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# MULTI OBJECTIVE FUZZY DECISION MAKING APPROACH FOR SELECTION OF TYPE OF CAISSON FOR BRIDGE FOUNDATION

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Thc approach illustrated through the problem is to decide an optimum candidate Caisson[1] by defining a fuzzy decision calculus that requires only ordinal information on the ranking of preferences and importance weights to multiple objectives[5]. The problem has been worked out in two different scenario with different set of preferences. A special procedure has been followed for resolving the situation where tie results. The optimum solution obtained uses well known max-min rule of the fuzzy logic.

**Key Words: Caisson; Numerical Tie; Multiple Objectives; Optimum Solution; Preference Scenario**

## 1. INTRODUCTION

The typical multi objective decision problem involves the selection of one alternative, $a_i$, from a universe of alternatives given a set, say $\{0\}$, of criteria or objective that are important to decision maker.

We wish to calculate how well each alternative, or choice, satisfies each objective by combining the weighted objectives into an overall decision function in some plausible way. This decision function essentially denotes a mapping of the laternatives in $A$ to an ordinal set of ranks. This process naturally demands subjective information from the decision authority regarding the importance of each objective.

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To develop this calculus we require some definitions.

Let us define a universe of $n$ alternatives, $A = \{a_1, a_2 \; .... \; a_n\}$ and a set of $r$ objectives, $0 = \{0_1, \; .... \; 0_r\}$. Here $0_i$ indicates the $i$th objective. Then the degree of membership of alternative $a$ in $0_i$ denoted by $\mu \, 0_i (a)$ is the degree to which alternative $a$ satisfies the criteria specified for this objective. We, now, seek a decision function that simultaneously satisfies all of the decision objectives; hence the decision function, $D$, is given by the intersection of all the objective sets,

$$D = 0_1 \cap 0_2 \cap \cdots 0_r. \qquad \text{... (1)}$$

Therefore, the grade of membership that the decision function, $D$, has for end alternative $a$ is given by

$$\mu_D (a) = \min \; [\mu_{01} (a), \; \mu_{02} (a), \; ....., \; \mu_{0r} (a)] \qquad \text{... (2)}$$

The optimum decision, $a^*$ will then be the alternative that satisfies

$$\mu_D (a^*) = \max_{a \in A} \; [\mu_D (a)]. \qquad \text{... (3)}$$

We now define a set preference, $\{P\}$, which we will constrain to being linear and ordinal. Elements of this preference set can be linguistic values such as none, low, medium, high, absolute, or perfect, or they could be values on the interval [0, 1]. These preference will be attached to each of the objectives to quantify the decision maker's feelings about the influence that each objective should have on the chosen alternative.

Let the parameter $b_i$, be contained on the set of preferences, $\{P\}$, where $i = 1, 2, \; ... \; r$. Hence we have for each objective a measure of how important it is to the decision maker for a given decision.

The decision function, $D$, now takes on a more general form when each objective is associated with a weight expressing its importance to the decision maker. This function is represented as the intersection of $r$-tuples, represented as a decision measure, $M (0_i, b_i)$, involving objectives and preferences.

$$D = M (0_1, b_1) \cap M (0_2, b_2) \cap \cdots \cap M (0_r, b_r). \qquad \text{... (4)}$$

Now the problem is which operation should relate each objective, $0_i$ and its importance, $b_i$ that maintains the linear ordering required of the preference set and at same time relating the two quantities in a logical way where negation is also accommodated. Fortunately the classical implication operator satisfies all of these requirements. Hence, the decision measure for a particular alternative $a$, can be replaced with a classical implication of the form,

$$M (0_i (a), b_i) = b_i \rightarrow 0_i (a) = \bar{b}_i \vee 0_i (a). \qquad \text{... (5)}$$

Justification of the implication[5] as an appropriate measure can be developed using an intuitive argument.

The statement "$b_i$ implies $0_i$" indicates a unique relationship between a preference and its associated objective. Whereas various objectives can have the same preference weighting in a cardinal sense, they will be unique in ordinal sense even though the equality situation $b_i = b_j$ for $i \neq j$ can exist for same objective. Ordering will be preserved because $b_i \geq b_j$ will include the equality case as a subset. Therefore, a reasonable decision model will be the joint intersection of $r$ decision measures,

$$D = \bigcap_{i=1}^{r} \bar{b}_i \cup O_i \quad \text{... (6)}$$

and the optimum solution, $a^*$, is alternative that maximizes $D$.

