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STONE TOPOLOGY OF THE SET OF PRIME FILTERS OF A 0-DISTRIBUTIVE LATTICE

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In this paper we study the Stone topology of the set of prime filters of a bounded 0-distributive lattice and 0 and 1-distributive lattice. We also obtain some necessary and sufficient conditions for the subspace of maximal filters to be normal.

Key Words: Maximal Filters; Normal Space; Regular Space; Sub Space; Comaximal Ideals

1. INTRODUCTION

A 0-distributive (1-distributive) lattice is a lattice L with 0(1) in which $a \wedge b = 0 = a \wedge c$ ($a \vee b = 1 = a \vee c$) implies $a \wedge (b \vee c) = 0$ ($a \vee (b \wedge c) = 1$) (Balasubramani *et al.*[1]). For the topological concepts which have now become commonplace the reader is referred to Kelly[4] and Hu[3]. For the lattice theoretic concepts the reader is referred to Gratzer[2].

Let X be a topological space. X is called T_o if distinct points of X have distinct closures. A point p of X is called a T_1 point if the closure of p contains no point other than p. A point p of X is called an anti-T_1 point if the closure of no point other than p contains p (for closure we use the notation Cl.). X is called T_1 if every point of X is T_1. X is called T_2 if any two distinct points of X have disjoint neighbourhoods.

A closed (open) subset of X is called a closed domain (open domain) if it is identical with the closure of its interior (interior of its closure). A closed (open) subset of X is called regular if it is an intersection (union) of closed domains (open domains) whose interiors (closures) contain (are contained in) it. X is called regular if every open subset of X is regular. The regularity of X may alternatively be expressed as follows. Given a nonempty closed subset C of X and a point of $p \notin C$, we can find closed subsets C_1, C_2 of X containing C, p respectively such that $p \notin C_1, C_1 \cap C_2 = \phi$ and $C_1 \cup C_2 = X$. A T_1 regular space is called a T_3-space. A subset A of X is called compact, if every open cover of A has a finite subcover. A subset A of X is said to be dense if Cl. $A = X$. X is said to be connected if X is not the union of two disjoint nonempty open subset of X. Otherwise, X is said to be disconnected.

Let A and B be any two disjoint subsets of X, we say A is weakly separable from B if there exists an open subset of X containing A and disjoint from B. Clearly A is weakly separable from B if and only if $A \cap Cl.\ B = \phi$. X is called π_0 if every nonempty open subset of X contains a nonempty closed subset. X is called normal if given any two disjoint closed subsets C_1, C_2 of X we can find subsets C_3, C_4 of X containing C_1, C_2 respectively such that $C_1 \cap C_4 = \phi = C_2 \cap C_3$ and $C_3 \cup C_4 = X$. Let E be a subspace of the space X. A map$\Upsilon : X \rightarrow E$ is called a retraction of X onto E if Υ/E is the identify map on E. A subspace E of X is called a retract of X if there is a retraction of X onto E.

1.1. Definition

Throughout this section, L denotes a bounded 0-distributive lattice. $L(F)$ denotes the lattice of all filters of L and p the set of prime filters of L. For any filter A of L, $F(A)$ denotes the set of all prime filters containing A and $F^1(A) = p - F(A)$. Since L is 0-distributive every proper filter of L is contained in a prime filter (Balasubramani *et al.*[1]). Hence $F(A)$ is nonempty if $A \neq L$.

***Theorem* 1.1** — *Let* $\{Ai/i \in I\}$ *be any family of filters of* L and $A_1, ..., A_n$ *be any finite number of filters of L. Th.. ı*

1. $F(\vee A_i) = \cap F(A_i)$

2. $F(A_1 \cap \cdots \cap A_n) = F(A_1) \cup \cdots \cup F(A_n)$

3. $F(L) = \phi$

4. $F([1]) = p$

PROOF 1 : Let B be a prime filter of L. Then $B \supseteq \vee A_i$ if and only if $B \supseteq A_i$ for all i. Hence the result.

