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WEAKLY PRIME SETS FOR VECTOR FUNCTION SPACES

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We have defined and discussed various types of weakly prime sets for vector function spaces. Some special types of vector function spaces and their weakly prime sets have been studied.

Key Words: Vector Function Spaces; Generalized Peak Sets; Weakly Prime Sets

1. WEAKLY PRIME SETS

The concept of a weakly prime set for a function algebra was introduced and studied by Ellis[2]. We generalized this notion for function spaces[7]. The collection (and hence the decomposition generated by it) of all maximal weakly prime sets is finer than the standard Bishop decomposition. In[6], we have studied various Bishop decompositions for a vector function space. Here, we define weakly prime sets in various ways, compare the decompositions obtained by them among themselves and also with Bishop decompositions. Finally, we study the weakly prime decompositions for some special function spaces.

Let X be a compact, Hausdorff space and B be a commutative Banach algebra with identity e. Let $C(X; B)$ denote the set of all B-valued, continuous functions on X. Then, $C(X; B)$ is a commutative Banach algebra with pointwise operations and the sup norm $\|f\| = \sup_{x \in X} \|f(x)\|$ $(f \in C(X; B))$. For $B = \mathbb{C}$, $C(X; B)$ is denoted by $C(X)$. For $f \in C(X)$ and $b \in B$, the function $f \otimes b$ is defined by $(f \otimes b)(x) = f(x)b$ $(x \in X)$. The space of all finite linear combinations of functions of the form $f \otimes b$ $(f \in C(X), b \in B)$ is dense in $C(X; B)$. The function of the form $1 \otimes b$ $(b \in B)$, is called a vector constant function.

Definitions 1.1 — (*i*) A vector function space on X is a closed subspace of $C(X; B)$ containing vector constant functions.

(*ii*) A vector function algebra on X is a closed subalgebra of $C(X; B)$ which separtes the points of X and contains vector constants.

If $B = \mathbb{C}$, then a vector function space (algebra) is a function space (function algebra).

Definition 1.2 — Let V be a vector function space on X. A closed subset F of X is called a peak set for V if there exists $f \in V$ such that $f_{|F} = e$ and $\|f(x)\| < 1$ for all $x \in X - F$. The intersection of peak sets is called a generalized peak set for V.

Now onwards, V denotes a vector function space on X.

For a closed subset E of X, define $M(V_{|E}) = \{f \in C(E;B) : fg \in V_{|E}, \forall g \in V_{|E}\}$. If $E = X$, then we write $M(V)$ for $M(V_{|E})$, i.e., $M(V) = \{f \in C(X;B) : fg \in V, \forall g \in V\}$. Then $M(V)$ is a closed subalgebra of $C(X;B)$ and $M(V) \subseteq V$. Further, if V is an algebra, then $M(V) = V$.

Definitions 1.3 — Let V be a vector function space on X.

(*i*) A closed subset K of X is called a weakly prime set for V if whenever G and H are generalized peak sets for $M(V_{|K})$ such that $G \bigcup H = K$, then either $K = G$ or $K = H$.

(*ii*) A closed subset K of X is called an (*M*)-weakly prime set for V if G and H are generalized peak sets for $M(V)_{|K}$ and $G \bigcup H = K$, then either $K = G$ or $K = H$.

It can be checked that each weakly prime set ((*M*)-weakly prime set) is contained in a maximal weakly prime set ((*M*)-weakly prime set) for V. The collection of all maximal weakly prime sets ((*M*)-weakly prime sets) for V is denoted by $\mathcal{P}(V)$ ($\mathcal{P}_M(V)$).

For a function space A, the weakly prime sets have been studied in[7]. In fact, as in case of the Bishop decompositions, here also we can define two different types of weakly prime sets for A. In[7], only one type of weakly prime sets has been discussed. We recall that definition here and also define another type of weakly prime sets for A, which corresponds to the Bishop decomposition defined by Edwards[1].

Definition 1.4[7] — A closed subset K of X is called an (*FP*)-weakly prime set for A if $K = G \bigcup H$, where G and H are generalized peak sets for $N(A_{|K})$, then either $K = G$ or $K = H$, where $N(A_{|K}) = \{f \in C(K) : fg \in A_{|K}, \forall g \in A_{|K}\}$.

We denote the collection of all maximal (*FP*)-weakly prime sets for A by $\mathcal{P}_{FP}(A)$.

