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T_H AND S_H-INTERVAL-VALUED FUZZY H_v-SUBGROUPS

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In this paper, T_H and S_H - interval-valued fuzzy H_v-subgroups are defined. Some of their basic properties and structural characteristics are discussed and studied. Furthermore, we obtain the relationship between T_H-interval-valued fuzzy H_v-subgroups, S_H-interval-valued fuzzy H_v-subgroups and level H_v-subgroups. Also the theorems of the homomorphic image and the inverse image are given[1].

Key Words : H_v-Group; H_v-Subgroup; t-Norm; t-Conorm; Fuzzy Set; Interval-Valued Fuzzy Set

1. INTRODUCTION

The concept of a fuzzy subset of a non-empty set first was introduced by Zadeh in 1965[13]. In[7], Rosenfeld formulated the concept of a fuzzy subgroup of a group. This work was the first fuzzification of any algebraic structure and thus opened a new direction, new exploration, new path of thinking to mathematicians, engineers, computer scientists, and many others in various ways of various tests

The concept of H_v-structures has been introduced by Vougiouklis in 1990 during the 4th AHA congress[8]. The concept of H_v-structures constitute a generalization of the well-known algebraic hyperstructures (semihypergroup, hypergroup, hyperring and so on. Actually some axioms concerning the above hyperstructures are replaced by their corresponding weak axioms, see[8,9,10]. In[3,4,5,6] the present author applied the concept of fuzzy sets theory in the theory of algebraic hyperstructures, for example in[4], we defined the concept of fuzzy H_v-subgroup of an H_v-group which is a generalization of the concept of fuzzy subgroup of a group.

In 1975, Zadeh[14] introduced the concept of interval-valued fuzzy subsets, where the values of the membership functions are intervals of numbers instead of the numbers. In[1], Biswas defined interval-valued fuzzy subgroups of the same nature of Rosenfeld's fuzzy subgroups. On the basis of Biswas[1] Li and Wang[11] introduced the idempotent interval t-norms T_H induced by a t-norm, and defined the T_H-interval-valued fuzzy subgroups. Also, in[12], they introduced the idempotent interval t-conorms S_H induced by a t-conorm, and defined the S_H-interval-valued fuzzy subgroups.

In this paper, T_H and S_H-interval-valued fuzzy H_v-subgroups are defined. In the meantime, some of their basic properties and structural characteristcs are discussed and studied. Furthermore, we obtain the relationship between T_H-interval-valued fuzzy H_v-subgroups, S_H-interval-valued fuzzy H_v-subgroups and level H_v-subgroups. Also the theorems of the homomorphic image and the inverse image are given.

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2. Basic Definitions

In this section, we recall some basic definitions for the sake of completeness.

Definition 2.1. (Vougiouklis[10]) — A hyperstructure is a non-empty set H together with a map. $: H \times H \to \mathcal{P}^*(H)$ called hyperoperation, where $\mathcal{P}^*(H)$ denotes the set of all non-empty subsets of H. A hyperstructure $(H, \cdot)$ is called an H_v-group if the following axioms hold :

(*i*) $(x \cdot y) \cdot z \cap x \cdot (y \cdot z) \neq 0$ for all x, y, z in H,

(*ii*) $a \cdot H = H \cdot a = H$ for all a in H.

If $x \in H$ and A, B be subsets of H, then by $A \cdot B, A \cdot x, x \cdot B$ we mean

$$A \cdot B = \bigcup_{x \in A, y \in B} x \cdot y, \quad A \cdot x = A \cdot \{x\}, \text{ and } x \cdot B = \{x\} \cdot B.$$

Definition 2.2 (Zadeh[13]) — Let X be a non-empty set. A fuzzy subset F defined on X is given by

$$F = \{(x, \mu_F(x)) \mid x \in X\} \text{ where } \mu_F : X \to [0, 1].$$

Definition 2.3 (Davvaz[3]) — A fuzzy subset F of an H_v-group H is called a fuzzy H_v-subgroup of H if the following conditions hold :

(*i*) $\min\{\mu_F(x), \mu_F(y)\} \leq \inf\limits_{\alpha \in x \cdot y} \{\mu_F(\alpha)\}$ for all $x, y \in H$,

