Indian J. pure appl. Math., 35(1) : 71-80, January 2004

g^*-CLOSED SETS IN BITOPOLOGICAL SPACES

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(*Received* 20 *August* 2002; *after revision* 29 *April* 2003; *accepted* 23 *July* 2003)

In this paper we introduce g^*-closed sets[12] in bitopological spaces. Properties of these sets are investigated and we introduce two new bitopological spaces $(i,j)-T^*_{1/2}$ and $(i,j)-{}^*T_{1/2}$ spaces as applications. Further we introduce and study g^*-continuity[12] in bitopological spaces.

Key Words : (i,j)–g^* **Closed Sets;** $(i,j)-T^*_{1/2}$ **Space;** $(i,j-{}^*T_{1/2}$ **Space and** $D^*\,(i,j)$**-Continuity**

1. INTRODUCTION

A triple (X, τ_1, τ_2) where X is a non-empty set and τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly[6] initiated the study of such spaces.In 1985, Fukutake[2] introduced the concepts of g-closed sets[8] in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Recently Veera Kumar[12] introduced and studied the concepts of g^*-closed set and g^*-continuity in topological spaces. g^*-closed set lies between closed set and g-closed set.

The purpose of this paper is to introduce the concepts of g^*-closed sets[12], $T^*_{1/2}$-spaces[12], ${}^*T_{1/2}$-spaces[12] and g^*-continuity[12] for bitopological spaces and investigate some of their properties.

2. PRELIMINARIES

If A is a subset of X with a topology τ, then the closure of A is denoted by τ-cl (A) or cl(A), the interior of A is denoted by τ-int (A) or int (A) and the complement of A in X is denoted by A^c.

Definition 2.1 — A subset A of a topological space (X, τ) is called

(*i*) a generalized closed set (briefly g-closed set[8]) if $cl(A) \subseteq U$ whenever $A \subseteq U$ ad U is open in X.

(*ii*) a generalized open set (briefly g-open set[8]) if A^c is g-closed in X.

(*iii*) a preclosed set[8] if cl(int (A)) $\subseteq A$.

(*iv*) a regular open set[11] if A = int (cl (A))

(*v*) a semi-open set[7] if $A \subseteq$ cl (int (A))

Definition 2.2 — The intersection of all pre closed sets containing A is called the pre-closure of A and it is denoted by τ-pcl(A) or pcl(A).

Throughout this paper X and Y always represent nonempty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) on which no separation axioms are assumed unless explicitly mentioned and the integers $i, j, k \in \{1, 2\}$. For a subset A of X, τ_i-cl (A) (resp. τ_i-int (A), τ_i-pcl(A)) denote the closure (resp. interior, preclosure) of A with respect to the topology τ_i. We denote the family of all g-open subsets of X with respect to the topology τ_i by $GO(X, \tau_i)$ and the family of all τ_j-closed sets is denoted by the symbol F_j. By (i, j) we mean the pair of topologies (τ_i, τ_j).

Definition 2.3 — A subset A of a topological space (X, τ_1, τ_2) is called

(*i*) (i, j)-g-closed[2] if τ_j-cl$(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \tau_i$.

(*ii*) (i, j)-rg-closed[1] if τ_j cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in τ_i.

(*iii*) (i, j)-gpr-closed[4] if τ_j-pcl $(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in τ_i..

(*iv*) (i, j)-wg-closed[3] if τ_j-cl $(\tau_i$-int $(A)) \subseteq U$ whenver $A \subseteq U$ and $U \in \tau_i$.

(*v*) (i, j)-ω-closed[4] if τ_j-cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in τ_i.

The family of all (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-wg-closed and (i, j)-ω-closed) subsets of a bitopological space (X, τ_1, τ_2) is denoted by $\boldsymbol{D}(i, j)$ (resp. $\boldsymbol{D}_r(i, j)$, $\zeta(i, j)$, $\boldsymbol{W}(i, j)$ and $C(i, j)$).

Definition 2.4 — (*i*) A bitopological space (X, τ_1, τ_2) is said to be (i, j)-$T^2_{1/2}$ if every (i, j)-g-closed sets is τ_j-closed.

(*ii*) A bitopological space (X, τ_1, τ_2) is said to be strongly pairwise $T^2_{1/2}$ if it is $(1, 2)$-$T_{1/2}$ and $(2, 1)$-$T_{1/2}$.

