Indian J. pure appl. Math., **35**(1) : 51-55, January 2004

HOMOLOGY AND CHAOTIC UNFOLDING OF CHAOS MANIFOLDS

EI ZOHNY - HABIBA

Mathematics Department, Faculty of Science for Girls, Al Azhar University

(*Received* 17 *January* 2003; *accepted* 25 *July* 2003)

In this paper we will discuss the homology group and the unfolding of chaotic manifolds. The relations between homology of the manifold and its unfolding are deduced.

Key Words : Homology; Unfolding; Manifolds

1. INTRODUCTION

In[16] the folding of a manifold was introduction by Robertson in 1977. In [2, 3] the chaotic unfolding of a manifold was introduced. More studies on the folding of manifold were discussed in [17, 18, 19, 20, 21, 22, 24]. The homology group of manifold was defined in [1, 2, 3, 5, 8, 9, 10, 11, 12, 14, 15]. Manifolds without boundaries are discussed in [4, 6, 7, 13]. In this article we will achieve, after chaotic unfolding of manifold, the homology group of the image manifold may or may not be isomorphic to the homology group of the original manifold. We will investigate these isomorphism relations.

Theorems governing these relations are obtained. The effect of retraction on the homology group of the chaotic unfolding of a manifold is discussed. For Chaos manifolds M and M of the same dimensions, a map $g : M \rightarrow M$ is said to be unfolding of M into M if, for every piecewise geodesic path $\gamma : I \rightarrow M$, I = [0, 1], the induced path $\gamma = g \, o \, \gamma : I \rightarrow M$ is piecewise geodesic but with length greater than that of γ. A chaotic unfolding is said to be regular if it preserves curvature; otherwise, it is called non-regular.

2. MAIN RESULTS

We will introduce the following definition

Definition — The perturbation of the manifold round some fixed points or not is a chaotic manifold. [See Figs. 1-4].

Local properties of a chaotic unfolding were discussed. Now, we will use homology groups to describe the global properties of a chaotic manifold.

***Theorem* 1** — *Let M be an n-dimensional manifold with boundary. Then, for each chaotic unfolding* $g : M \rightarrow M'$, *either*

$$H_n(M') \approx 0 \text{ or } H_n(M') \approx Z$$

Where $M' = g(M)$.

PROOF : We distinguish between cases

(*i*) The Chaotic unfolding Preserves Boundaries

In this case, $H_n(M') \approx 0$, regardless of regularity or non-regularity of the unfolding, (Fig. 1).