Definition 1.5 — A closed subset K of X is called an (*E*)-weakly prime set for A if $K = G \bigcup H$, where G and H are generalized peak sets for $N(A)_{|K}$, then either $K = G$ or $K = H$. (Note that $N(A) = N(A_{|X})$).

The collection of all maximal (*E*)-weakly prime sets for A is denoted by $\mathcal{P}_E(A)$.

It is clear from definitions that $\mathcal{P}_{FP}(A) < \mathcal{P}_E(A)$, i.e., $\mathcal{P}_{FP}(A)$ is finer than $\mathcal{P}_E(A)$. In general, $\mathcal{P}_{FP}(A) \neq \mathcal{P}_E(A)$[7]. But, if A is a function algebra on X, then $\mathcal{P}_{FP}(A) = \mathcal{P}_E(A)$.

Remarks 1.6 : (*i*) If $B = \mathbb{C}$, then for any $K \subset X, M(V_{|K}) = N(V_{|K})$ and so, $\mathcal{P}(V) = \mathcal{P}_{FP}(V)$ and $\mathcal{P}_M(V) = \mathcal{P}_E(V)$.

(*ii*) If V is an algebra, then $M(V_{|K}) = V_{|K} = M(V)_{|K}$ for any $K \subset X$. Hence $\mathcal{P}_M(V) = \mathcal{P}(V)$.

(*iii*) Since $M(V)_{|E} \subset M(V_{|E})$ for any $E \subset X$, $\mathcal{P}(V) < \mathcal{P}_M(V)$. But, in general $\mathcal{P}(V) = \mathcal{P}_M(V)$.

As in case of a function space, we shall show that $\mathcal{P}(V)$ has the (*GA*)-property for V and every member of $\mathcal{P}(V)$ is a p-set for V. We shall need the following concepts. For details see[5,6,7]

Let $M(X, B^*)$ denote the set of all weak* regular, B^*-valued measures of bounded variation on X. Then $M(X, B^*)$ is the dual of $C(X; B)$. For a vector function space V on X, $V^{\perp} = \{\mu \in M(X, B^*) : \int f d\mu = 0, \ \forall f \in V\}$.

First, we shall prove that members of $\mathcal{P}(V)$ and $\mathcal{P}_M(V)$ are p-sets for V.

Definition 1.7 — A closed subset F of X is called a p-set for V if $\mu \in V^{\perp}$ implies that $\mu_F \in V^{\perp}$, where $\mu_F(G) = \mu(F \cap G)$ for every Borel subset G of X.

It is known that, if A is a function space on X, then a p-set is a generalized peak set for A. But the converse is not true. However, if A is a function algebra, then both are same. For a vector function space V, we do not know whether a p-set will be a generalized peak set for V or not, but using the next result, we will get that a generalized peak set is a p-set for a vector function algebra V.

Proposition 1.8 — Let $E \subset X$ be a p-set for V and a closed subset $F \subset E$ be a generalized peak for for $M(V_{|E})$. Then F is a p-set for V.

PROOF : Let $\mu \in V^{\perp}$ and $\varepsilon > 0$. Then, there is an open set U in X with $F \subset U$ such that $|\mu|(U - F) < \varepsilon$. Clearly, $U \cap E$ is open in E and $F \subset U \cap E$. Since F is a generalised peak set for $M(V_{|E})$, there is a peak set K with peaking function f in $M(V_{|E})$ such that $F \subset K \subset U \cap E$. Then $|\mu|(K - F) < \varepsilon$. Define h on E by $h = e$ on K and $h = 0$ on $E - K$, where e is the identity for B. Then $\{f^n\}$ converges pointwise and boundedly to h on E.

Now, let $g \in V$. Since $f^n \in M(V_{|E})$, $f^n g_{|E} \in V_{|E}$. Thus, $\int_K g \, d\mu = \int_E gh \, d\mu = \lim_n \int_E gf^n \, d\mu$.

But E is a p set for $V^{\perp}$ and $\mu \in V^{\perp}$. So, $\mu_E \in V$. Since $f^n g_{|E} \in V_{|E}$, $\int_E gf^n \, d\mu = 0$. Therefore, $\int_K g \, d\mu = 0$. Also, $\left| \int_E g \, d\mu - \int_F g \, d\mu \right| \leq \| g \| (|\mu|(K - F)) < \varepsilon \| g \|$. Therefore,

$$\left| \int_F g \, d\mu \right| = \left| \int_K g \, d\mu - \int_F g \, d\mu \right| < \varepsilon \| g \|.$$

Since ε is arbitrary, $\int_F g \, d\mu = 0$, i.e., $\mu_F \in V^{\perp}$ or F is p-set for V.