Definition 2.5 — A map $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(*i*) $\tau_j - \sigma_k$-continuous[10] if $f^{-1}(V) \in \tau_j$ for every $V \in \sigma_k$.

(*ii*) $\boldsymbol{D}(i, j) - \sigma_k$-continuous[10] (resp. $\boldsymbol{D}_r(i, j) - \sigma_k$-continuous[1], $\zeta(i, j) - \sigma_k$-continuous[4], $\boldsymbol{W}(i, j) - \sigma_k$-continuous[3] and $\boldsymbol{C}(i, j) - \sigma_k$-continuous[4]) if the inverse image of every σ_k-closed set is (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-wg-closed and $(i, j) - \omega$-closed) set in (X, τ_1, τ_2).

3. (i, j)-g^* Closed Sets

In this section we introduce the concept of (i, j)-g^*-closed sets in bitopological spaces.

Definition 3.1 — A subset A of a bitopological space (X, τ_1, τ_2) is said to be an (i, j)-g^*-closed set if τ_j-cl$(A) \subseteq U$ whenever $A \subseteq U$ and $U \in GO(X, \tau_i)$.

We denote the family of all (i, j)-g^*-closed sets in (X, τ_1, τ_2) by $\boldsymbol{D}^*(i, j)$.

Remark 3.2 : By setting $\tau_1 = \tau_2$ in Definition 3.1, *a* (i, j)-g^*-closed set is a g^*-closed set[12].

Proposition 3.3 — If A is τ_j-closed subset of (X, τ_1, τ_2) then A is (i, j)-g^*-closed.

The converse of the above proposition is not true as seen from the following example.

Example 3.4 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{c\}, \{a, c\}, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$. Then the subset $\{b\}$ is $(1, 2)$-g^*-closed but not τ_2-closed in (X, τ_1, τ_2).

Proposition 3.5 — If A is both τ_i-g-open and (i, j)-g^*-closed, then A is τ_j-closed.

Proposition 3.6 — In a bitopological space (X, τ_1, τ_2), every (i, j)-g^*-closed set is *(i)*, (i, j)-g-closed, *(ii)* (i, j)-rg-closed, *(iii)* (i, j)-gpr-closed and *(iv)* (i, j)-wg-closed.

The following examples show that the reverse implications of the above proposition are not true.

Example 3.7 — Let $X = \{a,b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{a\}, \{a, b\}, X\}$. Then the subset $\{b\}$ is $(1, 2)$-g-closed but not $(1,2)$-g^*-closed.

Example 3.8 — Let $X = \{a, b, c, d, e\}$, $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$ and $\tau_2 = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, X\}$. Then the set $A = \{a, b\}$ is $(1, 2)$-rg-closed but not $(1, 2)$-g^*-closed.

Example 3.9 — Let X $\{a, b, c\}$, $\tau_1 = \{\phi, \{c\}, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$ and $A = \{c\}$. Then A is $(1, 2)$-gpr-closed but not $(1, 2)$-g^*-closed.

Example 3.10 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$. Then the subset $A = \{a\}$ is $(1, 2)$-wg-closed but not a $(1, 2)$-g^*-closed set.

Remark 3.11 : The following examples show that (i, j)-ω-closed sets and (i, j)-g^*-closed sets are independent.

Example 3.12 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the subset $A = \{c\}$ is $(1, 2)$-ω-closed set but not a $(1, 2)$-g^*-closed set.

Example 3.13 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, b\}, X\}$ and $B = \{a, c\}$. Then B is $(1, 2)$ g^*-closed set but not a $(1, 2)$-ω-closed set.

Remark 3.14 : (i, j)-g^*-closed sets and τ_j-g-closed sets are independent. In Example 3.10, the set $\{a\}$ is τ_2-g-closed but not a $(1, 2)$-g^*-closed set.

