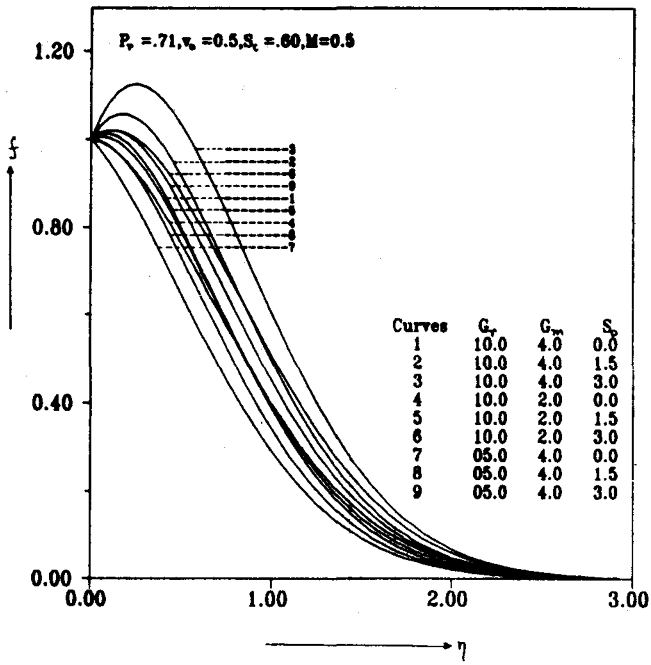

FIG. 1. Velocity profiles (cooling of the plate) for different values of G_r, G_m and S_0.

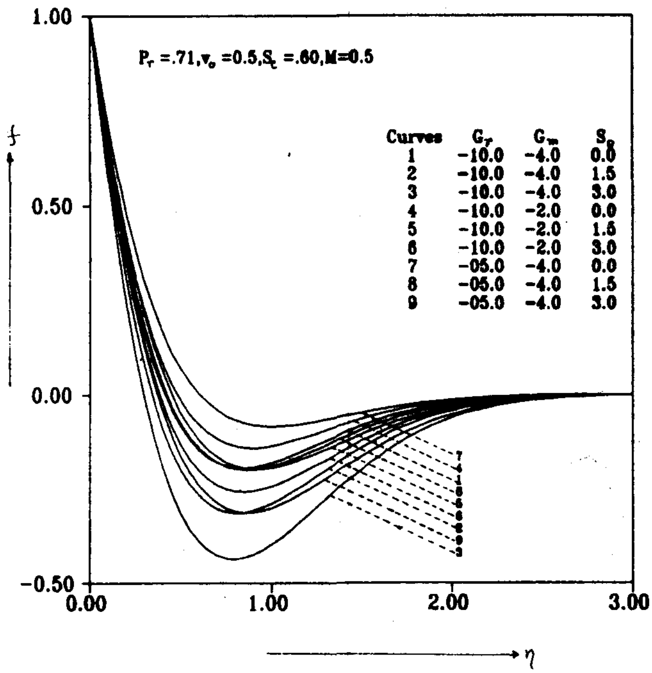

FIG. 2. Velocity profiles (heating of the plate) for different values of G_r, G_m and S_0.

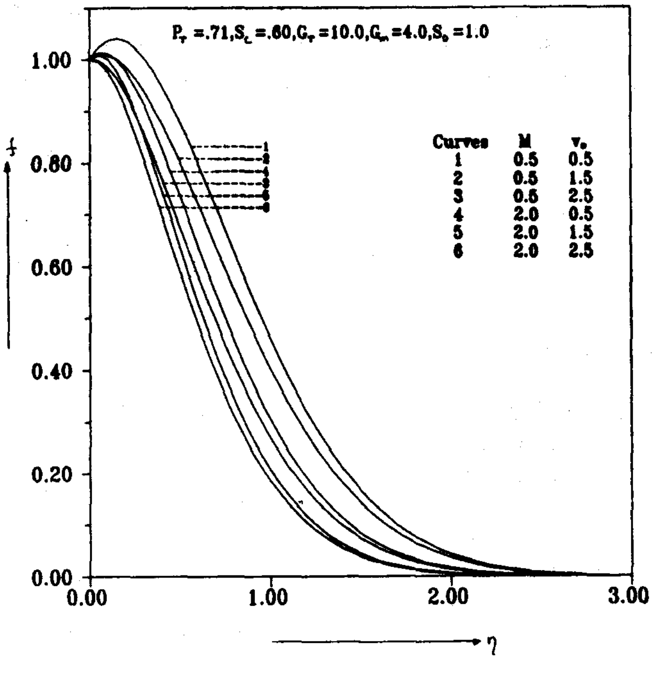

FIG. 3. Velocity profiles (cooling of the plate) for different values of M and v_0.