By taking $E = X$, we get the following.

Corollary 1.9 — Let V be a vector function space on X. If $F \subset X$ is a generalized peak set for $M(V)$, then F is a p-set for V.

Theorem 1.10 — *Each $E \in \mathcal{P}(V)$ is a p-set for V.*

PROOF : Let $E \in \mathcal{P}(V)$ and let F be the smallest p-set for V which contains E^4. It is enough to show that F is a weakly prime set for V.

Let $F = G \cup H$, wher G and H are generalized peak sets for $M(V_{|F})$. Let $G_1 = G \cap E$ and $H_1 = H \cap E$. Then $E = E \cap F = G_1 \cup H_1$ and G_1, H_1 are generalized peak sets for $M(V_{|E})$. Since E is a weakly prime set for V, either $E = G_1$ or $E = H_1$. So, $E \subset G$ or $E \subset H$, i.e., $E \subset G \subset F$ or $E \subset H \subset F$. Now, F is a p-set for V and G and H are generalized peak sets for $M(V_{|F})$. Therefore, by Proposition 1.8, G and H are p-sets for V. But, as F is the smallest p-set for V with $E \subset F$, wither $G = F$ or $H = F$. Thus, F is a weakly prime set for V.

Since $M(V)_{|E} \subset M(V_{|E})$ for any $E \subset X$, by the similar argument as given in Theorem 1.10, we can prove the following result.

Proposition 1.11 — Every member of $\mathcal{P}_M(V)$ is a p-set for V.

It can be proved that if E is a p-set for V, then E is a CR-set for V, i.e., $V_{|E}$ is closed in $C(E\,;B)$. Hence, every member of $\mathcal{P}(V)$ and $\mathcal{P}_M(V)$ is a CR-set for V.

Finally, we define the (GA)-property and show that $\mathcal{P}(V)$ and $\mathcal{P}_M(V)$ have the (GA)-property for V.

Definition 1.12 — Let V be a vector function space on X and $\mathcal{E}$ be a family of closed subsets of X. We say that $\mathcal{E}$ has the (GA)-property for V if for each $\mu \in b(V^{\perp})^e$, supp$\mu \subset E$ for some $E \in \mathcal{E}$, where $b(V^{\perp})^e$ denotes tne set of all extreme points of the unit ball of $V^{\perp}$.

Theorem 1.13 — *$\mathcal{P}(V)$ has the (GA)-property for V.*

PROOF : Let $\mu \in b(V^{\perp})^e$ and S = supp μ. It is enough to show that S is a weakly prime set for V.

Let $S = G \cup H$, where G and H are generalized peak sets for $M(V_{|S})$. Then, by Corollary 1.9, G and H are p-sets for $V_{|S}$. Let $\mu_1 = \mu_{|G}$ and $\mu_2 = \mu - \mu_1$. Now, $\mu \in b(V^{\perp}_{|s})$ and G is a p-set for $V_{|S}$. So $\mu_1 \in b(V^{\perp}_{|s})$ and hence $\mu_2 \in b(V^{\perp}_{|s})$. Further, $\|\mu\| = \|\mu_1\| + \|\mu_2\|$. Since μ is an extreme point of $b(V^{\perp})$, either $\mu = \mu_1$ or $\mu = \mu_2$, i.e., $S = G$ or $S = H$. Thus S is a weakly prime set for V.

Since $\mathcal{P}(V) < \mathcal{P}_M(V)$, and $\mathcal{P}(V)$ has the (GA)-property for V, the following can be proved easily.

Proposition 1.14 — $\mathcal{P}_M(V)$ has the (GA)-property for V.

2. More on Weakly Prime Decompositions

For a vector function space V, different types of Bishop decompositions have been defined and studied in[6]. The definitions of Bishop decompositions or antisymmetric set for V have been given in terms of subalgebras of $C(X)$ which are associated with V. Also, an antisymmetric set is defined with the help of $m(B)$, the maximal ideal space of B. So, in this section, we define and study weakly prime sets for V on these lines.