2. If B is a prime filter such that $B \supseteq A_1 \cap \cdots \cap A_n$, then $B \supseteq A_j$ for some $j \in \{1, ..., n\}$. Hence $F(A_1 \cap \cdots \cap A_n) \subseteq F(A_1) \cup \cdots \cup F(A_n)$. The reverse inclusion is obvious.

3. and 4 are obvious.

Hence the result.

Since $F'(A) = p - F(A)$ as a consequence of Theorem 1.1, we have

***Theorem* 1.2** — *Let* $\{Ai/i \in I\}$ *be any family of filters of* L and $A_1, ..., A_n$ *be any finite number of filters of L. Then*

1. $F^1(\vee A_i) = \cap F^1(A_i)$

2. $F^1(A_i \cap \cdots \cap A_n) = F^1(A_1) \cap \cdots \cap F^1(A_n)$

3. $F^1(L) = p$

4. $F^1([1]) = \phi$

By Theorem 1.2, it follows that $\{F^1(A)/A \in L(F)\}$ is a topology on p. We shall denote this topology by T and the resulting topological space (p, T) also by p when there is no ambiguity. The sets $F(S)$ are precisely the closed subsets of p.

Remark 1.3 : From Theorem 1.1 and Theorem 1.2 it follows that the mapping $A \to F^1(A)$ $(A \to F(A))$ is a homomorphism (dual homomorphism) of the lattice of open subsets (closed subsets) of p onto $L(F)$.

2. PROPERTIES

Theorem 2.1 — *If X is any subset of p, Cl. $X = F(X_0)$ where X_0 is the intersection of the members of X.*

PROOF : Clearly $F(X_0)$ is a closed subset of p and $X \subseteq F(X_0)$. If $X \subseteq F(A)$ for some filter A, then $A \subseteq X_0$ and so $F(X_0) \subseteq F(A)$. Hence the result.

Theorem 2.2 — p is T_0 and compact.

PROOF : From Theorem 2.1, it follows that the closure of a single point is the set of all prime filters containing it. Clearly of any two distinct (prime) filters there is one which does not contain the other. Hence distinct points of p have distinct closures. Thus p is T_0.

Let $p = \bigcup F^1(A_i)$. By Theorem 1.2, $p = F^1(\vee A_i)$. Since every proper filter of a 0-distributive lattice is contained in a prime filter (Balasubramani *et al.*[1]) it follows that $\vee A_i = L$. Hence there exists a finite number of elements $a_{i1}, ..., a_{in}$ $(a_{ij} \in A_{ij})$ such that $0 = a_{i1} \wedge ... \wedge a_{in}$. Consequently $A_{i1} \vee ... \vee A_{in} = L$ and so $p = F^1(A_{i1} \vee ... \vee A_{in}) = F^1(A_{i1}) \bigcup ... \bigcup F^1(A_{in})$. Thus p is compact.

Remark 2.3 : Since every maximal filter of a 0-distributive lattice is prime (Balasubramani *et al.*[1]) as an immediate consequence of Theorem 2.1, we have that the T_1 points of p are precisely the maximal filters of L and the closure of the set of T_1 points of p is $F(D)$ where D is the filter consisting of the dense elements of L.

Theorem 2.4 — *p is π_0 if and only if $D = [1)$.*

PROOF : Let $D = [1)$ and $F^1(A)$ be any nonempty open subset of p. Then $A \neq D$ and so $A \subseteq M$, for some maximal filter of M of L. $\{M\}$ is a closed subset of p by Remark 2.3 and clearly $F^1(A) \supseteq \{M\}$. Thus p is π_0.

Suppose $D \neq [1)$. Let $F(B)$ be any nonempty closed subset of p. Then $B \neq L$ and so $B \subseteq M$ for some maximal filter M. By Remark 2.3 $M \in p$, so that $M \in F(B)$. But $M \notin F^1(D)$. Hence $F^1(D) \not\supseteq F(B)$ and thus p is not π_0. Hence the theorem.

Let us denote the set of all maximal filters and the set of dense maximal filters of L by $\mathcal{M}$ and $\mathcal{M}_2$ respectively. By Remark 2.3, the subspaces $\mathcal{M}$ and $\mathcal{M}_2$ are T_1.