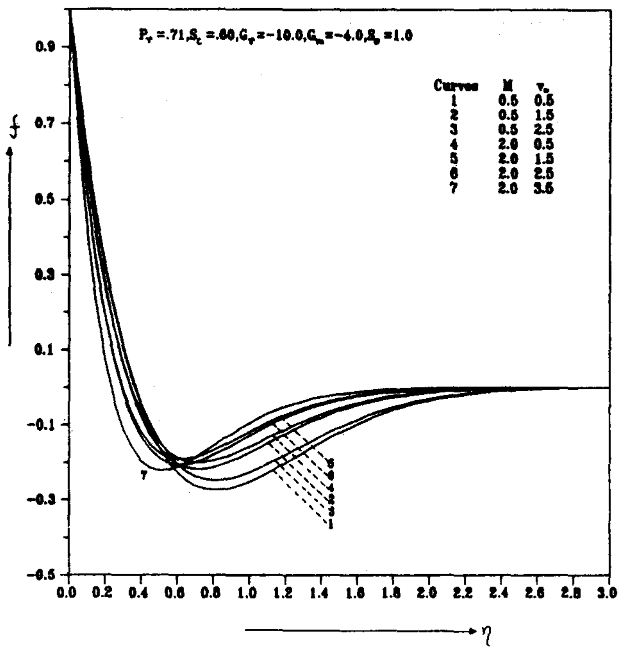

FIG. 4. Velocity profiles (heating of the plate) for different values of M and v_0.

The results of the numerical calculations are shown in Figs. 1-4. In Fig. 1, the velocity profiles are shown for different values of G_r, G_m and S_0 in case of cooling of the plate ($G_r > 0$). For this purpose the values of G_r are taken to be large ($G_r = 5$ and 10), since larger values of G_r correspond to a cooling problem that is generally encountered in nuclear engineering. From this figure it is observed that velocity increases with the increase of either of the parameters G_r, G_m and S_0. In Fig. 2 the heating effect of the plate on the velocity profiles are shown. Compairing the Fig. 2 with Fig. 1 it is apparent that the heating of the plate has an opposite effect to that of cooling of the plate. In Fig. 3 the effects of magnetic and suction parameters are shown for cooling of the plate. It is seen from this figure that as the magnetic parameter increases the velocity decreases. The same effect on the velocity profiles is also observed for increasing values of the suction parameters, which is usually expected. In Fig. 4 the effects of the magnetic and suction parameters are shown in case of heating of the plate. In this case velocity decreases with the increase of M close to the wall (approx. $\eta \leq 0.62$), whereas for roughly $\eta > 0.62$ the velocity increases with the increase of M. In presence of the magnetic field and in the case of heating of the plate (Fig. 4), there appears to be a peculiar effect of suction on the velocity field. In the range $\eta \leq 0.62$ velocity decreases (curves 1 and 2) and then increases (curves 2 and 3) with gradual increase of v_0. However, for larger values of v_0 the velocity has been found to decrease. But for $\eta > 0.62$ opposite effects are observed.

Finally the effects of various parameters on the skin friction are shown in Tables I and II. For brevity, the discussion of the effects of the parameters on the skin friction are not done here. Nevertheless, the said effects are self evident from the above mentioned tables.