(*ii*) for all $x, a \in H$ there exists $y \in H$ such that $x \in a \cdot y$ and

$$\min\{\mu_F(a), \mu_F(x)\} \leq \mu_F(y),$$

(*iii*) for all $x, a \in H$ there exists $z \in H$ such that $x \in z \cdot a$ and

$$\min\{\mu_F(a), \mu_F(x)\} \leq \mu_F(z).$$

Definition 2.4 — An interval number is $[a^-, a^+]$, where $0 \leq a^- \leq a^+ \leq 1$. Let $D[0, 1]$ denote the set of all interval numbers, i.e.,

$$D[0, 1] = \{[a^-, a^+] \mid 0 \leq a^- \leq a^+ \leq 1\},$$

where the elements in $D[0, 1]$ are called the interval numbers on $D[0, 1]$. For any $a \in [0, 1]$, if we define $a = [a, a]$, then we have $a \in D[0, 1]$. It means that the interval numbers are an extension of ordinary numbers.

Definition 2.5 — Let $\hat{a}_i \in D[0, 1]$, where $\hat{a}_i = [a_i^-, a_i^+]$ for all $i \in I$, I be an index set, then we define

$$\inf \hat{a}_i = \left[\bigwedge_{i \in I} a_i^-, \bigwedge_{i \in I} a_i^+ \right],$$

$$\sup \hat{a}_i = \left[\vee_{i \in I} a_i^-, \vee_{i \in I} a_i^+ \right].$$

In particular, whenever $\hat{a}_1, \hat{a}_2 \in D[0, 1], \hat{a}_1 = [a^-, a^+], \hat{a}_2 = [b^-, b^+]$, we define

1. $\hat{a}_1 \leq \hat{a}_2$ iff $a^- \leq b^-, a^+ \leq b^+$,
2. $\hat{a}_1 = \hat{a}_2$ iff $a^- = b^-, a^+ = b^+$,
3. $\hat{a}_1 < \hat{a}_2$ iff $\hat{a}_1 \leq \hat{a}_2$ and $\hat{a}_1 \neq \hat{a}_2$,
4. $k\hat{a}_1 = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.

Obviously, $(D[0, 1], \leq, \sup, \inf)$ constitutes a complete lattice with the least element $0 = [0, 0]$ and the greatest element $1 = [1, 1]$.

Definition 2.6 (Zadeh[14]) — Let X be a non-empty set. An interval-valued fuzzy subset F defined on X is given by

$$F = \left\{ (x, [\mu_F^-(x), \mu_F^+(x)]) \mid x \in X \right\},$$

where μ_F^- and μ_F^+ are two fuzzy subsets of X such that $\mu_F^-(x) \leq \mu_F^+(x)$ for all $x \in X$. Suppose $\hat{\mu}_F(x) = [\mu_F^-(x), \mu_F^+(x)]$, then the interval-valued fuzzy subset F is given by

$$F = \{(x, \hat{\mu}_F(x)) \mid x \in X\} \text{ where } \hat{\mu}_F : X \to D[0, 1].$$

Definition 2.7 — A mapping $T: [0, 1] \times [0, 1] \to [0, 1]$ is called a t-norm, if for every $x, y, z \in [0, 1]$, it satisfies the following conditions :

1. $T(x, 1) = x, T(0, 0) = 0$,
2. $T(x, y) = T(y, x)$,
3. $T(T(x, y), z) = T(x, T(y, z))$,
4. if $y \leq z$ then $T(x, y) \leq T(x, z)$.

Let T be a t-norm, if for arbitrary $x \in [0, 1]$, it satisfies $T(x, x) = x$, then T is called an idempotent t-norm.

Definition 2.8 (Xiaoping Li and Gujun Wang[11]) — Let T be an idempotent t-norm. Define the mapping $T_H : D[0, 1] \times D[0, 1] \to D[0, 1]$ by

$$(\hat{a}, \hat{b}) \to T_H(\hat{a}, \hat{b}) = [T(a^-, b^-), T(a^+, b^+)],$$

the T_H is called an idempotent interval t-norm.

Definition 2.9 — A mapping $S: [0, 1] \times [0, 1] \to [0, 1]$ is called a t-conorm, if for every $x, y, z \in [0, 1]$, it satisfies the following conditions :

1. $S(x, 0) = x, \quad S(1, 1) = 1$,
2. $S(x, y) = S(y, x)$,
3. $S(S(x, y), z) = S(x, S(y, z))$,
4. if $y \leq z$ then $S(x, y) \leq S(x, z)$.

Let S be a t-conorm, if for arbitrary $x \in [0, 1]$, it satisfies $S(x, x) = x$, then S is called an idempotent t-conorm.