Theorem 2.5 — *The subspace $\mathcal{M}$ is the smallest of the subspaces Y of p such that Y is not weakly separable from any point outside it.*

PROOF : Let $A \in p$ and $A \notin \mathcal{M}$. Clearly $Cl\{A\} = F(A)$. Also $A \subseteq M$ for some $M \in \mathcal{M}$ and hence $M \cap Cl\{A\} \neq \phi$. Thus $\mathcal{M}$ is not weakly separable from A.

Let X be any subspace of p such that $M \not\subseteq Y$. Then there exists $M \in \mathcal{M}$ such that $M \notin Y$. By Remark 2.3 $Cl.\{M\} = \{M\}$ and so $Y \cap Cl.\{M\} = \phi$. Thus Y is weakly separable from M.

Theorem 2.6 — *Let Y be any subset of p containing $\mathcal{M}$. Then Y is compact. In particular $\mathcal{M}$ is compact.*

PROOF : Let $Y \subseteq \bigcup F^1(A_i)$. Then $Y \subseteq F^1(\vee A_i)$ and so $\mathcal{M} \subseteq \bigcup F^1(A_i)$. Then $Y \subseteq F^1(\vee A_i)$ and so $\mathcal{M} \subseteq F^1(\vee A_i)$. It follows that $\vee A_i = L$. Hence there exists a finite number of elements $a_{i1}, \ldots, a_{in}$ $(a_{ij} \in A_{ij})$ such that $0 = a_{i1} \wedge \ldots \wedge a_{in}$. Consequently $A_{i1} \vee \ldots \vee A_{in} = L$ and so $Y \subseteq F^1(A_{i1} \vee \ldots \vee A_{in}) = F^1(A_{i1}) \cup \ldots \cup F^1(A_{in})$. Thus Y is compact.

Theorem 2.7 — *The following three conditions are equivalent.*

1. *p is a T_1-space*
2. *$p = M$*
3. *Every prime ideal of L is minimal prime.*

PROOF : By Remark 2.3. 1 $\Leftrightarrow$ 2.

2 $\Rightarrow$ 3. Suppose 2 holds. Let A be any prime ideal of L. Clearly L-A is a prime filter. By 2, $L - A \in M$. Hence $A = L\text{-}(L\text{-}A)$ is a minimal prime ideal (Balasubramani *et al.*[1]).

3 $\Rightarrow$ 2. Suppose 3 holds let $B \in p$. Clearly L-B is a prime ideal and is therefore a minimal prime ideal by 3. Since L is a 0-distributive lattice L-B is a minimal prime semi ideal. Clearly $B = L\text{-}(L - B)$ is a maximal filter and hence $B \in \mathcal{M}$. It follows that $p = M$.

Theorem 2.8 — *If L is Complemented p is T_2.*

PROOF : Let A and B be distinct points of p. Then $A \not\supseteq B$ or $B \not\supseteq A$. Without loss of generality, we may assume $A \not\supseteq B$. Let $x \in B - A$, $U = F^1([x))$ and $V = F^1([x^1))$ where x^1 is a complement of x. Clearly U and V are neighbourhoods of A and B respectively. Also $U \cap V = F^1([x)) \cap F^1([x^1)) = F^1([x \vee x^1)) = F^1([1)) = \phi$. Thus p is T_2.

Theorem 2.9 — *Let L be complemented. Then p is connected if and only if L is the two element chain.*

PROOF : Suppose L is the two-element chain. Then $p = [1)$ and so p is connected.

Suppose $L \neq \{0, 1\}$. Let $x \in L$ such that $x \neq 0$, 1 and x^1 be a complement of x. Then $F^1([x)) \cup F^1([x^1)) = F^1([x) \vee [x^1)) = F^1([x \wedge x^1)) = F^1(L) = p$ and $F^1([x)) \cap F^1([x^1)] = F^1([x) \cap [x^1)) = F^1([x \vee x^1) = F^1([1)) = \phi$. Since $F^1([x)) = F([x^1))$ and $F^1([x^1)) = F([x))$ and L is 0-distributive, $F^1([x))$ and $F^1([x))$ are nonempty (Balasubramani *et al.*[1]). It follows that p is disconnected.

