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EFFECT OF SUSPENDED PARTICLES ON COUPLE-STRESS FLUID HEATED FROM BELOW IN THE PRESENCE OF ROTATION AND MAGNETIC FIELD

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The thermal instability of a couple-stress fluid with suspended particles is considered. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For stationary convection, couple-stress is found to postpone the onset of convection whereas suspended particles hasten it. It is found that the principle of exchange of stabilities is satisfied. The thermal instability of a couple-stress fluid with suspended particles, in the presence of rotation and magnetic field separately, is also considered. The magnetic field and rotation are found to have stabilizing effects on the stationary convection and introduce oscillatory modes in the system. The sufficient conditions for the non-existence of overstability are also considered.

Key Words: Thermal Convection; Couple-Stress Fluid; Fine Dust; Rotation; Magnetic Field

1. INTRODUCTION

The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside, plays an important role in geophysics, interior of the Earth, oceanography and atmospheric physics etc. A detailed account of the theoretical and experimental study of the onset of Bénard convection in Newtonian fluids, under varying assumptions of hydrodynamics, has been given by Chandrasekhar[1]. The use of Boussinesq approximation has been made throughout which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. Sharma[2] has considered the effect of rotation and magnetic field on the thermal instability in compressible fluids. The fluid has been considered to be Newtonian in all the above studies. Chandra[3] observed that in an air layer, convection occurred at much lower gradients than predicted if the layer depth was less than 7 mm and called this motion "columnar instability". However for layers deeper than 10 mm, a Bénard-type cellular convection was observed. Thus there is a contradiction between the theory and experiment. Scanlon and Segel[4] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure fluid was supplemented by that of the particles. Palaniswamy and Puroshotham[5] have considered that stability of shear flow of stratified fluids with fine dust and have found the effect of fine dust to increase the region of instability.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. Stokes[6] proposed and postulated the theory of couple-stress fluid. One of the applications of the couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded-bearing synovial joints of the human body and these joints have a low-friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is a viscous, non-Newtonian fluid. According to the theory of Stokes[6], couple-stresses are found to appear in noticeable magnitude in fluids with very large molecules. Since the long chain hylauronic acid molecules are found as additives in synovial fluid, Walicki and Walicka[7] modelled synovial fluid as a couple-stress fluid in human joints. Goel *et al.*[8] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture having vertical temperature and concentration gradients with rotation. Sharma and Sharma[9] have studied the couple-stress fluid heated from below in porous medium. An electrically conducting couple-stress fluid heated from below in porous medium in the presence of uniform horizontal magnetic field has also been studied by Sharma and Sharma[10] The use of magnetic field is being made for the clinical purposes in detection and cure of certain diseases with the help of magnetic field devices/instruments. Sharma and Thakur[11] have studied the thermal convection in couple-stress fluid in porous medium in hydromagnetics.

Environmental pollution is the main cause of dust to enter into human body. The metal dust which filters into the blood stream of those working near furnace causes extensive damage to the chromozomes and genetic mutations so observed are likely to breed cancer as malformations in the coming progeny. Therefore, it is very essential to study the blood flow with dust particles. Considering blood as couple-stress fluid and dust particles as micro-organisms, Rathod and Thippeswamy[12] have studied the gravity flow of pulsatile blood through closed rectangular inclined channel with micro-organisms.

Keeping in mind the importance of non-Newtonian fluids and convection in fluid layer heated from below, the present paper is devoted to study the effect of suspended particles on the couple-stress fluid heated from below. The effects of magnetic field and rotation, separately, having relevance and importance in geophysics and bio-mechanics are also considered.

2. Formulation of the Problem — Perturbation Equations

Consider an infinite, horizontal, incompressible couple-stress fluid layer of thickness d, heated from below so that, the temperature and density at the bottom surface $z = 0$ are T_0, ρ_o respectively and at the upper surface $z = d$ are T_d, ρ_d and that a uniform adverse temperature gradient $\beta = \left(\left| \frac{dT}{dz} \right| \right)$ is maintained. Let ρ, p, T and $\vec{q}(u, v, w)$ denote respectively the density, pressure, temperature and velocity of the fluid; $\vec{q}_q\, (\bar{x}, t)$ and $N(\bar{x}, t)$ denote the velocity and number density of suspended particles respectively. Then the momentum balance, mass balance equations of the couple-stress fluid (Stokes[6], Chandrasekhar[1] and Scanlon and Segel[4]) are