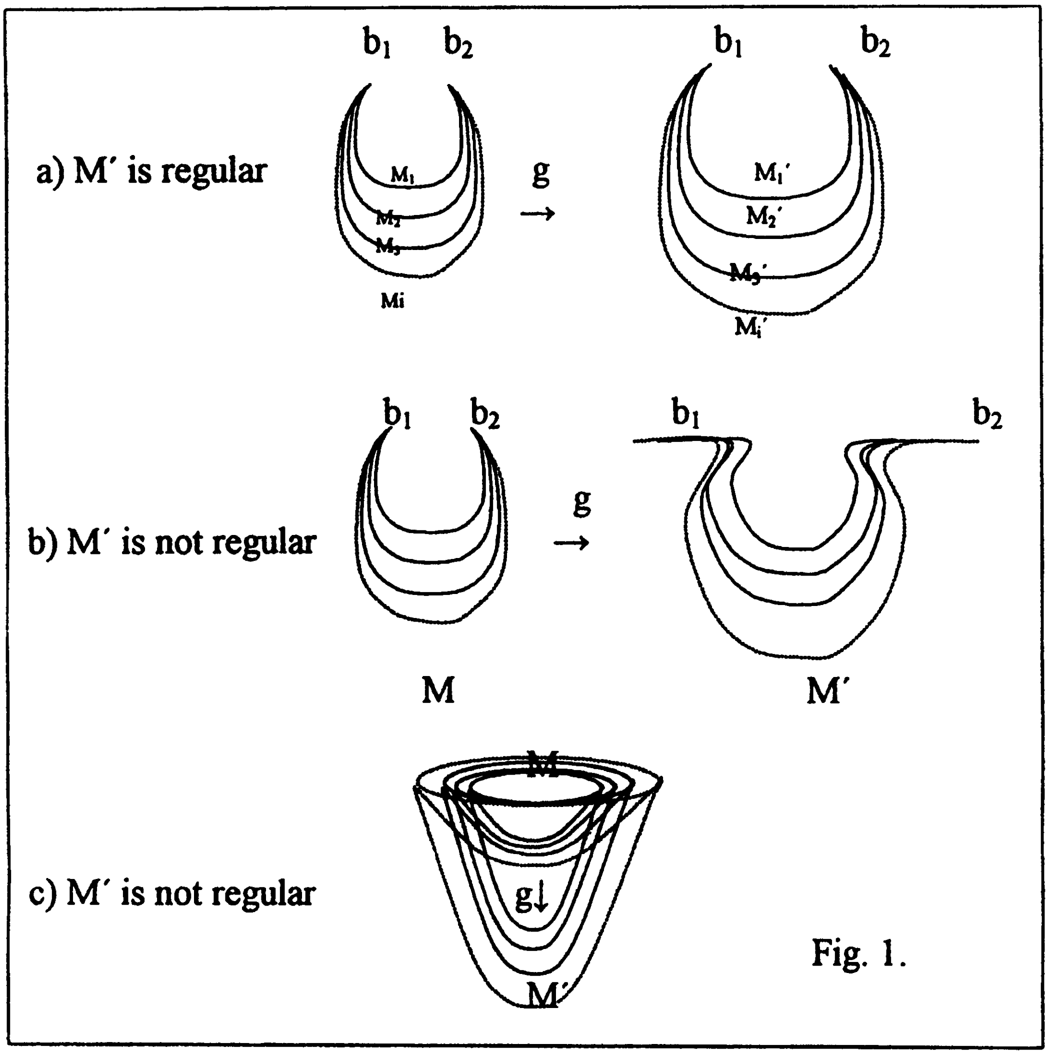


Fig. 1.

(*ii*) The Chaotic unfolding Does Not preserve Boundaries.

$H_n(M') \approx Z$, regardless of regularity or non-regularity of the chaotic unfolding (Fig. 2).

Thus $\quad H_n(M) \approx 0 \neq \underline{Z} \approx H_n(M')$.

Remark : In general, the homology group of a chaotic unfolding is not necessarily isomorphic to that of the manifold.

***Theorem* 2** — *Let M be an oriented n-dimensional manifold without boundary. Then for each chaotic unfolding $g : M \to M'$, either*

$$H_n(M) \approx \underline{Z} \approx H_n(M')$$

or

$$H_n(M) \approx 0 \approx H_n(M')$$

Where $M' = g(M)$.

PROOF : Clearly, M is homeomorphic to a sphere S^n, a connected sum of n-spheres, a torus, or a connected sum of turi[7]. In the first two cases we have that,

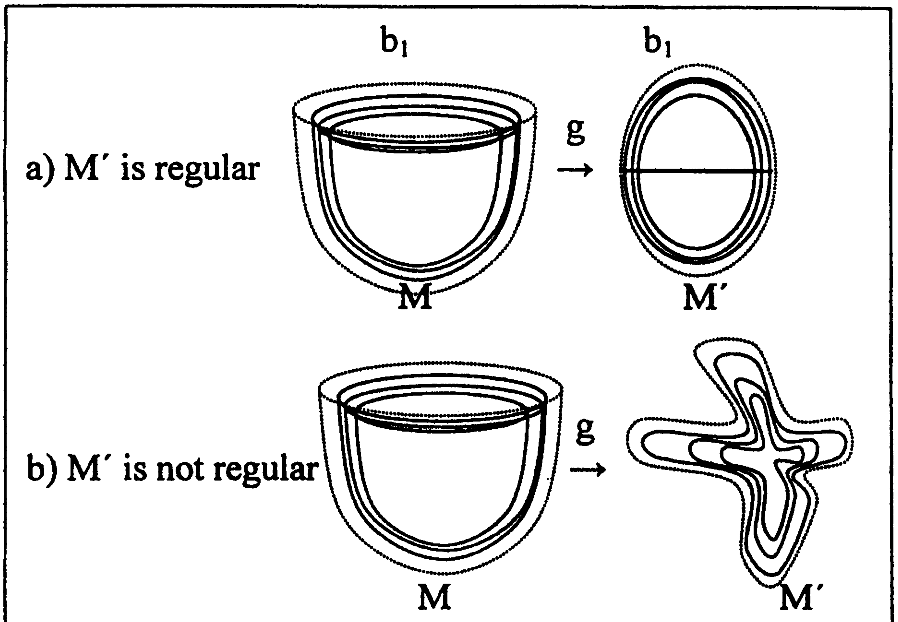


Fig. 2

$$H_n(M) \approx Z \approx H_n(M')$$

and in the last two cases we have that

$$H_n(M) \approx 0 \approx H_n(M')$$

***Theorem* 3** — *Let M be an oriented n-dimensional manifold. Then for each chaotic unfolding $g: M \to M'$, a necessary and sufficient condition for the property*

$$H_n(M) \approx H_n(M')$$

is that g (boundary of M) = boundary of M′

where $M' = g(M)$.