If we define

$$C_i = \bar{b}_i \cup O_i$$

hence
$$\mu C_i(a) = \max \; [\mu \bar{b}_i(a), \; \mu_{O_i}(a)] \quad \text{... (7)}$$

then the optimum solution, expressed in membership form, is given by

$$\mu_D(a^*) = \max_{a \in A} \left[ \min \left\{ \mu C_1(a), \; \mu C_2(a) \ldots \mu C_r(a) \right\} \right]. \quad \text{... (8)}$$

This model is intuitive in the following manner. As the $i$th objective becomes more important in the final decision, $bi$ increases causing $\bar{b}$ to decrease, which in turn causes $C_i(a)$ to decrease, thereby increasing the likelihood that $C_i(a) = 0_i(a)$, where now $0_i(a)$ will be the value of the decision function, $D$, representing alternative $\boldsymbol{a}$ [see eq. (6)]. As we repeat this process for other alternatives, $\boldsymbol{a}$; eq. (8) reveals that the largest value $0_i(a)$ for other alternatives will eventually will result in the choice of optimum solution, $a^*$.

A special procedure[5] should be followed in the event of a numerical tie between two or more alternatives. If two alternatives $x$, and $y$ are tied, their respective decision values are equal, i.e. $D(x) = D(y)$

$$= \max_{a \in A} \; [D(a)] \text{ where } a = x = y.$$

Since $D(a) = \min_i [C_i(a_i)]$ there exists same alternative $K$ such that $C_k(x) = d(x)$ and some alternative of such $C_g(y) = D(y)$.

Let

$$\hat{D}(x) = \min_{i \neq k} \; [C_i(x)] \text{ and } \hat{D}(y) = \min_{i \neq g} \; [C' \; y)]. \quad \text{... (9)}$$

Then, we compare $\hat{D}(x)$ and $\hat{D}(y)$ and if for example, $\hat{D}(x) > \hat{D}(y)$ we select $x$ as our optimum alternative. However, if a tie still persists i.e. if $\hat{D}(x) = \hat{D}(y)$ then there exists some other alternatives $j$ and $h$ such that $\hat{D}(x) = c_j(x)$ and $\hat{D}(y) = C_h(y)$.

Then we formulate

$$\hat{\hat{D}}(x) = \min_{i \neq k, j} [C_i(x)] \text{ and } \hat{\hat{D}}(y) = \min_{i \neq g, h} [C_i(y)] \quad \ldots (10)$$

and compare $\hat{\hat{D}}(x)$ and $\hat{\hat{D}}(y)$. The tie-breaking procedure continues in this manner until an unambiguous optimum alternative emerges or all the alternatives have been exhausted. In the latter case, where a tie still results some other tie-breaking procedure, such as a refinement in this preference scales, can be used.

*Selection of type of Caisson for Bridge Foundation* : Many a time, when undertaking a big project like construction of a bridge, docks, break waters and other structures for shore protection, when the soil contains large boulders and massive structure is required to extend to our below the river or lake bed to provide resistance against destructive force due to floating objects, sand scour, etc. and when the foundation is subjected to large lateral forces, Caissons[1] are the only choice left.

A big construction company specialized in this type of work undertakes the project of laying Caisson for a bridge which henceforth will be termed as decision maker.

After surveying the site, engineer has to decide which type of Caisson to select for the purpose. Among the many alternative designs available, engineer reduces the list of candidates for bridge foundation design to three alternatives : (1) Open Caisson; (2) Pneumatic Caisson and (3) Box Caisson.

The decision maker has defined five objectives that affect his decision making process : (*i*) Cost, (*ii*) Maintainability of the Caisson, (*iii*) whether the design is a standard one, (*iv*) requirement of skilled workers and (*v*) time taken for the completion of project.