Definition 2.10 (Xiapong Li and Guijun Wang[12]) — Let S be an idempotent t-conorm. Define the mapping $S_H : D[0, 1] \times D[0, 1] \to D[0, 1]$ by

$$(\hat{a}, \hat{b}) \to S_H(\hat{a}, \hat{b}) = [S(a^-, b^-), S(a^+, b^+)]$$

the S_H is called an idempotent interval t-conorm.

Definition 2.11 (Xiaoping Li and Guijun Wang[11]) — Let G be a group, T_H be an idempotent interval t-norm. An interval-valued fuzzy subset F of G is called an interval-valued fuzzy subgroup with respect to T_H, if the following conditions are fulfilled.

(*i*) $T_H(\hat{\mu}_F(x), \hat{\mu}_F(y)) \leq \hat{\mu}_F(xy)$ for all $x, y \in G$,

(*ii*) $\hat{\mu}_F(x) \leq \hat{\mu}_F(x^{-1})$ for all $x \in G$.

Definition 2.12 (Xiaoping Li and Guijun Wang[12]) — Let G be a group, S_H be an idempotent interval t-conorm. An interval-valued fuzzy subset F of G is called an interval-valued fuzzy subgroup with respect to S_H, if the following conditions are fulfilled.

(*i*) $(\hat{\mu}_F(xy), \hat{\mu}_F(x)) \leq S_H \hat{\mu}_F(y)$, for all $x, y \in G$,

(*ii*) $\hat{\mu}_F(x) \leq \hat{\mu}_F(x)$ for all $x \in G$.

3. T_H and S_H-Interval Valued Fuzzy H_v-Subgroups

In this section, we first define the T_H and S_H-interval valued fuzzy H_v-subgroups, and then we give some results.

Definition 3.1 — Let H be an H_v-group, T_H be an idempotent interval t-norm. An interval-valued fuzzy subset F of H is called an interval-valued fuzzy H_v-subgroup with respect to T_H, if the following conditions hold :

(*i*) $T_H(\hat{\mu}_F(x), \hat{\mu}_F(y)) \leq \inf_{\alpha \in x \cdot y} \left\{ \hat{\mu}_F(\alpha) \right\}$ for all $x, y \in H$,

(*ii*) for all $x, a \in H$ there exists $y \in H$ sucht hat $x \in a \cdot y$ and

$$T_H(\hat{\mu}_F(x), \hat{\mu}_F(a)) \leq \hat{\mu}_F(y),$$

(*iii*) for all $x, a \in H$ there exists $z \in H$ such that $x \in z \cdot a$ and

$$T_H(\hat{\mu}_F(x), \hat{\mu}_F(a)) \leq \hat{\mu}_F(z).$$

Definition 3.2 — Let H be an H_v-group, S_H be an idempotent interval t-conorm. An interval-valued fuzzy subset F of H is called an interval-valued fuzzy H_v-subgroup with respect to S_H, if the following conditions hold:

(*i*) $\sup_{\alpha \in x \cdot y} \{\hat{\mu}_F(\alpha)\} \leq S_H(\hat{\mu}_F(x), \hat{\mu}_F(y))$ for all $x, y \in H$,

(*ii*) for all $x, a \in H$ there exists $y \in H$ such that $x \in a \cdot y$ and

$$\hat{\mu}_F(y) \leq S_H(\hat{\mu}_F(x), \hat{\mu}_F(a)),$$

(*iii*) for all $x, a \in H$ there exists $z \in H$ such that $x \in z \cdot a$ and

$$\hat{\mu}_F(z) \leq S_H(\hat{\mu}_F(x), \hat{\mu}_F(a)).$$

Definition 3.3 — Let X be a non-empty set and F an interval-valued fuzzy subset of X. Then we define

$$F_{[t,s]} = \left\{ x \in X \mid \hat{\mu}_F(x) \geq [t\ s] \right\},$$

and
$$\overline{F}_{[t,s]} = \left\{ x \in X \mid \hat{\mu}_F(x) \leq [t, s] \right\}.$$

Theorem 3.4 — *Let H be an H_v-group and T_H an idempotent interval t-norm. Let F be an interval-valued fuzzy subset of H, then F is an interval-valued fuzzy H_v-subgroup with respect to T_H if and only if for every t, s where $0 \leq t \leq s \leq 1$, $F_{[t,s]} (\neq 0)$ is an H_v-subgroup of H.*