PROOF : *Sufficiency*: By assumption we have that g (boundary of M) = boundary of M' as in Fig. 3.

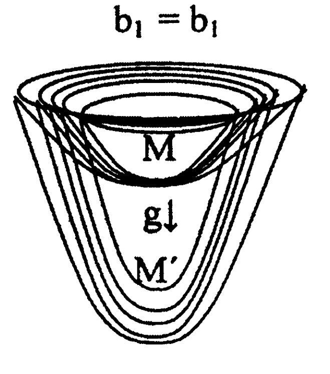


Fig. 3.

Clearly, $H_n(M) \approx H_n(M')$.

Necessity — Let g (boundary of M) $\neq$ boundary of M', as in Fig. 4.

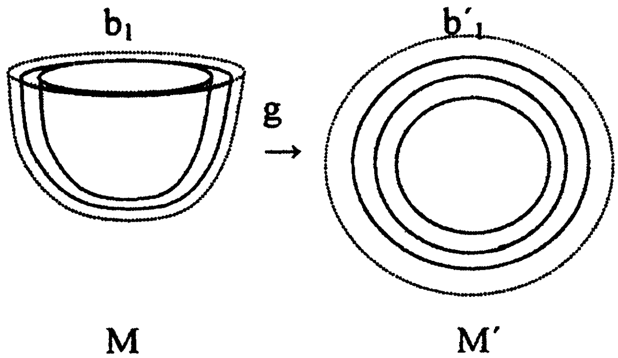


Fig. 4.

Fig. 5

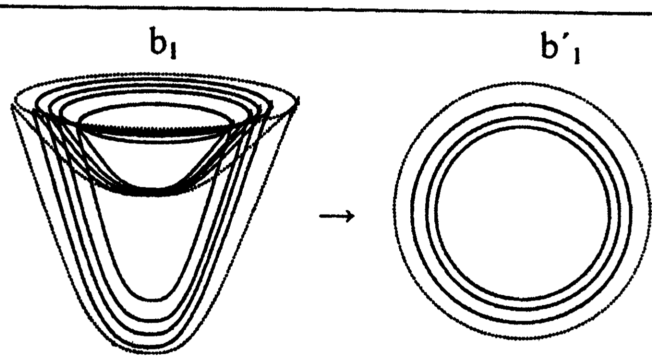


Fig. 6

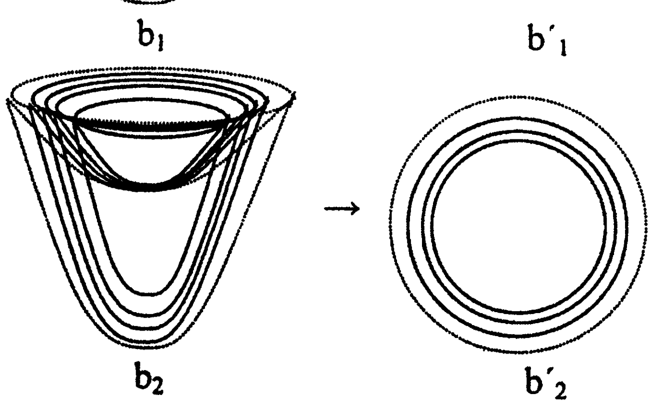
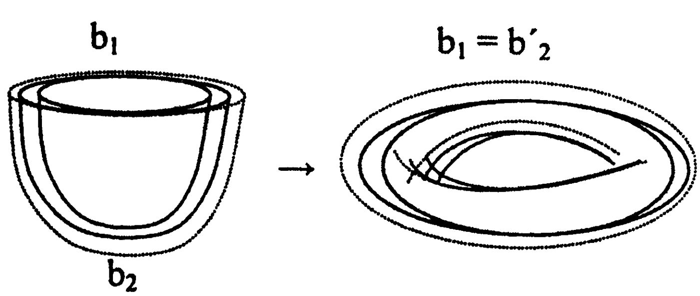


Fig. 7



Then, $H_n(M) \approx 0 \approx H_n(M')$.

Remark : A chaotic unfolding M' of an oriented n-dimensional manifold M is homeomorphic to M if $H_n(M) \approx H_n(M')$.

***Theorem* 4** — *The limit of the chaotic unfoldings of a section of hypersphere is either a shpere or a torus.*

PROOF : We need only consider Figs. 4, 5, 6, 7 above.

Corollary — The limit of chaotic unfoldings of a section of a hypertorus is either a sphere or a torus.

PROOF : We need only to consider Figs. 4, 6, 7 above.

Theorem — *The homology of the chaos retraction of the chaotic unfolding of a manifold is isomorphic to the homology of the manifold.*

PROOF : The proof is clear.

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