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THE EFFECT OF VARIABLE VISCOSITY ON LAMINAR FLOW DUE TO A POINT SINK

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The steady laminar boundary layer flow for a point sink with variable viscosity effects has been investigated. The similarity solutions of the two point boundary value problem governed by coupled parabolic partial differential equations have been obtained numerically using an implicit finite difference scheme. The temperature dependent viscosity is found to have a pronounced effect on momentum and thermal transport in the boundary layer region.

Key Words: Point Sink; Variable Viscosity; Skin Friction; Heat Transfer

1. INTRODUCTION

The analysis of laminar flow and convective heat transfer, under the influence of thermo-physical properties, has gained importance in many scientific and engineering applications. For the flows in internal passages, it is known that the radial temperature distribution significantly modifies the velocity distributions through temperature viscosity coupling[1]. Hence, like other thermo-physical properties, temperature-dependent viscosity also plays a vital role in surface friction and heat transfer rate near the wall[2-5]. In spite of its importance in many applications, the aspect of temperature-dependent viscosity has received rather little attention of researchers. This prompted the authors to investigate the influence of variation of viscosity with temperature on the axisymetric flow inside a cone due to a point sink. The present study is relevant in conical nozzle and diffuser flow problems.

2. FORMULATION AND GOVERNING EQUATIONS

Consider the steady laminar axisymmetric forced convection flow of an incompressible fluid inside a cone with a hole at the vertex. The hole can be regarded as a three-dimensional point sink[6]. The cone has been taken as semi-infinite so that it can be regarded as independent of the length r. The fluid properties are assumed to be isotropic and constant, except for the fluid viscosity (μ) which is assumed to be an inverse linear function of the temperature (T) (see Lai and Kulacki[7]), viz.,

$$1/\mu = 1/\mu_\infty [1 + \beta (T - T_\infty)] \qquad \text{... (1)}$$

or

$$1/\mu = a (T - T_e)] \qquad \text{... (2)}$$

where

$$a = \beta/\mu_\infty \text{ and } T_e = T_\infty - 1/\beta. \qquad \text{... (3)}$$

Here, both a and T_e are constants and their values depend on the reference state and the thermal property of the fluid i.e., (β). In general, $a > 0$ for liquids and $a < 0$ for gases.

Under the foregoing assumptions, the boundary layer equations governing the momentum and heat transfer are:

$$\partial (ru)/\partial r + \partial (rw)/\partial z = 0 \quad \text{... (4)}$$

$$u\, \partial u/\partial r + w\, \partial u/\partial z = U\, \partial U/\partial r + 1/\rho_\infty\, \partial/\partial z\, (\mu\, \partial u/\partial z) \quad \text{...(5)}$$

$$u\, \partial T/\partial r + w\, \partial T/\partial z = \alpha\, \partial^2 T/\partial z^2 \quad \text{... (6)}$$

where $U = -m/r^2, m > 0.$ ... (7)

The boundary conditions are given by

$$\left.\begin{array}{l} u\,(r, 0) = 0,\ w\,(r, 0) = 0,\ T\,(r, 0) = Tw \\ u\,(r, \infty) = U,\ T\,(r, \infty) = T_\infty \end{array}\right\}. \quad \text{... (8)}$$

Introducing the following transformations

$$\eta = (m/2\nu r^3)^{1/2} z, \quad \psi(r, z) = -(2m\ \nu r)^{1/2} f(\eta)$$

and

$$ru = (\partial \psi/\partial z),\ rw = -(\partial \psi/\partial r),\ G\,(\eta) = (T - T_\infty/T_w - T_\infty),$$

$$Pr = (\nu/\alpha),\ \nu = \mu/\rho \quad \text{... (9)}$$

to eqs. (4-6), we find eq. (4) is identically satisfied and eqs. (5) and (6) reduce, respectively, to

$$F'' + [1 - (G\ /\ Ge)\ \{4(1 - F^2) - fF'\}]$$
$$+ (G\ /\ Ge)\ \{\ F'G'\ /\ (1 - G\ /\ Ge)\} = 0 \quad \text{... (10)}$$

and

$$G'' - Pr\, G' = 0 \quad \text{... (11)}$$

where (′) denotes the derivative with respect to η and

$$u = UF\,(\eta),\ F = f',\ w = (m\nu/2r^3)^{1/2}\ \{f - 3\eta F\}$$

$$f = \int_0^\eta F d\eta, \quad Ge = T_e - T_\infty/T_w - T_\infty = -1/\beta\,(T_w - T_\infty) \quad \text{... (12)}$$