PROOF : Suppose that F is an interval-valued fuzzy H_v-subgroup of H. For every $x, y \in F_{[t,s]}$ we have $\hat{\mu}_F(x) \geq [t, s]$ and $\hat{\mu}_F(y) \geq [t, s]$, hence $T_H(\hat{\mu}_F(x), \hat{\mu}_F(y)) \geq [t, s]$ and so $\inf_{\alpha \in x \cdot y} \left\{ \hat{\mu}_F(\alpha) \right\} \geq [t, s]$. Therefore for every $\alpha \in x \cdot y$ we have $\alpha \in F_{[t,s]}$, so $x \cdot y \subseteq F_{[t,s]}$. Hence for every $a \in F_{[t,s]}$ we have $a \cdot F_{[t,s]} \subseteq F_{[t,s]}$. Now let $x \in F_{[t,s]}$ then there exists $y \in H$ such that $x \in a \cdot y$ and $T_H(\hat{\mu}_F(x), \hat{\mu}_F(a)) \leq \hat{\mu}_F(y)$. From $x \in F_{[t,s]}, a \in F_{[t,s]}$ we get $T_H(\hat{\mu}_F(x), \hat{\mu}_F(a)) \geq [t, s]$ and so $\hat{\mu}_F(y) \geq [t, s]$ or $y \in F_{[t,s]}$ and this proves that $F_{[t,s]} \subseteq a \cdot F_{[t,s]}$. Similarly, we get $F_{[t,s]} = F_{[t,s]} \cdot a$.

Conversely, assume that for every $[t\ s] \in D[0, 1]$, $F_{[t,s]} (\neq 0)$ is an H_v-subgroup with respect to T_H. For every $x, y \in H$ if we put $[t_0, s_0] = T_H(\hat{\mu}_F(x), \hat{\mu}_F(y))$ then $x \in F_{[t_0,s_0]}$, $y \in F_{[t_0,s_0]}$ and so $x \cdot y \subseteq F_{[t_0,s_0]}$. Therefore for every $\alpha \in x \cdot y$ we have $\alpha \in F_{[t_0,s_0]}$ implying $\inf_{\alpha \in x \cdot y} \left\{ \hat{\mu}_F(\alpha) \right\} \geq T_H(\hat{\mu}_F(x), \hat{\mu}_F(y))$ and in this way the condition (i) of Definition 3.1 is verified. To verify the second condition if for every $a, x \in H$ we put $[t_1, s_1] = T_H(\hat{\mu}_F(a), \hat{\mu}_F(x))$ then $x \in F_{[t_1,s_1]}$ and $a \in F_{[t_1,s_1]}$, so there exists $y \in F_{[t_1, s_1]}$ such that $x \in a \cdot y$. On the other hand since $y \in F_{[t_1, s_1]}$ then $\hat{\mu}_F(y) \geq [t_1, s_1]$ and hence $T_H(\hat{\mu}_F(a), \hat{\mu}_F(x)) \leq \hat{\mu}_F(y)$. In the similar way third condition of Definition 3.1 is valid. □

***Theorem* 3.5** — *Let H be an H_v-group and S_H an idempotent interval t-conorm. Let F be an interval-valued fuzzy subset of H, then F is an interval-valued fuzzy H_v-subgroup with respect to S_H if and only if for every t, s where $0 \leq t \leq s \leq 1$, $\overline{F}_{[t, s]} (\neq 0)$ is an H_v-subgroup of H.*

PROOF : The proof is similar to the proof of Theorem 3.4 and we omit it. □

Definition 3.6 — Let f be a mapping from a non-empty set X into a non-empty set Y. Let A be an interval-valued fuzzy subset of X and B an interval-valued fuzzy subset of Y. Then the image $f[A]$ of A is the interval-valued fuzzy subset of Y with the membership function defined by

$$\hat{\mu}_{f[A]}(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} \left\{ \hat{\mu}_A(z) \right\} & \text{if } f^{-1}(y) \neq 0 \\ & \forall, y \in Y \\ [0, 0] & \text{otherwise} \end{cases}$$

The inverse image $f^{-1}[B]$ of B is the interval-valued fuzzy subset of X with the membership function defined by

$$\hat{\mu}_{f^{-1}[B]}(x) = \hat{\mu}_B(f(x)) \text{ for all } x \in X.$$

Applying the definition, clearly we have

$$\hat{\mu}_{f[A]}(y) = \left[\bigwedge_{x \in f^{-1}(y)} \mu_A^-(x), \bigwedge_{x \in f^{-1}(y)} \mu_A^+(x) \right] \text{ for all } y \in Y,$$

$$\hat{\mu}_{f^{-1}[B]}(x) = [\mu_B^-(f(x)), \mu_B^+(f(x))] \text{ for all } x \in X.$$

Definition 3.7 (Vougiouklis[10]) — Let H_1 and H_2 be two H_v-groups. A function $f: H_1 \to H_2$ is called an H_v-homomorphism if it satisfies the condition $f(x \cdot y) \cap f(x) \cdot f(y) \neq 0$ for all $x, y \in H_1$; f is called a strong homomrphism if $f(x \cdot y) = f(x) \cdot f(y)$ for all $x, y \in H_1$. A strong homomorphism is called epimorphism if it is onto.