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} = -\frac{1}{\rho_o} \nabla p$$

$$+ \vec{g}\left(1 + \frac{\delta \rho}{\rho_o}\right) + \left(\nu - \frac{\mu'}{\rho_o}\nabla^2\right)\nabla^2 \vec{q} + \frac{KN}{\rho_o}(\vec{q}_d - \vec{q}), \qquad \ldots (1)$$

$$\nabla \cdot \vec{q} = 0. \qquad \ldots (2)$$

The equation of state is

$$\rho = \rho_o\, [1 - \alpha(T - T_o)], \qquad \ldots (3)$$

where the suffix zero refers to the values at the reference level $z = 0$. Here $\vec{g}\,(0, 0, -g)$ is acceleration due to gravity, $\bar{x} = (x, y, z)$ and $K = 6\pi \mu \eta'$, η' being particle radius, is the Stokes' drag coefficient. Assuming uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term, in the equation of motion (1), proportional to the velocity difference between particles and fluid.

The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. Interparticle reactions are ignored for we assume that the distances between the particles are large compared with their diameters. If mN is the mass of the particles per unit volume, then the equations of motion and continuity for the particles are:

$$mN\left[\frac{\partial \vec{q}_d}{\partial t} + (\vec{q}_d \cdot \nabla)\vec{q}_d\right] = KN[\vec{q} - \vec{q}_d], \quad \text{... (4)}$$

$$\frac{\partial N}{\partial t} + \nabla \quad (N\vec{q}_d) = 0. \quad \text{... (5)}$$

Let c_v, c_{pt} denote the heat capacity of the fluid at constant volume and the heat capacity of the particles. Assuming that the particles and fluid are in thermal equilibrium, the equation of heat conduction gives

$$\rho_0 c_v\left(\frac{\partial}{\partial t} + \vec{q} \cdot \nabla\right)T + mNc_{pt}\left(\frac{\partial}{\partial t} + \vec{q}_d \cdot \nabla\right)T = q\nabla^2 T,$$

or

$$\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)T + \frac{mNc_{pt}}{\rho_0 c_v}\left(\frac{\partial}{\partial t} + \vec{q}_d \cdot \nabla\right)T = \kappa\nabla^2 T. \quad \text{... (6)}$$

The kinematic viscosity ν, couple-stress viscosity μ', thermal diffusivity κ and coefficient of thermal expansion α are all assumed to be constants.

The basic motionless solution is

$$\vec{q} = (0, 0, 0),\ \vec{q}_d = (0, 0, 0),\ T = T_o - \beta z,\ \rho = \rho_o(1 + \alpha\beta z), \quad \text{... (7)}$$

$N = N_o$, a constant.

Assume small perturbations around the basic solution and let $\delta\rho, N, \delta p, \theta, \vec{q}(u, v, w)$ and $\vec{q}_d\ (l, r, s)$ denote respectively the perturbations in density, suspended particles number density N_o, pressure p, temperature T, couple-stress fluid velocity (0, 0, 0) and particles velocity (0, 0, 0). The change in density $\delta\rho$ caused mainly by the perturbation θ in temperature is given by

$$\delta\rho = -\alpha\rho_o\theta. \quad \text{... (8)}$$

Then the linearized perturbation equations of the couple-stress fluid become

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_o}\nabla\delta p - \vec{g}\alpha\theta + \left(\nu - \frac{\mu'}{\rho_o}\nabla^2\right)\nabla^2\vec{q} + \frac{KN_o}{\rho_o}(\vec{q}_d - \vec{q}), \quad \text{... (9)}$$

$$\nabla \cdot \vec{q} = 0, \quad \text{... (10)}$$

$$mN_0\frac{\partial \vec{q}_d}{\partial t} = KN_o(\vec{q} - \vec{q}_d), \quad \text{... (11)}$$

$$(1+h)\frac{\partial \theta}{\partial t} = \beta (w + hs) + \kappa \nabla^2 \theta, \qquad \text{... (12)}$$

where

$$\kappa = \frac{q}{\rho_0 c_v} \quad \text{and} \quad h = \frac{mN_0 c_{pt}}{\rho_0 c_v}.$$

3. The Dispersion Relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, \theta] = [W(z), \Theta(z)] \exp (ik_x x + ik_y y + nt), \qquad \text{... (13)}$$

where k_x, k_y are wave numbers along x- and y-directions respectively, $k = \left(\sqrt{k_x^2 + k_y^2}\right)$ is the resultant wave number and n is, in general, a complex constant.