Let V be a vector function space on X. For a closed subset E of X, we define $N(V_{|E}) = \{f \in C(E) : (f \otimes e)g \in V_{|E},\ \forall g \in V_{|E}\}$. For $E = X$, we denote it by $N(V)$. It is clear that $N(V)$ is a closed subalgebra of $C(X)$.

Definitions 2.1 — (*i*) A closed subset K of X is said to be an (FP)-weakly prime set for V if $K = G \cup H$, where G and H are generalized peak sets for $N(V_{|K})$, then either $K = G$ or $K = H$.

(*ii*) A closed subset K of X is said to be an (E)-weakly prime set for V if $K = G \cup H$, where G and H are generalized peak sets for $N(V)_{|K}$, then either $K = G$ or $K = H$.

The collection of all maximal (FP)-weakly prime sets ((E)-weakly prime sets) for V is denoted by $\mathcal{P}_{FP}(V)$ ($\mathcal{P}_E(V)$).

Remarks 2.2 : (*i*) If $B = \mathbb{C}$, then definitions 2.1(*i*) and (*ii*) coincide with Definitions 1.4 and 1.5.

(*ii*) If $\mathcal{K}_{FP}(V)$ and $\mathcal{K}_E(V)$ denote the Bishop decompositions in (FP)-sense and (E)-sense respectively for V[6], then it can be checked that $\mathcal{P}_{FP}(V) < \mathcal{K}_{FP}(V)$ and $\mathcal{P}_E(V) < \mathcal{K}_E(V)$.

Let K be a closed subset of X and $F \subset K$. Then, it can be verified that, if F is a peak set for $N(V_{|K})$, then F is a peak set for $M(V_{|K})$. Hence, the following propositions can be proved easily.

Proposition 2.3 — $\mathcal{P}(V) < \mathcal{P}_{FP}(V) < \mathcal{P}_E(V)$ and $\mathcal{P}_M(V) < \mathcal{P}_E(V)$. Hence, $\mathcal{P}_{FP}(V)$ and $\mathcal{P}_E(V)$ also have the (GA)-property for V.

Proposition 2.4 — Each member of $\mathcal{P}_{FP}(V)$ and $\mathcal{P}_E(V)$ is a p-set for V.

Next, we give some conditions under which some of these families are equal.

For a vector function space V on X and $\phi \in m(B)$, define $V_\phi = \{\phi o f : f \in V\}$. Then V_ϕ is a closed subspace of $C(X)$ and it is clear that $N(V) \subset M(V)_\phi$ for every $\phi \in m(B)$.

Proposition 2.5 — If $N(V_{|K}) = M(V_{|K})_\phi$ for every closed subset K of X and for some ϕ in $m(B)$, then $\mathcal{P}_{FP}(V) = \mathcal{P}(V)$. Also, if $N(V) = M(V)_\phi$ for some $\phi \in m(B)$, then $\mathcal{P}_E(V) = \mathcal{P}_M(V)$,

PROOF : Suppose $N(V_{|K}) = M(V_{|K})_\phi$ for some $\phi \in m(B)$. By Proposition 2.3, $\mathcal{P}(V) < \mathcal{P}_{FP}(V)$. Let $K \in \mathcal{P}_{FP}(V)$. Suppose that $K = G \cup H$, where G and H are generalized peak sets for $M(V_{|K})$. Now, it can be checked that if $S \subset K$ is a peak set with a peakmg function f in $M(V_{|K})$, then $\phi o f$ will also be a peaking function for S and $\phi o f \in M(V_{|K})_\phi$. Thus, G and H are generalized peak sets for $M(V_{|K})_\phi$ also. Since $M(V_{|K})_\phi = N(V_{|K})$ and $K \in \mathcal{P}_{FP}(V)$, either $K = G$ or $K = H$. Therefore, K is a weakly prime set for V and $\mathcal{P}_{FP}(V) < \mathcal{P}(V)$.

Corollary 2.6 — If V is a vector function algebra on X and if for some $\phi \in m(B)$, $M(V_{|K})_\phi = N(V_{|K})$ for every $K \subset X$, then $\mathcal{P}(V) = \mathcal{P}_M(V) = \mathcal{P}_{FP}(V) = \mathcal{P}_E(V)$.

Finally, we define weakly prime sets for V with the help of $m(B)$.

Let us recall the definition of the Bishop decomposition for V, which involves $m(B)$.