The transformed boundary conditions are

$$\left.\begin{array}{l} F = 0;\ G = 1 \ \text{ at } \ \eta = 0 \\ F = 1;\ G = 0 \ \text{ as } \ \eta \to \infty \end{array}\right\}. \quad \text{... (13)}$$

Here η is the transformed similarity variable; u and w are velocity components along r- and z-directions, respectively; ψ and f are the dimensional and dimensionless stream functions, respectively; F (or f') is dimensionless velocity component; G is the dimensionless temperature; Pr is the Prandtl number; ρ, ν, α are respectively, density, kinematic viscosity and thermal diffusivity;

U is the inviscid flow velocity; m is the strength of the point sink; the subscripts w and ∞ denote, respectively, conditions at the wall ($\eta = 0$) and in the free stream ($\eta \to \infty$) respectively; the subscript e represents the conditions at the edge of the boundary layer.

The value of the viscosity variation parameter Ge is determined by the viscosity of the fluid under consideration and the operating temperature difference between the wall. If Ge is large (i.e. $Ge \to \infty$), the effects of variable viscosity can be neglected. On the other hand, for a smaller value of Ge, either the fluid viscosity changes markedly with temperature or operating temperature difference is high. In either case, the variable viscosity effect is expected to become very important. Also, it may be noted here that, the liquid viscosity varies differently with temperature than that of gas and therefore it is important to note that $Ge < 0$ for liquids and $Ge > 0$ for gases.

It is worth mentioning here that, when $\beta \to 0$ i.e., if $\mu = \mu_\infty$ then $Ge \to \infty$ and the eq. (10) reduces to

$$F'' + 4(1 - F^2) - fF' = 0 \qquad \text{... (14)}$$

which is precisely the equation considered by Roseanhead[6] who studied, only the momentum transfer inside the boundary layer with constant viscosity. It may be remarked here that the boundary layer approximations is not valid in the immediate neighbourhood of the vertex of the cone[1].

The physical quantities of interest are the surface friction (C_1) and Nusselt number (Nu_r) defined as

$$C_f = 2\tau_w / \rho U^2 = 2^{\frac{1}{2}} (Re)^{-\frac{1}{2}} [(Ge/1 + Ge)] F_w \qquad \text{... (15)}$$

and

$$Nu_r = [rq_w/k(T_w - T_\infty)] = -2^{1/2} (Re_r)^{1/2} G'_w \qquad \text{... (16)}$$

where $\tau_w = \mu (\partial u/\partial z)_w$, $q_w = -k(\partial T/\partial z)_w$, $\mathrm{Re}_r = (m/\nu r)$. Here τ_w and q_w are the shear stress and heat transfer at the wall respectively; Re_r is the local Reynold's number, k is the thermal conductivity. Further, F'_w and G'_w represent, respectively, the skin friction parameter and heat transfer rate at the wall.

3. Results and Discussion

The transformed dimensionless coupled non-linear eqs. (10) and (11) together with boundary conditions (13) are solved using a stable finite difference method described in Choi[8]. To ensure the convergence of the finite difference scheme the step size ($\Delta \eta$) has been optimized and the results presented here are independent of step sizes or (η_∞) up to 4th decimal places. The calculations have been carried out with ($\Delta \eta$) = 0.05 and (η_∞) = 4.0

To verify the proper treatment of the problem the velocity field (F) for $Pr = 0.7$ has been compared with that of the corresponding constant-viscosity case[6], by setting $Ge \to \infty$, and the results are in excellent agreement. For sake of brevity, the comparison is not shown here. Our skin friction parameter F'_w for constant-viscosity case is found to be 2.2728 as compared to 2.273 reported in Roseanhead[6].

Table I reveals the influence of *Ge*, the viscosity variation parameter, in the flow and temperature fields, for $Pr = 0.7$ (air) and $Pr = 7.0$ (water). Results indicate that as *Ge* increases, the momentum boundary layer thickness decreases, whereas the thermal boundary layer thickness increases. This phenomenon is more pronounced for the case of water as compared to that of air. Further, for $Ge > 0$ ($Ge = 6.0$) it is observed that velocity shoots up little beyond the edge of the boundary layer, indicating the significant effect of variation of viscosity with temperature.