Lemma 2.10 — The set $U(a) = \{M \in \mathcal{M} / a \in M\}$ is a closed subset of M.

PROOF : Clearly $U(a) = F([a)) \cap \mathcal{M}$. Hence $U(a)$ is a closed subset of M.

We denote $\mathcal{M} - U(a)$ by $U^1(a)$.

Definition 2.11 — *(Pawar*[5]*). Two ideals* A and B of a lattice L are said to be weakly comaximal if $(A \vee B) \cap D \neq \phi$.

Definition 2.12 — (Pawar[5]). Two ideals A and B of a lattice L are said to be comaximal if $A \vee B = L$.

Lemma 2.13 — Let N be any nondense minimal prime ideal of L. Then $N = (x)^*$ for some $x \in N^*$.

PROOF : Since N is nondense $x \neq 0$ for some $x \in N^*$. Let $y \in N$. Then $y \wedge x = 0$ and so $y \in (x)^*$. Let $a \in (x)^*$. Then $a \wedge x = 0$. Clearly $x \in L - N$ and so $a \in N$. It follows that $N = (x)^*$.

The following theorem extends the result of Pawar[4].

Theorem 2.14 — *Suppose every minimal prime ideal of L is nondense and* $(x)^* \vee (y)^* = (x \wedge y)^*$ *for all x, y* in *L. Then the following statements are equivalent.*

1. *Each* $A \in F(D)$ *is contained in a unique maximal filter.*
2. *For* $M_1 \neq M_2$ *in* $\mathcal{M}$ *there exist* $a_1 \in L - M_1$ *and* $a_2 \in L - M_2$ *such that* $a_1 \vee a_2 \in D$.
3. $\mathcal{M}$ *is* T_2.
4. $\mathcal{M}$ *is normal.*
5. *Any two distinct minimal prime ideals of L are weakly comaximal.*

PROOF : 1 ⇒ 2. Let $M_1, M_2 \in \mathcal{M}$ and $M_1 \neq M_2$. Since L is 0-distributive $L - M_1$ and $L - M_2$ are minimal prime ideals (Balasubramani *et al.*[1]). By Lemma 2.13, $L - M_1 = (x)^*$ and $L - M_2 = (y)^*$ for some $x \in (L - M_1)^*$ and $y \in (L - M_2)^*$ such that $x \neq 0$ and $y \neq 0$. Clearly

$(L-M_1)\vee(L-M_2) = (x)^*\vee(y)^*=(x\wedge y)^*$. We claim $((L-M_1)\vee(L-M_2))\bigcap D\neq\phi$. Suppose $((L-M_1)\vee(L-M_2))\bigcap D=\phi$. Hence $(x\wedge y)^*\bigcap D=\phi$. Since L is 0-distributive for each a in L and each filter A disjoint from $(a)^*$, there is a prime filter containing A and disjoint from $(a)^*$ (Balasubramani *et al.*[1]). Hence there exists $A\in F(D)$ such that $((L-M_1)\vee(L-M_2))\bigcap A=\phi$ and consequently $A\subseteq M_1\bigcap M_2$ which is a contradiction to 1. Hence $((L-M_1)\vee(L-M_2))\bigcap D\neq\phi$. Let $t\in((L-M_1)\vee(L-M_2))\bigcap D$. Then $t\leq a_1\vee a_2$ for some $a_1\in L-M_1$ and $a_2\in L-M_2$. Since $t\in D, a_1\vee a_2\in D$.

2 $\Rightarrow$ 3. Suppose (2) holds. Let $M_1, M_2\in\mathcal{M}$ such that $M_1\neq M_2$. By (2) there exists $a_1\in L-M_1$ and $a_2\in L-M_2$ such that $a_1\vee a_2\in D=\bigcap\mathcal{M}$. It follows that $U^1(a_2), U^1(a_2)$ are neighbourhoods of M_1, M_2 respectively and $U^1(a_1)\bigcap U^1(a_2)=\phi$.