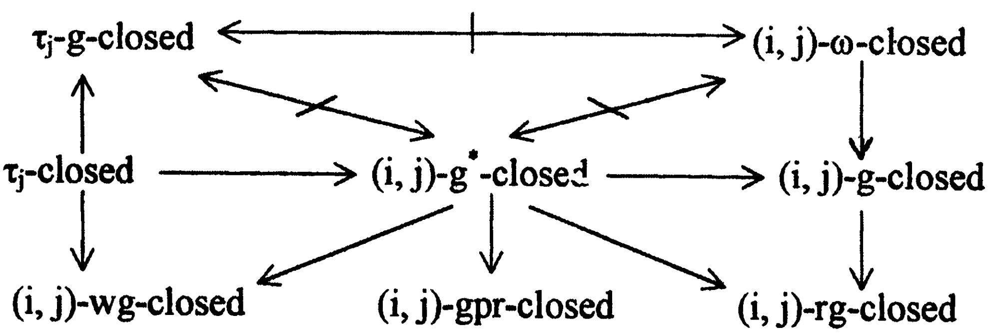


Diagram 3.16

Example 3.15 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, \{b, c\}, X\}$ and $\tau_2 = \{\phi, \{a, b\}, X\}$. Then the set $\{a, b\}$ is (1, 2)-g^*-closed but not a τ_2-g-closed.

where $A \rightarrow B$ (resp. $A \not\leftrightarrow B$) represents A implies B but not conversely (resp. A and B are independent).

Proposition 3.17 — If $A, B \in D^* (i,j)$, then $A \cup B \in D^* (i,j)$.

Remark 3.18 : The intersection of two (i, j)-g^*-closed sets need not be (i, j)-g^*-closed as seen from the following example.

Example 3.19 —If $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{b, c\}, X\}$ and $\tau_2 = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$, then $\{a, b\}$ and $\{b, c\}$ are (2, 1)-g^*-closed but $\{a, b\} \cap \{b, c\} = \{b\}$ is not (2, 1)-g^*-closed.

Remark 3.20 : D^* (1, 2) is generally not equal to D^* (2, 1). For example $D^*(1, 2) \neq D^* (2, 1)$ in Example 3.19.

Proposition 3.21 — If $\tau_1 \subseteq \tau_2$ in (X, τ_1, τ_2) then $D^* (2, 1) \subseteq D^* (1, 2)$.

The converse of the above proposition is not true as seen from the following example.

Example 3.22 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$. Then $D^* (2, 1) \subseteq D^* (1, 2)$ but τ_1 is not contained in τ_2.

Proposition 3.23 — For each element x of (X, τ_1, τ_2), $\{x\}$ is τ_i-g-closed or $\{x\}^c$ is (i, j)-g^*-closed.

Proposition 3.24 — If A is (i, j)-g^*-closed, then τ_j-cl(A)- A contains no non-empty τ_i-$g$closed set.

PROOF : Let A be an (i, j)-g^*-closed set and F be a τ_i-g-closed set such that $F \subseteq \tau_j$-cl(A) - A. Since $A \in D^*(i, j)$, we have τ_j-cl $(A) \subseteq F^c$. Thus $F \subseteq \tau_j$-cl(A) $\cap$ $(\tau_j$-cl $(A))^c = \phi$.

The converse of the above proposition is not true as seen from the following example.

Example 3.25 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$. If $A = \{b\}$, then τ_2-cl (A) - $A = \{c\}$ does not contain any nonempty τ_1-g closed set. But A is not $(1, 2)$-g^*-closed.

Corollary 3.26 — If A is (i, j)-g^*-closed set in (X, τ_1, τ_2), then A is τ_j-closed if and only if τ_j-cl $(A) - A$ is τ_i-g-closed.

PROOF : Necessity : If A is τ_j-closed, then τ_j-cl $(A) = A$. i.e., τ_j-cl $(A) - A = \phi$ and hence τ_j-cl $(A) - A$is τ_i-g-closed.

Sufficiency : If τ_j-cl $(A) - A$ s τ_i-g-closed, then by Proposition 3.24, τ_j-cl$(A) - A = \phi$, since A is (i, j)-g^*-closed. Therefore A is τ_j-closed.

Proposition 3.27 — If A is an (i, j)-g^*-closed set, then τ_i-cl$(x) \cap A \neq \phi$ holds for each $x \in \tau_j$-cl (A).

The converse of the above proposition is not true. The subset $A = \{b\}$ in (X, τ_1, τ_2) of Example 3.7 is not $(1, 2)$-g^*-closed. However τ_1-cl$(x) \cap A \neq \phi$ holds for each $x \in \tau_2$-cl(A).

Proposition 3.28 — If A is an (i, j)-g^*-closed set of (X, τ_i, τ_j) such that $A \subseteq B \subseteq \tau_j$-cl$(A)$, then B is also an (i, j)-g^*-closed set of (X, τ_i, τ_j).

Proposition 3.29 — Let $A \subseteq Y \subseteq X$ and suppose that A is (i, j)-g^*-closed in X. Then A is (i, j)-g^*-closed relative to Y.