Moreover, the owner (decision maker) also decides to rank his preferences for these objectives on the unit interval. Hence the engineer sets up his problem, as follows :

$$A = \{OC, PNC, BC\} = \{a_1, a_2, a_3\}$$

$O$ = {Cost, Plumbness contro, Depth, Area of site, Nonavailability of skilled workers, Time required}

Each objective here deserves to be calssified i.e.

| | | |
|---|---|---|
| Cost | - | Lower cost has larger membership |
| Plumbness control | - | Accuracy of the plumbness has larger membership |
| Depth | - | Fixed quantity and has been assigned membership 1 |
| Area of site | - | Lesser area has larger membership |

| | | |
|---|---|---|
| Non-availability of skilled worker | - | Lesser availability has larger membership |
| Time required | - | Shorter time has larger membership |

$$P = \{b_1, b_2, b_3, b_4, b_5, b_6\} \rightarrow [0, 1].$$

From previous experience with various foundation designs, engineer first rates the Caissons with respect to objectives given below. These ratings are fuzzy sets expressed in Zadeh's notation,

$$\begin{matrix} \underset{\sim}{O}_1 \\ \underset{\sim}{O}_2 \\ \underset{\sim}{O}_3 \\ \underset{\sim}{O}_4 \\ \underset{\sim}{O}_5 \\ \underset{\sim}{O}_6 \end{matrix} \begin{bmatrix} 0.9/OC & 0.3/PNC & 0.8/BC \\ 0.3/OC & 0.9/PNC & 0.6/BC \\ 0.8/OC & 0.9/PNC & 0.4/BC \\ 0.4/OC & 0.3/PNC & 1.0/BC \\ 0.9/OC & 0.4/PNC & 0.7/BC \\ 0.2/OC & 0.7/PNC & 0.9/BC \end{bmatrix}$$

These are the fuzzy membership values shown graphically in Fig. 1 for the said criteria which at a glance does not provide any clear idea as to which type of Caisson should be laid. Therefore, engineer employs the decision making procedure as described in this paper.

Now, the engineer wishes to investigate two decision scenarios. Each decision scenario propogates a different set of preferences from the decision maker (Construction Company) who wishes to determine the sensitivity of the optimum solutions to his preference ratings.

*Scenario I*

In the first scenario, the decision maker lists his preferences for each of the six objectives as shown in Fig. 2. From there preference values the following calculations result.

| $b_1$ | $\bar{b}_1$ | $b_2$ | $\bar{b}_2$ | $b_3$ | $\bar{b}_3$ | $b_4$ | $\bar{b}_4$ | $b_5$ | $\bar{b}_5$ | $b_6$ | $\bar{b}_6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.8 | 0.2 | 0.6 | 0.4 | 1.0 | 0.0 | 0.5 | 0.5 | 0.6 | 0.4 | 0.9 | 0.1 |

$$D(a_1) = D(OC) = (0.2 \vee 0.9) \wedge (0.4 \vee 0.3) \wedge (0.0 \vee 0.8)$$

$$\wedge (0.5 \vee 0.4) \wedge (0.4 \vee 0.9) \wedge (0.1 \vee 0.2)$$

$$= 0.9 \wedge 0.4 \wedge 0.8 \wedge 0.5 \wedge 0.9 \wedge 0.2$$

$$= 0.2$$

$$D(a_2) = D(PNC) = (0.2 \vee 0.3) \wedge (0.4 \vee 0.9) \wedge (0.0 \vee 0.9)$$

$$\wedge (0.5 \vee 0.3) \wedge (0.4 \vee 0.3) \wedge (0.1 \vee 0.7)$$

$$= 0.3 \wedge 0.9 \wedge 0.9 \wedge 0.5 \wedge 0.4 \wedge 0.7$$

$$= 0.3$$

$$D(a_3) = D(BC) = (0.2 \vee 0.8) \wedge (0.4 \vee 0.6) \wedge (0.0 \vee 1.0)$$

$$\wedge (0.5 \vee 0.7) \wedge (0.4 \vee 0.9) \wedge (0.1 \vee 0.9)$$

$$= 0.8 \wedge 0.6 \wedge 1 \wedge 0.7 \wedge 0.9 \wedge 0.9$$

$$= 0.6.$$

Therefore

$$D^* = \max \{0.2, 0.3, 0.6\} = 0.6.$$

Thus the Engineer chooses the third alternative, $a_3$, the Box Caisson one as his/her foundation design under preference scenario 1.