TABLE I

Numerical values of τ for $P_r = 0.71$ and $S_c = 0.60$

G_r	G_m	S_0	M	v_0	τ
10.0	4.0	1.0	0.5	0.5	0.8945
10.0	4.0	1.0	0.5	1.5	0.5903
10.0	4.0	1.0	0.5	2.5	0.5918
10.0	4.0	1.0	2.0	0.5	0.4054
10.0	4.0	1.0	2.0	1.5	0.1287
10.0	4.0	1.0	2.0	2.5	0.1500
−10.0	−4.0	1.0	0.5	0.5	−6.4405
−10.0	−4.0	1.0	0.5	1.5	−7.4218
−10.0	−4.0	1.0	0.5	2.5	−8.8559
−10.0	−4.0	1.0	2.0	0.5	−6.4997
−10.0	−4.0	1.0	2.0	1.5	−7.4768
−10.0	−4.0	1.0	2.0	2.5	−8.8926
−10.0	−4.0	1.0	2.0	3.5	−11.0092

TABLE II

Numerical values of τ for $P_r = 0.71$ and $S_c = 0.60$

G_r	G_m	S_0	M	v_0	τ
10.0	4.0	0.0	0.5	0.5	0.5052
10.0	4.0	1.5	0.5	0.5	1.0892
10.0	4.0	3.0	0.5	0.5	1.6735
10.0	2.0	0.0	0.5	0.5	0.1889
10.0	2.0	1.5	0.5	0.5	0.3105
10.0	2.0	3.0	0.5	0.5	0.6023
05.0	4.0	0.0	0.5	0.5	0.6459
05.0	4.0	1.5	0.5	0.5	0.6093
05.0	4.0	3.0	0.5	0.5	0.5225
–10.0	–4.0	0.0	0.5	0.5	–6.0508
–10.0	–4.0	1.5	0.5	0.5	–6.6354
–10.0	–4.0	3.0	0.5	0.5	–7.2199
–10.0	–2.0	0.0	0.5	0.5	–5.5635
–10.0	–2.0	1.5	0.5	0.5	–5.8558
–10.0	–2.0	3.0	0.5	0.5	–6.1481
–05.0	–4.0	0.0	0.5	0.5	–4.8989
–05.0	–4.0	1.5	0.5	0.5	–5.4835
–05.0	–4.0	3.0	0.5	0.5	–6.0681

REFERENCES

1. H. Alfven, *Cosmical Electrodynamics*, Oxford Univ. Press, London, 1950
2. E. R. G. Eckert and R. M. Drake, *Analysis of Heat and Mass Transfer*, McGraw-Hill Book Co. New York, 1972.
3. G. A. Georgantopoulos, *Astrophys. Space Sci.* **65** (2) (1979), 433.
4. G. A. Georgantopoulos and N. D. Nanousis, *Astrophys. Space Sci.* **67** (1) (1980), 229.
5. K. R. Cramer and Shih-I Pai, *Magneto fluid Dynamics for Engineers and Applied Physics*, McGraw-Hill Book Co., New York, 1973.
6. D. D. Haldavnekar and V. M. Soundalgekar, *Acta Phys. Acad. Sci. Hung.* **43** (3/4) (1977), 243.
7. B. K. Jha and A. K. Singh, *Astrophys. Space Sci.* **173** (1990), 251.
8. N. G. Kafoussias, *Astrophys. Space Sci.* **192** (1992), 11.
9. A. T. Y. Lui (ed.), *Magnetotail Physiscs*, The Johns Hopkins Univ. Press, Baltimore and London, 1987.
10. N. D. Nanousis and C. L. Goudas, *Astrophys. Space Sci.* **66** (1) (1979), 13.

11. N. D. Nanousis, G. A. Georgantopoulos and A. I. Papaionnou, *Astrophys. Space Sci.* **70** (1980), 377.
12. T. Y. Na, *Computational Method in Engineering Boundary Value Problem*, Academic Press, New York 1979.
13. A. Raptis and A. K. Singh, *Mech. Res. Comm.* **12** (1985), 31.
14. M. A. Sattar and M. M. Hossain, *Can. J. Phys.* **70** (1992), 369.
15. M. A. Sattar, *Indian. J. pure appl. Math.*, (submitted).
16. V. M. Soundalgekar and S. K. Gupta, *Int. J. Energy Sci.* 3(1) (1979), 59.
17. H. Schlichting, Boundary Layer Theory, McGrow Hill Book Co., New York 1968.