Theorem 3.30 — *In a bitopological space (X, τ_1, τ_2), $GO(X, \tau_i) \subseteq F_j$ if and only if every subset of X is an (i, j)-g^*-closed set.*

PROOF : Suppose that $GO(X, \tau_i) \subseteq F_j$. Let A be a subset of X such that $A \subseteq U$, where $U \in GO(X, \tau_i)$. Then τ_j-cl$(A) \subseteq \tau_j$-cl $(U) = U$ and hence A is (i, j)-g^*-closed.

Conversely, suppose that every subset of X is (i, j)-g^*-closed. Let $U \in GO(X, \tau_i)$. Since U is (i, j)-g^*-closed, we have τ_j-cl$(U) \subseteq U$. Therefore $U \in F_j$ and hence $GO(X, \tau_i) \subseteq F_j$.

4. (i, j)-$T^*_{1/2}$ SPACES AND (i, j)-${}^*T_{1/2}$ SPACES

In this section, we introduce $(i,j)-T^*_{1/2}$ and $(i,j)-{}^*T_{1/2}$ bitopological spaces and in Theorem 4.19 we prove that the $(i,j)-{}^*T_{1/2}$ spaces is the dual of the class of $(i,j)-T^*_{1/2}$ spaces to the class of $(i,j)-T_{1/2}$ spaces.

Definition 4.1 — A bitopological space (X, τ_1, τ_2) is said to be an $(i,j)-T^*_{1/2}$ space if every (i, j)-g^*-closed set is τ_j-closed.

Proposition 4.2 — If (X, τ_1, τ_2) is $(i,j)-T_{1/2}$ space, then it is an $(i,j)-T^*_{1/2}$ space but not conversely.

Example 4.3 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{X\}$ and $\tau_2 = \{\phi, \{a\}, \{b, c\}, X\}$. Then (X, τ_1, τ_2) is a $(1, 2)$-$T^*_{1/2}$ space but not a $(1, 2)$-$T_{1/2}$-space.

***Theorem* 4.4** — *A bitopological space (X, τ_1, τ_2) is an $(i,j)-T^*_{1/2}$ space if and only if $\{x\}$ is τ_j-open or τ_i-g-closed for each $x \in X$.*

PROOF : Suppose that $\{x\}$ is not τ_i-g-closed. Then $\{x\}^c$ is (i, j)-g^*-closed by Proposition 3.23. Since (X, τ_1, τ_2) is an $(i,j)-T^*_{1/2}$ space, $\{x\}^c$ is τ_j-closed. Therefore $\{x\}$ is τ_j-open.

Conversely, let F be an (i, j)-g^*-closed set. By assumption, $\{x\}$ is τ_j-open or τ_i-g-closed for any $x \in \tau_j$-cl(F).

Case (i) — Suppose $\{x\}$ is τ_j-open. Since $\{x\} \cap F \neq \phi$, we have $x \in F$.

Case (ii) — Suppose $\{x\}$ is τ_j-g-closed. If $x \notin F$, then $\{x\} \subseteq \tau_j$ -cl (F) − F, which is a contradiction to Proposition 3.24. Therefore $x \in F$.

Thus in both cases, we conclude that F is τ_j-closed. Hence (X, τ_1, τ_2) is an $(i,j)-T^*_{1/2}$ space.

Remark 4.5 : (X, τ_1)-space is not generally $T^*_{1/2}$-space even if (X, τ_1, τ_2) is $(1, 2)$-$T^*_{1/2}$ space as shown in the following Example 4.6. Also (X, τ_1, τ_2) is not generally $(1, 2)$-$T^*_{1/2}$ space even if both (X, τ_1) and (X, τ_2) are $T^*_{1/2}$-spaces. This is shown in Example 4.7.

Example 4.6 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then (X, τ_1) is not $T^*_{1/2}$ but (X, τ_1, τ_2) is $(1, 2)$-$T^*_{1/2}$.

Example 4.7 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{a\},$

$\{a, b\}$, $\{a, c\}$, $X\}$. Then both (X, τ_1) and (X, τ_2) are $T^*_{1/2}$ but (X, τ_1, τ_2) is not $(1, 2)$-$T^*_{1/2}$.

Definition 4.8 — A bitopological space (X, τ_1, τ_2) is said to be strongly pairwise $T^*_{1/2}$-space if it is both $(1, 2)$-$T^*_{1/2}$ and $(2, 1)$-$T^*_{1/2}$.