Using (13), eqs. (9) and (12), on using (10) and (11), in non-dimensional form, become

$$\left(D^2 - a^2\right)\left[\sigma\left(1 + \frac{M}{1+\tau_1 \sigma}\right) + F(D^2 - a^2)^2 - (D^2 - a^2)\right] W$$

$$= -\frac{g\alpha d^2 a^2 \Theta}{\nu}, \qquad \text{... (14)}$$

$$\left(D^2 - a^2 - Hp_1 \sigma\right)\Theta = -\frac{\beta d^2}{\kappa}\frac{(H + \tau_1 \sigma)}{(1+\tau_1 \sigma)} W, \qquad \text{... (15)}$$

where

$$a = kd,\ \sigma = \frac{nd^2}{\nu},\ \tau = \frac{m}{K},\ \tau_1 = \frac{\tau \nu}{d^2},\ M = \frac{mN_0}{\rho_0},$$

$$p_1 = \frac{\nu}{\kappa},\ H = 1 + h,\ F = \frac{\mu'}{\rho_0 d^2 \nu}$$

and

$$D = \frac{d}{dz}.$$

Since both the boundaries are maintained at constant temperature, the perturbations in the temperature are zero at the boundaries. The case of two free boundaries is little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions with respect to which eqs. (14) and (15) must be solved are

$$W = 0,\ \Theta = 0 \text{ at } z = 0 \text{ and } 1. \qquad \text{... (16)}$$

The constitutive equations for the couple-stress fluid are

$$\tau_{ij} = (2\mu - 2\mu' \nabla^2)\, e_{ij},$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).$$

The conditions on a free surface are

$$\tau_{xz} = (\mu - \mu' \nabla^2)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = 0,$$

$$\tau_{yz} = (\mu - \mu' \nabla^2)\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) = 0. \qquad \text{... (17)}$$

From the equation of continuity (10) differentiated with respect to z, we conclude that

$$\left[\mu - \mu' \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right] \frac{\partial^2 w}{\partial z^2} = 0, \qquad \text{... (18)}$$

which implies that

$$\frac{\partial^2 w}{\partial z^2} = 0, \; \frac{\partial^4 w}{\partial z^4} = 0, \text{ at } z = 0 \text{ and } d. \qquad \text{... (19)}$$

Using (13), the boundary conditions (19) in non-dimensional form transform to

$$D^2 W = D^4 W = 0 \text{ at } z = 0 \text{ and } 1. \qquad \text{... (20)}$$

It can be shown from eqs. (14), (15) and boundary conditions (16), (20) that all the even order derivatives of W must vanish for $z = 0$ and 1 and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \qquad \text{... (21)}$$

where W_o is a constant.

Eliminating Θ between eqs. (14) and (15) and substituting the proper solution (21) in the resultant equation, we obtain the dispersion relation

$$R_1 = \left(\frac{1 + i\,\tau_1\,\sigma_1\,\pi^2}{H + i\tau_1\,\sigma_1\,\pi^2} \right) \left(\frac{1+x}{x} \right) \times$$

$$\left[i\sigma_1 \left(1 + \frac{M}{1 + i\tau_1\,\sigma_1\,\pi^2} \right) + (1+x) + F_1\,(1+x)^2 \right] (1 + x + iH\,p_1\,\sigma_1) \qquad \text{... (22)}$$

where

$$R_1 = \frac{R}{\pi^4},\; x = \frac{a^2}{\pi^2},\; i\sigma_1 = \frac{\sigma}{\pi^2},\; F_1 = \pi^2 F.$$

4. The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (22) reduces to

$$R_1 = \frac{(1+x)^3}{xH}\,[1 + F_1\,(1+x)]. \qquad \text{... (23)}$$

To study the effect of couple-stress and suspended particles, we examine the nature of $\frac{dR_1}{dF_1}$ and $\frac{dR_1}{dH}$ analytically. Eq. (23) gives

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{xH}. \qquad \text{... (24)}$$

which is always positive meaning thereby that for the stationary convection, the couple-stress postpones the onset of convection and has a stabilizing effect. Eq. (24) yields

$$\frac{dR_1}{dH} = -\frac{(1+x)^3}{xH^2}\,[1 + F_1\,(1+x)], \qquad \text{... (25)}$$

which is always negative meaning thereby that for the stationary convection, the suspended particles hasten the onset of convection and have a destabilizing effect.