*Scenario II*

Now in the second scenario engineer was given a different set of preferences by the company as shown in Fig. 3. From there following calculations result.

| $b_1$ | $\bar{b}_1$ | $b_2$ | $\bar{b}_2$ | $b_3$ | $\bar{b}_3$ | $b_4$ | $\bar{b}_4$ | $b_5$ | $\bar{b}_5$ | $b_6$ | $\bar{b}_6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.6 | 0.4 | 0.5 | 0.5 | 1.0 | 0.0 | 0.6 | 0.4 | 0.7 | 0.3 | 0.6 | 0.4 |

$$D(a_1) = (0.4 \vee 0.9) \wedge (0.5 \vee 0.3) \wedge (0.0 \vee 0.8)$$

$$\wedge (0.4 \vee 0.4) \wedge (0.3 \vee 0.9) \wedge (0.4 \vee 0.2)$$

$$= 0.9 \wedge 0.5 \wedge 0.8 \wedge 0.4 \wedge 0.9 \wedge 0.4$$

$$= 0.4$$

$$D(a_2) = (0.4 \vee 0.3) \wedge (0.5 \vee 0.9) \wedge (0.0 \vee 0.9)$$

$$\wedge (0.4 \vee 0.3) \wedge (0.3 \vee 0.4) \wedge (0.4 \vee 0.7)$$

$$= 0.4 \wedge 0.9 \wedge 0.9 \wedge 0.4 \wedge 0.4 \wedge 0.7$$

$$= 0.4$$

$$D(a_3) = (0.4 \vee 0.8) \wedge (0.5 \vee 0.6) \wedge (0.0 \vee 0.4)$$

$$\wedge (0.4 \vee 1) \wedge (0.3 \vee 0.7) \wedge (0.4 \vee 0.9)$$

$$= 0.8 \wedge 0.6 \wedge 0.4 \wedge 1 \wedge 0.7 \wedge 0.9$$

$$= 0.4.$$

Therefore

$$D^* = \max \{D(a_1), D(a_2), D(a_3)\}$$

$$= \max \{0.4, 0.4, 0.4\} = 0.4.$$

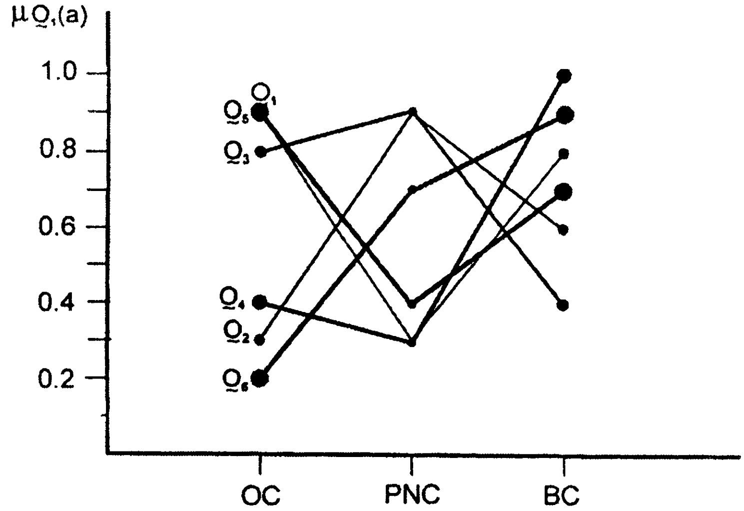


FIG. 1

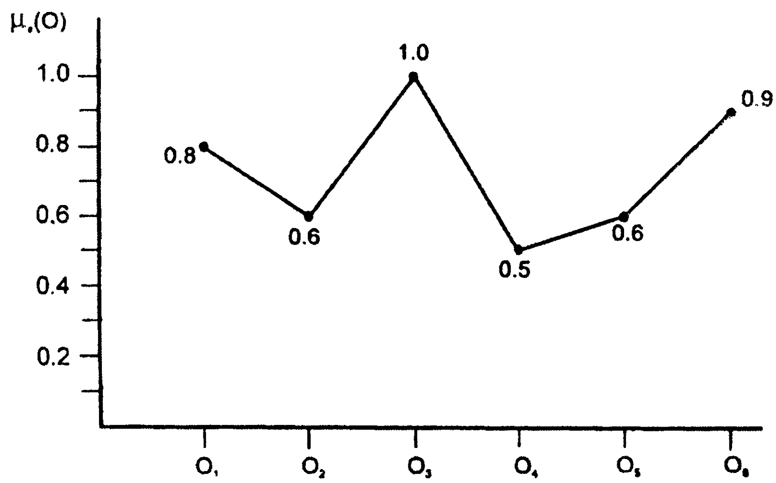


FIG. 2. Preference in Scenario I

Here, there is a tie between all the three alternatives.

To resolve this tie, the engineer implements the eq. (9). After looking closely at $D(a_1)$, $D(a_2)$ and $D(a_3)$ and noting that the decision value of 0.4 for $D(a_1)$ comes from the IV and VI term, for $D(a_2)$ comes from I, IV and V term and for $D(a_3)$ comes from III term. Then the calculations proceed again between tied choices $a_1, a_2$ and $a_3$.