Lemma 3.8 — Let H_1 and H_2 be two H_v-groups and the map $f: H_1 \to H_2$ be an epimorphism. If K be an H_v-subgroup of H_1, then $f(K)$ is an H_v-subgroup of H_2.

***Theorem* 3.9** — *Let H_1 and H_2 be two H_v-groups and f be a strong homomorphism from H_1 into H_2 and T_H be an idempotent interval t-norm.*

(i) Let A be an interval-valued fuzzy H_v-subgroup of H_1 with respect to T_H. Then the image $f[A]$ of A is an interval-valued fuzzy H_v-subgroup of H_2 with respect to T_H.

(ii) Let B be an interval-valued fuzzy H_v-subgroup of H_2 with respect to T_H. Then the inverse image $f^{-1}[B]$ of B is an interval-valued fuzzy H_v-subgroup of H_1 with respect to T_H.

PROOF : (*i*) Suppose that A be an interval-valued fuzzy H_v-subgroup of H_1. By Theorem 3.4, for every $[t, s] \in D[0, 1]$, $A_{[t, s]} (\neq 0)$ is an H_v-subgroup of H_1. Therefore by Lemma 3.8, $f(A_{[t, s]})$ is an H_v-subgroup of H_2. Since $(f[A])_{[t, s]} = f(A_{[t, s]})$, we get $(f[A])_{[t, s]}$ is an H_v-subgroup of H_2. Therefore $f[A]$ is an interval-valued fuzzy H_v-subgroup of H_2.

(*ii*) For arbitrary $x, y \in H_1$ and for every $\alpha \in x \cdot y$, we have

$$\hat{\mu}_{f^{-1}[B]}(\alpha) = \hat{\mu}_B(f(\alpha))$$

$$\geq T_H(\hat{\mu}_B(f(x)), \hat{\mu}_B(f(y)))$$

$$= T_H(\hat{\mu}_{f^{-1}[B]}(x), \hat{\mu}_{f^{-1}[B]}(y))$$

Therefore $\inf_{\alpha \in x \cdot y} \left\{ \hat{\mu}_{f^{-1}[B]}(\alpha) \right\} \geq T_H(\hat{\mu}_{f^{-1}[B]}(x), \hat{\mu}_{f^{-1}[B]}(y))$, and in this way the first condition of Definition 3.1 is verified. To verify the second condition, let $x, a \in H_1$ then by reproduction axiom there exists $y \in H_1$ such that $x \in a \cdot y$ and so $f(x) \in f(a) \cdot f(y)$. Now we have

$$T_H(\hat{\mu}_{f^{-1}[B]}(x), \hat{\mu}_{f^{-1}[B]}(a)) = T_H(\hat{\mu}_B(f(x)), \hat{\mu}_B(f(a)))$$

$$\leq \hat{\mu}_B(f(y))$$

$$= \hat{\mu}_{f^{-1}[B]}(y).$$

In the similar way the third condition of Definition 3.1 is valid. □

***Theorem* 3.10** — *Let H_1 and H_2 be two H_v-groups and f be a strong homomorphism from H_1 into H_2, S_H be an idempotent interval t-conorm.*

(i) Let A be an interval-valued fuzzy H_v-subgroup of H_1 with respect to S_H. Then the image $f[A]$ of A is an interval-valued fuzzy H_v-subgroup of H_2 with respect to S_H.

(ii) Let B an interval-valued fuzzy H_v-subgroup of H_2 with respect to S_H. Then the inverse image $f^{-1}[B]$ of B is an interval-valued fuzzy H_v-subgroup of H_1 with respect to S_H.