Proposition 4.9 — If (X, τ_1, τ_2) is strongly pairwise $T_{1/2}$-space then it is strongly pairwise $T^*_{1/2}$-space but not conversely.

Example 4.10 — Let X, τ_1 and τ_2 be as in Example 4.3. Then (X, τ_1, τ_2) is also a $(2, 1)$-$T^*_{1/2}$ space and therefore it is strongly pairwise $T^*_{1/2}$ space. But (X, τ_1, τ_2) is not a strongly pairwise $T_{1/2}$-space, since it is not a $(1, 2)$-$T^*_{1/2}$ space.

We now introduce the following definition.

Definition 4.11 — A bitpological space (X, τ_1, τ_2) is said to be an $(i, j$- ${}^*T_{1/2}$ space if every (i, j)-g-closed set is (i, j)-g^*-closed.

Proposition 4.12 — Every (i, j)-$T_{1/2}$ space is an (i, j)- ${}^*T_{1/2}$ space but not conversely.

Example 4.13 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{a, c\}, X\}$ and $\tau_2 = \{\phi, \{a, b\}, X\}$. Then (X, τ_1, τ_2) is a $(1, 2)$- ${}^*T_{1/2}$ space but not a $(1, 2)$-$T_{1/2}$ space.

Remark 4.14 : (i, j)-$T^*_{1/2}$ and (i, j)- ${}^*T_{1/2}$ spaces are independent as seen from the following two examples.

Example 4.15 — Let X, τ_1 and τ_2 be as in Example 4.3. Then (X, τ_1, τ_2) is not a $(1, 2)$-$T^*_{1/2}$ space, but it is $(1, 2)$-$T^*_{1/2}$ space.

Example 4.16 — Let X, τ_1 and τ_2 be as in Example 4.3. Then (X, τ_1, τ_2) is not a $(1,2)$- ${}^*T_{1/2}$ space, however it is a $(1, 2)$- ${}^*T_{1/2}$ space as in Example 4.13.

Theorem 4.17 — *A bitopological space (X, τ_1, τ_2) is an (i, j)-$T_{1/2}$ space if and only if it is both (i, j)-$T^*_{1/2}$ and (i, j)-$T^*_{1/2}$.*

PROOF : Suppose that (X, τ_1, τ_2) is an (i, j)-$T_{1/2}$ space. Then by Proposition 4.12 and Proposition 4.2, (X, τ_1, τ_2) is (i, j)- ${}^*T_{1/2}$ space and (i, j)-$T^*_{1/2}$. space.

Conversely, suppose that (X, τ_1, τ_2) is both (i, j)-${}^*T_{1/2}$ and (i, j)-$T^*_{1/2}$. Let A be an (i, j)-closed set of (X, τ_1, τ_2). Since (X, τ_1, τ_2) is an (i, j)-${}^*T_{1/2}$ space, A is an (i, j)-g^*-closed set. Since (X, τ_1, τ_2) is an (i, j)-$T^*_{1/2}$ space. A is τ_j-closed set of (X, τ_1, τ_2). Therefore (X, τ_1, τ_2) is an (i, j)-$T_{1/2}$ space.

5. g^*-Continuous Maps

In this section we introduce g^*-continuous maps in bitopological spaces.

Definition 5.1 — A map $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called D^* (i, j)-σ_k-continuous if the inverse image of every σ_k-closed set is an (i, j)-g^*-closed set.

Proposition 5.2 — If $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tau_j - \sigma_k$-continuous, then it is D^* (i, j)-σ_k-continuous but not conversely.

Example 5.3 — Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$ and $Y = \{p, q\}$, $\sigma_1 = \{\phi, \{p\}, Y\}$ and $\sigma_2 = \{\phi, \{q\}, Y\}$. Define a map $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = p$, $f(b) = f(c) = q$. Then f is D^* $(2, 1) - \sigma_2$-continuous, but not $\tau_1 - \sigma_2$-continuous.