3 $\Rightarrow$ 4. Suppose (3) holds. Then $\mathcal{M}$ is T_2. By Theorem 2.6. $\mathcal{M}$ is compact. It follows that $\mathcal{M}$ is normal.

4 $\Rightarrow$ 1. Suppose there exists $A\in F(D)$ such that $A\subseteq M_1\in\mathcal{M}$, $A\subseteq M_1\in\mathcal{M}$ and $M_1\neq M_2$. Clearly $\{M_1\}$ and $\{M_2\}$ are disjoint closed subsets of $\mathcal{M}$. Let $F^1(A_1)$ be any neighbourhood of $\{M_1\}$ and $F^1(A_2)$ be any neighbourhood of $\{M_2\}$. Then $A\in F^1(A_1)\bigcap F^1(A_2)$ and $F^1(A_1)\bigcap F^1(A_2)\neq\phi$. Thus $\mathcal{M}$ is not normal.

2 $\Rightarrow$ 5. Suppose 2 holds. Let N_1 and N_2 be any two distinct minimal prime ideals of L. Clearly $L-N_1$ and $L-N_2$ are proper filters. Hence $L-N_1\subseteq M_1$ and $L-N_2\subseteq M_2$ for some maximal filters M_1 and M_2 of L. Since L is 0-distributive $L-M_1$ and $L-M_2$ are minimal prime ideals (Balasubramani *et al.*[1]). Also $L-M_1\subseteq N_1$ and $L-M_2\subseteq N_2$. It follows that $L-M_1=N_1$ and $L-M_2=N_2$. Since L is 0-distributive, N_1 and N_2 are minimal prime semi ideals. Hence $L-N_1$ and $L-N_2$ are distinct maximal filters. By (2) there exist $a_1\in N_1$ and $a_2\in N_2$ such that $a_1\vee a_2\in D$. It follows that $(N_1\vee N_2)\bigcap D\neq\phi$.

5 $\Rightarrow$ 2. Suppose 5 holds. Let $M_1, M_2\in\mathcal{M}$ and $M_1\neq M_2$. Since L is 0-distributive $L-M_1$ and $L-M_2$ are minimal prime ideals (Balasubramani *et al.*[1]). By 5, $((L-M_1)\vee(L-M_2))\bigcap D\neq\phi$. Hence there exist $a_1\in L-M_1$ and $a_2\in L-M_2$ such that $a_1\vee a_2\in D$.

We give below an example of a nondistributive lattice (in fact a nonmodular lattice) which is 0-distributive and in which every minimal prime ideal is nondense and $(x)^*\vee(y)^*=(x\wedge y)^*$ for all x, y.

Example 2.15 — Let L be the five element nonmodular lattice. Then L is 0-distributive. Also every minimal prime ideal of L is nondense and $(x)^* \vee (y)^* = (x \wedge y)^*$ for all x, y in L.

3. Topology on 1-Distributive Lattice

Throughout this section L denotes a 1-distributive lattice.

For any ideal A of L, let $G(A)$ denote the set of all prime filters disjoint from A and let $G^1(A) = p–G(A)$. Since L is a 1-distributive lattice, 0-distributive and 1-distributive lattices are duals of each other by the similar arguments as in section 2 we can show that $\{G^1(A)/A \in L(I)\}$ is a topology on p. We shall denote this topology by $\check{T}$. The sets $G(A)$ are precisely the closed subsets of $(p, \check{T})$.

***Theorem* 3.1** — *If X is any subset of p, Cl. $X = G(X_0)$ where X_0 is the intersection of the set complements of the members of X.*

Proof : Clearly $G(X_0)$ is a closed subset of p and $X \subseteq G(X_0)$. If $X \subseteq G(A)$ for some ideal A, then $A \subseteq X_0$ and so $G(X_0) \subseteq G(A)$. Hence the result.

Since L is a 1-distributive lattice, using duality property $L(F)$ is Pseudocomplemented (Balasubramani *et al.*[1]). A filter A of L is called simple if $A \vee A^{\#} = L$ ($A^{\#}$ is the Pseudocomplement of A). The set of simple maximal filters of L is denoted by $\mathcal{M}_1$. Let $\mathcal{N}$ and $\mathcal{N}_1$ denote the set of all minimal prime filters and the set of normal prime filters. Since L is 1-distributive by the duality properly $\mathcal{N}_1 \subseteq \mathcal{N}$ (Balasubramani *et al.*[1]).