TABLE I
Effect of viscosity variation on velocity and temperature fields

	Pr = 0.7				Pr = 0.70			
Ge	– 6.0		6.0		– 6.0		6.0	
η	F	G	F	G	F	G	F	G
0.0	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
0.4	0.4996	0.9418	0.3668	0.9705	0.5247	1.0000	0.3670	1.0000
0.8	0.7261	0.8789	0.7246	0.8776	0.7924	1.0000	0.8171	1.0000
1.2	0.8865	0.7887	0.9226	0.7900	0.8561	0.9998	0.9318	1.0000
1.6	0.9138	0.6722	0.9870	0.5923	0.9017	0.8828	0.9842	0.9999
2.0	0.9466	0.5873	1.0240	0.4788	0.9324	0.6457	1.0026	0.9951
2.4	0.9947	0.3112	1.03002	0.3173	0.9816	0.4127	1.0930	0.6745
2.8	0.9995	0.0755	1.0046	0.0776	0.9992	0.2285	1.1130	0.2895
3.2	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000

TABLE II
Viscosity variation effects on skin friction and heat transfer

Pr = 7.0			Pr = 0.7		
Ge	F'_w	G'_w	Ge	F'_w	G'_w
– 10.0	2.71256	– 1.10290	2.0	1.23421	– 0.32486
– 8.0	2.83849	– 0.12065	4.0	1.49534	– 0.26454
– 6.0	3.06403	– 0.11036	6.0	1.78338	– 0.22384
– 4.0	3.57983	– 0.10984	8.0	1.96723	– 0.12286
– 2.0	3.96203	– 0.0244	10.0	2.21619	– 0.12266
			....	2.27283	– 0.12236
			∞	2.27283	

Table 2 contains numerical results for skin friction parameter (F'_w) and heat transfer rate G'_w at the wall for both air and water for different values of *Ge* varying from –10 to 10. The results show that F'_w increases when *Ge* (< 0 or > 0) increases whereas for increasing values of *Ge*, G'_w decreases. Further in the case of water ($Pr = 7.0$), when $Ge = -10.0$, the value of F'_w is 2.7126 whereas it is 3.9620 if $Ge = -2.0$ showing about 46% increase in the skin friction.

Similarly, when Ge = –10.0, the value of G'_w is –1.10290 whereas it is 0.0244 if Ge = –2.0 showing about 80% decrease in the heat transfer. On the other hand, in the case of air (Pr = 0.7), there is about 44% increase in skin friction and about 62% decrease in heat transfer due to the change from the values of Ge = 2.0 to Ge = 6.0. Further more, when Pr =0.7, we see that the value F'_w reaches 2.2728 as $Ge \to \infty$. Hence, the skin friction at the wall asymptotically approaches to that of constant-viscosity case which implies either β or $(T_w - T_\infty)$ are very small. This is expected since large Ge implies that effect of variable viscosity can be neglected.

4. CONCLUSIONS

From the present study, it is found that the temperature dependent viscosity has a substantial effect on the drag and heat transfer characteristics as well as the velocity and temperature distributions within the boundary layer. Hence, it can be concluded that when the viscosity of a working fluid is sensitive to temperature variation or the temperature difference between the wall and the ambient fluid is moderate or large, the variable viscosity effect has to be taken into consideration. Otherwise, a considerable error in the prediction of surface skin friction factor and heat transfer rate may occur.

REFERENCES

1. H. Schlichting, *Boundary Layer Theory*, McGraw Hill, New York, 1960, p. 143.
2. A. Pantokratoras, *Int. J. Heat Mass Transfer*, **45** (2002), 963-77.
3. P. Saikrishnan and S. Roy, *Int. J. Engg. Sci.*, **41** (2003), 1351-65.
4. P. Saikrishnan and S. Roy, *Acta Mechanica*, **157** (2002), 187-99.
5. S. Roy and P. Saikrishnan, *Int. J. Heat Mass Transfer*, **46** (2003), 3389-96.
6. L. Roseanhead, (Ed.) *Laminar Boundary Layer*, Oxford University Press, Oxford, 1963, p. 427.
7. F. C. Lai and F. A. Kulacki, *Int. J. Heat Mass Transfer*, **33** (1990), 1028-31.
8. I. G. Choi, *Int. J. Heat Mass Transfer*, **25** (1982), 597-03.