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LETTER TO THE EDITOR

SOME REMARKS ON DELBOSCO'S BEST APPROXIMATION AND FIXED POINTS

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Some remarks on Delbosco's work (ref. 5) are presented.

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Using a result of Lin[7] (Theorem 1), Delbosco[5] proved the following theorem (Theorem 2):

Let C be a nonempty convex subset of a normed linear space X and $f: C \to X$ be a continuous function. Let $g: C \to C$ be a continuous almost quasi convex and onto map. Further, if C has a non-empty compact convex subset C_0 such that the set $B = \{y \in C : \| gx - fy \| \geq \| gy - fy \|$ for all $x \in C_0\}$ is compact, then there is a point $y_0 \in C$ such that $\| gy_0 - fy_0 \| = d(fy_0, C) = \{\| fyo - x \| : x \in C\}$.

It may be pointed out that this result was proved in Carbone[2] (Theorem 2.1) by applying Lin's theorem (Lin[7]-Theorem 1) and in Carbone[3] (Theorem 2) by applying Allen's Theorem (Allen[1] Theorem 2). Carbone's theorem and so Delbosco's theorem is also stated in the paper of Carbone and Singh[4] (Theorem 9). In fact a strengthened version of Carbone's theorem and so of Delbosco's theorem is given in Park, Singh and Watson[8] (Theorem 1). Moreover, the proof given by Delbosco[5] is same as that given by Carbone[2].

It may also be pointed out that above theorem of Delbosco holds not only in normed linear spaces but also in metric linear spaces (Every normed linear space is a metric linear space but there are plenty of metric linear spaces which are not normed linear spaces - see Rolewicz[9]). The proof given in Carbone[2,3] (Delbosco[5]) can easily be extended to metric linear spaces.

Remarks 1 : As per Delbosco[5], Theorem 3 (Ky Fan) and Theorem 4 (Lin) are useful in the proof of Theorem 2. But Theorem 3 (Ky Fan) has nowhere been used in proving Delbosco's theorem.

2. Theorem 4 (Lin) is stated in Delbosco[5] for Banach spaces whereas it is used in proving Delbosco - Theorem 2 stated in normed linear spaces. Infact Lin's theorem (Theorem 1 - Lin[7]) hold good in Hausodorff topological vector spaces. So is the case with Theorem of Ky Fan[6] stated in Delbosco[5].

3. In the statement and also in the proof of Delbosco - Theorem 2, it is claimed that by Theorem 4 of Lin we get the point $y_0 \in C_0$ but actually Lin's theorem ensures the existence of a point $y_0 \in B$ and so in C and not necessarily in C_0 as stated in Delbosco[5]. So in Delbosco - Theorem 2, the point $y_0 \in C$ and not necessarily in C_0.

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