Definition 3.2 — A lattice which is both 0-distributive and 1-distributive is called a 0 and 1-distributive lattice.

Throughout the remaining of this section L will denote a 0 and 1-distributive lattice.

As a consequence of Theorem 3.1. We have

***Theorem* 3.3** — *The anti-T_1 points $(p, \check{T})$ are precisely the maximal filters of L. Thus the T_1-points of (p, T) are identical in their totality with the anti-T_1 points of $(p, \check{T})$.*

***Theorem* 3.4** — *The subspace $\mathcal{N}$ of $(p, \check{T})$ is T_3.*

Proof : Since no minimal prime filter contains any other minimal prime filter, $\mathcal{N}$ is T_1. Let $C = \mathcal{N} \cap F(A)$ be any closed subset of $\mathcal{N}$ and $Q \notin C$ $(Q \in \mathcal{N})$. Then $Q \not\supseteq A$ and so there exists $a \notin A - Q$. Since $Q \not\supseteq [a), Q \supseteq [a)^{\#}$ Let $C_1 = \mathcal{N} \cap F([a))$ and $C_2 = \mathcal{N} \cap (F([A)^{\#}))$. Clearly

$C_1 \supseteq C, Q \notin C$, and $Q \in C_2$. Now $C \cap C_2 = \mathcal{N} \cap (F(A) \cap F([a)^{\#}) \subseteq \mathcal{N} \cap (F([a)) \vee F([a)^{\#})) \phi$. Also $C_1 \cup C_2 = \mathcal{N} \cap (F([a)) \cup F[a)^{\#}) = \mathcal{N} \cap F([a) \cap [a)^{\#} = \mathcal{N} \cap F([1)) = \mathcal{N}$. Hence $\mathcal{N}$ is regular. It follows that $\mathcal{N}$ is T_3.

Theorem 3.5 — *The minimal prime filters of L are precisely the T_1 (anti-T_1) points of* $(p, \check{T})$ (p, T).

PROOF : The proof is straightforward and follows by Theorem 2.1 and Theorem 3.1.

Theorem **3.6** — *T is stronger (weaker) that $\check{T}$ at a point Q of p if and only if Q is a T_1 (anti-T_1) point of T.*

PROOF : Suppose Q is T_1 point of (p, T). Then by Theorem 3.3 Q is a maximal filter of L. Let $F^1(A)$ be any neighbourhood of Q. Then $Q \not\trianglerighteq A$ and so there exists $a \in A - Q$. As Q is maximal $Q \vee [a) = L$. Hence $q \vee a = 0$ for some $q \in Q$. Clearly $q \in (a]^*$ and consequently $Q \in G^1((a]^*)$. Since $a \in A$, $G^1((a]^*) \subseteq F^1(A)$. Thus any T-neighbourhood of Q contains a $\check{T}$-neighbourhood of Q and so T is stronger than $\check{T}$ at Q.

Suppose Q is not a T_1 point of $(p\ T)$. Then by Theorem 3.3, Q is not a maximal filter and so there exists a maximal filter M of L such that $M \supset Q$. Clearly $F^1(M)$ is a T neighbourhood of Q. Let $G^1(A)$ be any $\check{T}$-neighbourhood of Q. Since $Q \in G^1(A)$ and $Q \subset M$, it follows that $M \in G^1(A)$. Therefore $G^1(A) \not\trianglelefteq F^1(M)$. Thus $F^1(M)$ contains no $\check{T}$-neighbourhood of Q. Hence T is not stronger than $\check{T}$ and Q.