Definition 2.7[6] — A subset K of X is said to be a (B)-antisymmetric set for V if $f \in M(V)$ and $(\phi o f)_{|K}$ is real-valued for each $\phi \in m(B)$, then $f_{|K}$ is constant. The collection of all maximal (B)-antisymmetric sets for V is denoted by $\mathcal{K}_B(V)$.

Definition 2.8 — A closed subset K of X is called a (B)-weakly prime set for V if $K = G \cup H$, where G and H are generalized peak sets for $M(V_{|K})_\phi$ for each $\phi \in m(B)$, then either $K = G$ or $K = H$.

The collection of all maximal (B)-weakly prime sets for V is denoted by $\mathcal{P}_B(V)$.

Remarks 2.9 — (*i*) Unlike the earlier two families, the families $\mathcal{K}_B(V)$ and $\mathcal{P}_B(V)$ are not comparable. For example, if V is a function algebra on X, then $\mathcal{P}_B(V) = \mathcal{P}(V) < \mathcal{K}(V) = \mathcal{K}_B(V)$. But if $V = V_0 = \{f \in C(X; B) : \phi_o \, o f \in A\}$ with A to be a function algebra, B is an antisymmetric algebra and $\phi_o \in m(B)$, then $\mathcal{K}_B(V) = \{\{x\} : x \in X\}^6$ whereas, $\mathcal{P}_B(V) = \mathcal{P}(A)$ (see Proposition 3.5).

(*ii*) It is clear that $\mathcal{P}_B(V) < \mathcal{P}_M(V)$.

(*iii*) If $M(V)_\phi = N(V)$ for some $\phi \in m(B)$, then as proved in the Proposition 2.5, $\mathcal{P}_E(V) = \mathcal{P}_B(V)$. Hence, $\mathcal{P}_B(V) = \mathcal{P}_M(V) = \mathcal{P}_E(V)$.

We do not know whether $\mathcal{P}_B(V)$ has the (*GA*)-property for V or not.

We can also define weakly prime sets by considering the algebra $M(V_{|K})_\phi$ and obtain similar results.

3. Special Vector Function Spaces

For a function space A, we can associate various vector function spaces with A such as $A \hat{\otimes} B$ and $A \# B$. We shall see the relation between weakly prime sets for A and for $A \hat{\otimes} B$ and $A \# B$.

Let A be a function space on X, B be a commutative Banach algebra with identity e and $m(B)$ denote the maximal ideal space of B. Then the tensor product $A \hat{\otimes} B$, is the closure of the linear span of functions $\{f \otimes b : f \in A, b \in B\}$ in $C(X; B)$. The slice product $A \# B$ is defined as

$$A \# B = \{f \in C(X; B) : \phi \, o f \in A, \quad \forall \phi \in m(B)\}.$$

Then $A \hat{\otimes} B$ and $A \# B$ are vector function spaces on X and $A \hat{\otimes} B \subset A \# B$. If A is a function algebra on X, then $A \hat{\otimes} B$ and $A \# B$ are vector function algebras on X.

Proposition 3.1 — If K is a *CR*-set for $A \hat{\otimes} B$, then, $M((A \# B)_{|K})_\phi = N(A_{|K}) = M((A \hat{\otimes} B)_{|K})_\phi$ for all $\phi \in m(B)$.

Proof : Let $\phi \in m(B)$. Let $f \in M((A \# B)_{|K})$. To show that $\phi \, o f \in N(A_{|K})$, let $g \in A_{|K}$. Then, $g \otimes e \in (A \# B)_{|K}$ and so, $f(g \otimes e) \in (A \# B)_{|K}$. Therefore, $\phi \, o \, (f(g \otimes e)) \in A_{|K}$, i.e., $(\phi \, o f) g \in A_{|K}$. Hence $\phi \, o f \in N(A_{|K})$. Thus, $M((A \# B)_{|K})_\phi \subset N(A_{|K})$.

Next, let $f \in N(A_{|K})$. It is enough to show that $f \otimes e \in M((A \hat{\otimes} B)_{|K})$. Let $g = \Sigma (g_i \otimes b_i)_{|K} \in (A \otimes B)_{|K}$. Then $(f \otimes e) g = (f \otimes e)(\Sigma (g_i \otimes b_i)_{|K}) = \Sigma f g_{i\,|K} \otimes b_i \in A_{|K} \otimes B = (A \otimes B)_{|K} \subset (A \hat{\otimes} B)_{|K}$. Now, let $g \in A \hat{\otimes} B$. Then $g = \lim g_n$ with $g_n \in A \otimes B$. Also, $(f \otimes e) g_{|K} = \lim$

$(f \otimes e)g_{n\,|K} \in (A \hat{\otimes} B)_{|K}$, as K is a CR-set for $A \hat{\otimes} B$. Thus, $f \otimes e \in M((A \hat{\otimes} B)_{|K})$ or $f \in M((A \hat{\otimes} B)_{|K})_\phi$. Hence $N(A_{|K}) \subset M((A \hat{\otimes} B)_{|K})_\phi$.