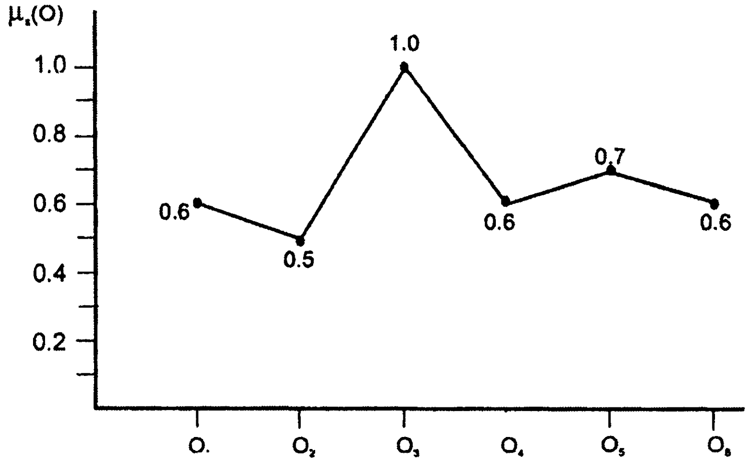


FIG. 3. Preference in Scenario II

$$\hat{D}(a_1) = \hat{D}(OC) = (0.4 \vee 0.9) \wedge (0.5 \vee 0.3) \wedge (0.0 \vee 0.8) \wedge (0.3 \vee 0.9)$$

$$= 0.9 \wedge 0.5 \wedge 0.8 \wedge 0.9$$

$$= 0.5.$$

$$\hat{D}(a_2) = \hat{D}(PNC) = (0.5 \vee 0.9) \wedge (0.0 \vee 0.9) \wedge (0.4 \vee 0.7)$$

$$= 0.9 \wedge 0.9 \wedge 0.7$$

$$= 0.7.$$

$$\hat{D}(a_3) = \hat{D}(BC) = (0.4 \vee 0.8) \wedge (0.5 \vee 0.6) \wedge (0.4 \vee 1)$$

$$\wedge (0.3 \vee 0.7) \wedge (0.4 \vee 0.9)$$

$$= 0.8 \wedge 0.6 \wedge 1.0 \wedge 0.7 \wedge 0.7$$

$$= 0.6.$$

Then

$$D^* = \max \{\hat{D}(a_1), \hat{D}(a_2), \hat{D}(a_3)\}$$

$$\max \{0.5, 0.7, 0.6\} = 0.7.$$

Hence the tie is broken and the engineer chooses second one i.e. Pneumatic Caisson $\boldsymbol{a_2}$ for foundation design under preference Scenario 2.

## CONCLUSION

Our approach involving multiple objective decision making situation represent an interesting class of problem that takes into account optimisation in decision[2] as do multiattribute decision problems[3].

The present work can be expanded to variety of foundation design problems by clearly choosing the criteria and the preferences. A problem on the selection of type of retaining wall design has already been undertaken by earlier researchers[4].

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