Suppose Q is an anti-T_1 point of (p, T). Then by Theorem 3.5, Q is a minimal prime filter of L. Let $G^1(A)$ be a $\check{T}$-neighbourhood of Q. Then $Q \cap A \neq \varnothing$, so that there exists $a \in Q \cap A$. As L is 1-distributive, by the duality property, $L(F)$ is Pseudocomplemented (Balasubramani *et al.*[1]). Since $a \in Q$ and Q is minimal prime, $Q \not\trianglerighteq [a)^{\#}$ and so $Q \in F^1([a)^{\#})$. Also any prime filter not containing $[a)^{\#}$ intersects A. Hence $F^1([a)^{\#}) \subseteq G^1(A)$. Thus any $\check{T}$-neighbourhood of Q contains a T-neighbourhood of Q and so T is weaker than $\check{T}$ at Q.

Suppose Q is not an anti-T_1 point of (p, T). Then by Theorem 3.5, Q is not a minimal prime filter of L. Since L is 1-distributive by, duality property there exists a minimal prime filter N such that (Balasubramani *et al.*[1]). Now $L - N$ is a prime ideal intersecting Q. Clearly $G(L - N)$ is a $\check{T}$-neighbourhood of Q. Let $F(A)$ be any T neighbourhood of Q. Since $Q \in F(A)$ and $Q \supset N$, it follows that $N \in F(A)$. Therefore, $F(A) \not\trianglelefteq G(L - N)$. Thus $G(L - N)$ contains no T-neighbourhood of

Q. Hence T is not weaker than T at Q.

The following theorem extends some results of Varlet[7] and Pawar[6].

Theorem 3.7 — *The following statements are equivalent.*

1. *Every prime filter of L is contained in a unique maximal filter.*
2. *Every prime ideal of L contains a unique minimal prime ideal.*
3. *Any two distinct minimal prime ideals of L are comaximal.*
4. *For any two distinct maximal filters M_1 and M_2 of L there exist $a_1 \in L - M_1$ and $a_2 \in L - M_2$ with $a_1 \vee a_2 = 1$.*
5. *For any maximal filter M, M is unique maximal filter containing the filter*

$$W(M) = \{x \in L/x \vee y = 1 \text{ for some } y \in L - M\}.$$

6. *$\mathcal{M}$ is T_2.*
7. *$\mathcal{M}$ is normal.*

PROOF : 1 $\Rightarrow$ 2. Suppose there is a prime filter Q such that $Q \subseteq M_1$ and $Q \subseteq M_2$ for some distinct maximal filters M_1 and M_2. Clearly $L - M_1$ and $L - M_2$ are distinct minimal prime ideals (Balasubramani *et al.*[1]). Clearly $L - Q$ is a prime ideal and $L - M_1 \subseteq L - Q$ and $L - M_2 \subseteq L - Q$. It follows that 1 $\Rightarrow$ 2.

2 $\Rightarrow$ 1. Let Q be any prime ideal of L. It follows that $Q \supseteq N_1$ for some minimal prime ideal N (Balasubramani *et al.*[1]). Suppose $Q \supseteq N_1$ for some minimal prime ideal $N_1 \neq N$. Clearly $L - Q$ is a prime filter, $L - Q \subseteq L - N$ and $L - Q \subseteq L - N_1$. It is easily seen that $L - N$ and $L - N_1$ are maximal filters. Thus 2 $\Rightarrow$ 1.

2 $\Rightarrow$ 3. Suppose there exist distinct minimal prime ideals N_1, N_2 such that $N_1 \vee N_2 = Q \neq L$. Since L is 1-distributive by the dual of a Theorem (Balasubramani *et al.*[1]), there exists a prime filter R such that $R \cap Q = \phi$. Clearly $R \cap N_1 = \phi$ and $R \cap N_2 = \phi$. Hence $R \subseteq L - N_1$ and $R \subseteq L - N_2$. Also $L - N_1$ and $L - N_2$ are maximal filters. It follows that 2 $\Rightarrow$ 3.

3 $\Rightarrow$ 2. Suppose there is a prime filter Q such that $Q \subseteq M_1$ and $Q \subseteq M_2$ for some distrinct maximal filters M_1 and M_2. Clearly $L - Q$ is a prime ideal. Also $L - M_1 \subseteq L - Q$ and $L - M_2 \subseteq L - Q$ and hence $(L - M_1) \vee (L - M_2) \subseteq L - Q \neq L$. By a Theorem (Balasubramani *et al.*[1]), $L - M_1$ and $L - M_2$ are minimal prime ideals. Thus 3 $\Rightarrow$ 2.