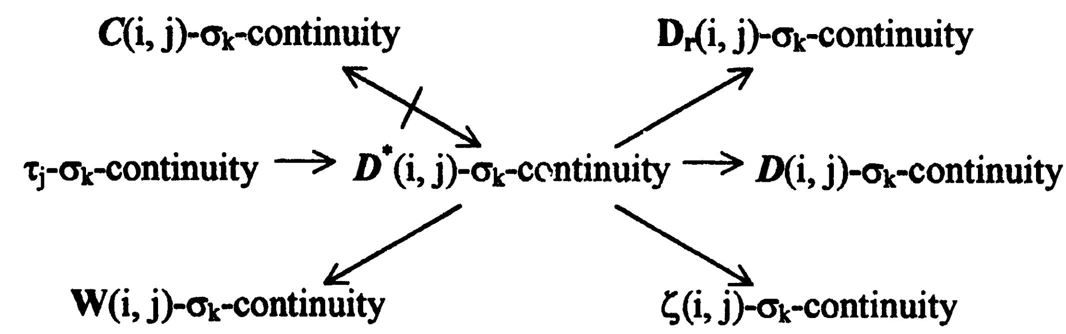
Proposition 5.4 — If $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $D^*(i,j) - \sigma_k$-continuous then it is (i). $\boldsymbol{D}(i,j) - \sigma_k$-continuous, (ii) $\boldsymbol{D}_r(i,j) - \sigma_k$-continuous, (iii) $\zeta(i,j) - \sigma_k$-continuous and (iv) $W(i,j) - \sigma_k$-continuous.

However the reverse implications of the above proposition are not true in general as seen from the following examples.

Example 5.5 — The map f in example 5.3 is $\boldsymbol{D}(1,2) - \sigma_1$-continuous but not $\boldsymbol{D}^*(1,2) - \sigma_1$-continuous.

Example 5.6 — Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{a, b\}, X\}$ and $\sigma_1 = \{\phi, \{a\}, \{b, c\}, Y\}$, $\sigma_2 = \{\phi, \{b\}, c\}, \{b, c\}, \{a \{c\}, Y\}$. Define $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then the map f is $\boldsymbol{D}_r(2, 1) - \sigma_2$-continuous but not $\boldsymbol{D}^*(2, 1) - \sigma_2$-continuous.

Example 5.7 — Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{c\}, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{a, b\}, X\}$ and $\sigma_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$, $\sigma_2 = \{\phi, \{a\}, Y\}$. Then the identity map f on X is $\zeta(1, 2) - \sigma_2$-continuous but not $\boldsymbol{D}^*(1, 2) - \sigma_2$-continuous.



Example 5.8 — Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi\ \{a, b\}, X\}$, $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$ and $\sigma_1 = \{\phi, \{a\}, \{b, c\}, Y\}$, $\sigma_2 = \{\phi, \{b, c\}, Y\}$. Then the identity map f on X is $W(1, 2)$-σ_2-continuous but not $\boldsymbol{D}^*(1, 2) - \sigma_2$-continuous.

Remark 5.9 $C(i, j) - \sigma_k$-continuous and $\boldsymbol{D}^*(i, j) - \sigma_k$-continuous are independent.

Example 5.10 — Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be the bitopological spaces of Examples 3.12 and 5.3 respectively. Define a map $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = f(b) = q$ and $f(c) = p$. Then f is $C(1, 2)$-σ_2-continuous but not $\boldsymbol{D}^*(1, 2) - \sigma_2$-continuous.

Example 5.11 — Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be the bitopological spaces of Examples 3.13 and 5.3 respectively. Define a map $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by $f(a) = f(c) = p$ and $f(b) = q$. Then f is $\boldsymbol{D}^*(1, 2) - \sigma_2$-continuous but not $\boldsymbol{C}(1, 2) - \sigma_2$-continuous.

Remark 5.12 : The following diagram summarizes the above discussions :

where $A \to B$ (resp. $A \not\leftrightarrow B$) represents A implies B but not conversely (resp. A and B are independent).

***Theorem* 5.13** — *Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a map*

(i) If (X, τ_1, τ_2) is an (i, j)-$T_{1/2}$ space then f is $\boldsymbol{D}(i, j) - \sigma_k$-continuous if and only if it is $\boldsymbol{D}^*(i, j) - \sigma_k$-continuous.

(ii) If (X, τ_1, τ_2) is an (i, j)-$T^*_{1/2}$ space then f is $\tau_j - \sigma_k$-continuous if and only if it is $\boldsymbol{D}^*(i, j) - \sigma_k$-continuous.

ACKNOWLEDGEMENT

The first author wishes to thank the UGC, New Delhi for the award of research fellowship to him under CFDP during the IX plan period. The authors also express their grateful thanks to the referee for reading the manuscript and making helpful comments.

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