Finally, let $\phi\, o f \in M((A \hat{\otimes} B)_{|K})_\phi$ with $f \in M((A \hat{\otimes} B)_{|K})$. We shall show that $f \in M((A \# B)_{|K})$. Let $g \in (A \# B)_{|K}$. Fix $\Psi \in m(B)$. Then, $\Psi\, o\, g \in A_{|K}$. Also, as $M((A \hat{\otimes} B)_{|K}) \subset (A \hat{\otimes} B)_{|K} \subset (A \# B)_{|K}$, $f \in (A \# B)_{|K}$ and as above, we get $\Psi\, o f \in N(A_{|K})$. So $\Psi\, o\,(fg) = (\Psi\, o f)(\Psi\, o\, g) \in A_{|K}$. Since $\Psi \in m(B)$ is arbitrary, $fg \in A_{|K} \# B$, i.e., $fg \in (A \# B)_{|K}$. Hence, $f \in M((A \# B)_{|K})$ and consequently, $M((A \hat{\otimes} B)_{|K})_\phi \subset M((A \# B)_{|K})_\phi$.

Proposition 3.2 — If K is a CR-set for $A \hat{\otimes} B$, then $N(A_{|K}) = N((A \hat{\otimes} B)_{|K}) = N((A \# B)_{|K})$.

PROOF : We know that for any vector function space $V, N(V) \subset M(V)_\phi$ for each ϕ. So, by Proposition 3.1, $N((A \hat{\otimes} B)_{|K}) \subset N(A_{|K})$. Conversely, let $f \in N(A_{|K})$. Then it is proved in the Proposition 3.1 that, $f \otimes e \in M((A \hat{\otimes} B)_{|K})$. Thus, $(f \otimes e)\, g \in (A \hat{\otimes} B)_{|K}$, $\forall g \in (A \hat{\otimes} B)_{|K}$. Hence $f \in N(A \hat{\otimes} B)_{|K})$. So, $N(A_{|K}) \subset N((A \hat{\otimes} B)_{|K})$. Similarly, we can show that $N(A_{|K}) = N((A \# B)_{|K})$.

Using Propositions 2.5, 3.1 and 3.2, we get

***Theorem* 3.3** — *Let A be a function space on X. Then*

$$\mathcal{P}_{FP}(A) = \mathcal{P}_{FP}(A \hat{\otimes} B) = \mathcal{P}(A \hat{\otimes} B) = \mathcal{P}(A \# B) = \mathcal{P}_{FP}(A \# B)$$

and

$$\mathcal{P}_E(A) = \mathcal{P}_E(A \hat{\otimes} B) = \mathcal{P}_M(A \hat{\otimes} B) = \mathcal{P}_B(A \hat{\otimes} B)$$

$$= \mathcal{P}_E(A \# B) = \mathcal{P}_M(A \# B) = \mathcal{P}_B(A \# B).$$

Corollary 3.4 — If A is a function algebra on X, then all the families of maximal weakly prime sets for A, $A \hat{\otimes} B$ and $A \# B$ coincide.

Next, we associate one more vector function space with the function space A.

Fix $\phi_o \in m(B)$ and define $V_o = \{f \in C(X; B): \phi_o\, o f \in A\}$. Then, it is clear that $A \# B \subset V_o$ and V_o is a vector function space on X. Also, it can be checked that $M(V_{o\,|K})_{\phi_o} = N(A_{|K}) = N(V_{o\,|K})$ and $M(V_{o\,|K})_\phi = C(K)$ for each $\phi \neq \phi_o$. Hence, using Proposition 2.5, we obtain.

Proposition 3.5 — $\mathcal{P}_{FP}(A) = \mathcal{P}_{FP}(V_o) = \mathcal{P}(V_o)$ and $\mathcal{P}_E(A) = \mathcal{P}_E(V_o) = \mathcal{P}_M(V_o) = \mathcal{P}_B(V_o)$.

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