3 $\Rightarrow$ 4. Suppose 3 holds. Let M_1 and M_2 be distinct maximal filters of L. Hence $L - M_1$ and $L - M_2$ are distinct minimal prime ideals (Balasubramani *et al.*[1]). By 3, $(L - M_1) \vee (L - M_2) = L$. Hence there exist $a_1 \in L - M_1$ and $a_2 \in L - M_2$ such that $a_1 \vee a_2 = 1$.

$4 \Rightarrow 5$. First we shall prove that $W(M)$ is a filter. Let $a \in W(M)$ and $b \geq a$. Clearly $b \in W(M)$. Let $a_1, a_2 \in W(M)$. Then there exist $c_1, c_2 \in L - M$ such that $a_1 \vee c_1 = 1 = a_2 \vee c_2$. Clearly $a_1 \vee (c_1 \vee c_2) = 1 = a_2 \vee (c_1 \vee c_2)$. Also $L - M$ is a minimal prime ideal (Balasubramani *et al.*[1]), and so $c_1 \vee c_2 \in L - M$. Since L is 1-distributive $(c_1 \vee c_2) \vee (a_1 \wedge a_2) = 1$. It follows that $a_1 \wedge a_2 \in W(M)$. Thus $W(M)$ is a filter. It is easily seen that $W(M) \subseteq M$. Suppose $W(M) \subseteq M_1$ for some maximal filter $M_1 \neq M$. Let $a \in L - M$ and $b \in L - M_1$. If $a \vee b = 1$, then $b \in W(M) \subseteq M_1$ which is not true. Hence $a \vee b \neq 1$. It follows that $4 \Rightarrow 5$.

$5 \Rightarrow 3$. Suppose there exist two distinct minimal prime ideas N_1 and N_2 such that $N_1 \vee N_2 \neq L$. Then $a \vee b \neq 1$ for all $a \in N_1$ and $b \in N_2$. Let $M_1 = L - N_2$ and $M_2 = L - N_1$. It is easily seen that M_1 and M_2 are distinct maximal fillers of L. Let $x_1 \in W(M_1)$. Then $x_1 \vee y_1 = 1$ for some $y_1 \in N_2$ and so $x \in L - N_1 = M_2$. Hence $W(M_1) \subseteq M_2$.

$3 \Rightarrow 6$. Suppose 3 holds. Let $M_1, M_2 \in \mathcal{M}$ and $M_1 \neq M_2$. Hence $L - M_1$ and $L - M_2$ are distinct minimal prime ideals (Balasubramani *et al.*[1]). By 3, $(L - M_1) \vee (L - M_2) = L$. Hence here exist $a_1 \in L - M_1$ and $a_2 \in L - M_2$ such that $a_1 \vee a_2 = 1$. It follows that $U^1(a_1)$ and $U^1(a_2)$ are neighbourhoods of M_1 and M_2 respectively and that $U^1(a_1) \cap U^1(a_2) = \phi$. Thus $\mathcal{M}$ is T_2.

$6 \Rightarrow 7$. Suppose 6 holds, then $\mathcal{M}$ is T_2. By Theorem 2.6 $\mathcal{M}$ is compact. It follows that $\mathcal{M}$ is normal.

$7 \Rightarrow 1$. Suppose there is a prime filter Q of L such that $Q \subseteq M_1$ and $Q \subseteq M_2$ for some $M_1, M_2 \in M$ with $M_1 \neq M_2$. Clearly $\{M_1\}$ and $\{M_2\}$ are disjoint closed subsets of $\mathcal{M}$. Let $F_1(A)$ be any neighbourhoods of $\{M_1\}$ and $F^1(B)$ any neighbourhood of $\{M_2\}$. Clearly $Q \in F^1(A) \cap F^1(B)$ and so $F^1(A) \cap F^1(B) \neq \phi$. It follows that $7 \Rightarrow 1$.

Hence the result.

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