PROOF : The proof is similar to the proof of Theorem 3.9 and we omit it. □

Now let $(H, \cdot)$ be an H_v-group. The relation β^* is the smallest equivalence relation on H such that the quotient H/β^* is a group. β^* is called the fundamental equivalence relation on H. This relation is studied by Corsini[2], see also[10]. Suppose $\beta^*(a)$ is the equivalence class containing

$a \in H$. According to[10] the product $\odot$ on H/β^* is defined as follows :

$$\beta^*(a) \odot \beta^8(b) = \beta^*(c) \text{ for all } c \in \beta^*(a) \cdot \beta^*(b).$$

Definition 3.11 — Let $(H, \cdot)$ be an H_υ-group and F an interval-valued fuzzy subset of H. The interval-valued fuzzy subset F_{β^*} is defined as follows :

$$\mu F_{\beta^*} : H/\beta^* \to D[0, 1]$$

$$\mu F_{\beta^*}(\beta^*(x)) = \underset{a \in \beta^*(x)}{d\sup} \left\{ \hat{\mu}_F(a) \right\}.$$

***Theorem* 3.12** — *Let F be an interval-valued fuzzy H_υ-subgroup of an H_υ-group H with respect to T_H. Then F_{β^*} is an interval-valued fuzzy subgroup of H/β^* with respect to T_H.*

PROOF : We know every group is an H_υ-group. Therefore, first, we consider H/β^* as an H_υ-group. Then by Theorem 3.9, F_{β^*} is an interval-valued fuzzy H_υ-subgroup of H/β^* with respect to T_H, i.e., we have

$$(i)\ T_H(\mu F_{\beta^*}(\beta^*(x)), \mu F_{\beta^*}(\beta^*(y))) \leq \inf_{\beta^*(\alpha) \in \beta^*(x) \odot \beta^*(y)} \left\{ \mu F_{\beta^*}(\beta^*(\alpha)) \right\},$$

$$\forall \beta^*(x), \beta^*(y) \in H/\beta^*$$

(*ii*) for all $\beta^*(x), \beta^*(a) \in H/\beta^*$ there exists $\beta^*(y) \in H/\beta^*$ such that

$$\beta^*(x) = \beta^*(a) \odot \beta^*(y) \text{ and } T_H(\mu F_{\beta^*}(\beta^*(x)), \mu F_{\beta^*}(\beta^*(a)) \leq \mu F_{\beta^*}(\beta^*(y)).$$

Now for all $\beta^*(x)$ in H/β^* we prove that $\mu F_{\beta^*}(\beta^*(x)) \leq \mu F_{\beta^*}(\beta^*(x)^{-1})$. Since $\beta^*(x) \in H/\beta^*$, by considering $\beta^*(a) = \beta^*(x)$ which is obtained from the second condition there exists $\beta^*(y_1)$ in H/β^* such that $\beta^*(x) = \beta^*(x) \odot \beta^*(y_1)$ and $T_H(\mu F_{\beta^*}(\beta^*(x), \mu F_{\beta^*}(\beta^*(x))) \leq \mu F_{\beta^*}(\beta^*(y_1))$. From $\beta^*(x) = \beta^*(x) \odot \beta^*(y_1)$ we obtain $\omega_H = \beta^*(y_1)$, where ω_H denotes the unit of the group H/β^*. Therefore

$$(I)\ \mu F_{\beta^*}(\beta^*(x)) \leq \mu F_{\beta^*}(\omega_H)$$

Now considering $\beta^*(x)$, ω_H in H/β^*, by condition (ii) above there exists $\beta^*(y_2)$ in H/β^* such that $\omega_H = \beta^*(x) \odot \beta^*(y_2)$ and $T_H(\mu F_{\beta^*}(\omega_H), \mu F_{\beta^*}(\beta^*(x))) \leq \mu F_{\beta^*}(\beta^*(y_2))$. From $\omega_H = \beta^*(x) \odot \beta^*(y_2)$ we obtain $\beta^*(y_2) = \beta^*(x)^{-1}$, so

$$(II)\ T_H(\mu F_{\beta^*}(\omega_H), \mu F_{\beta^*}(\beta^*(x))) \leq \mu F_{\beta^*}(\beta^*(x)^{-1})$$

By (I) and (II) the inequality $\mu F_{\beta^*}(\beta^*(x)) \leq \mu F_{\beta^*}(\beta^*(x)^{-1})$ is obtained. □

***Theorem* 3.13** — *Let F be an interval-valued fuzzy H_v-subgroup of an H_v-group H with respect to S_H. Then F_{β^*} is an interval-valued fuzzy subgroup of H/β^* with respect to S_H.*

PROOF : The proof is similar to the proof of Theorem 3.12 and we omit it. □

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