5. Principle of Exchange of Stabilities

Multiplying eq. (14) by W , the complex conjugate of W and using (15) together with boundary conditions (16) and (20), we obtain

$$\sigma\left(1 + \frac{M}{1+\tau_1\,\sigma}\right)I_1 + I_2 + FI_3 = \frac{g\alpha a^2 \kappa}{\nu\beta}\left(\frac{1+\tau_1\,\sigma^*}{H+\tau_1\,\sigma^*}\right)(I_4 + Hp_1\,\sigma^*\,I_5), \qquad \text{... (26)}$$

where

$$I_1 = \int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz,$$

$$I_2 = \int_0^1 \left(|D^2 W|^2 + 2a^2|DW|^2 + a^4|W|^2\right)dz,$$

$$I_3 = \int_0^1 \left(|D^3 W|^2 + 3a^2|D^2W|^2 + 3a^4|DW|^2 + a^6|W|^2\right)dz,$$

$$I_4 = \int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz,$$

$$I_5 = \int_0^1 |\Theta|^2 dz, \qquad \text{... (27)}$$

and are all positive definite.

The real and imaginary parts of (26) must vanish separately and the vanishing of the imaginary part gives

$$I_m(\sigma)\left[\left(1 + M\frac{1 - i\tau_1\sigma_i}{1 + \tau_1^2\sigma_i^2}\right)I_1 + \frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{\tau_1(H-1)}{H^2 + \tau_1^2\sigma_i^2}I_4 + \frac{H + \tau_1^2\sigma_i^2}{H^2 + \tau_1^2\sigma_i^2}Hp_1 I_5\right)\right] = 0.$$

But the quantity inside brackets is positive definite. Hence

$$I_m(\sigma) = 0.$$

This establishes that σ is real and that the principle of exchange of stabilities is valid for the couple-stress fluid with suspended particles heated from below.

6. Effect of Rotation, Perturbation Equations and Dispersion Relation

Here we consider an infinite, horizontal, incompressible couple-stress fluid layer with suspended particles of thickness d, heated from below. The fluid is acted on by a uniform rotation $\vec{\Omega}(0, 0, \Omega)$ and gravity force $\vec{g}(0, 0, -g)$. Then the linearized perturbation equations are

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_o}\nabla\delta p + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q} + \vec{g}\frac{\delta\rho}{\rho_o} + 2(\vec{q}\times\vec{\Omega}) + \frac{KN}{\rho_0}(\vec{q}_d - \vec{q}), \qquad \text{... (29)}$$

together with eqs. (10), (11) and (12)

Within the framework of Boussinesq approximation, eqs. (29), (10) (11) and (12) give

$$\left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\left[\frac{\partial}{\partial t}\nabla^2 w - g\alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + 2\Omega\frac{\partial\zeta}{\partial z}\right]$$

$$+\frac{mN}{\rho_o}\frac{\partial}{\partial t}\nabla^2 w = \left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\left(\nu - \frac{\mu'}{\rho_o}\nabla^2\right)\nabla^4 w, \qquad \text{... (30)}$$

$$\left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\left[\frac{\partial\zeta}{\partial t} - 2\Omega\frac{\partial w}{\partial z}\right] + \frac{mN_o}{\rho_o}\frac{\partial\zeta}{\partial t} = \left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\left(\nu - \frac{\mu'}{\rho_o}\nabla^2\right)\nabla^2\zeta, \qquad \text{... (31)}$$

together with (12), where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ denotes the z-component of vorticity.

Analyzing the disturbances into normal modes, we assume that ζ is also of the form

$$[\zeta] = \{Z(z)] \exp[ik_x x + ik_y y + nt], \qquad \text{... (32)}$$

together with (13), the non-dimensional form of the equations becomes

$$\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)\left(D^2-a^2\right)W+\frac{g\alpha a^2 d^2\Theta}{\nu}+\sqrt{T_A}\,dDZ\right]$$

$$=\left[1-F\left(D^2-a^2\right)\right]\left(D^2-a^2\right)^2 W, \qquad \text{... (33)}$$

$$\left[\left\{1-F\left(D^2-a^2\right)\right\}\left(D^2-a^2\right)-\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)\right]Z$$

$$=-\frac{\sqrt{T_A}}{d}DW, \qquad \text{... (34)}$$

$$\left(D^2-a^2-Hp_1\sigma\right)\Theta=-\frac{\beta d^2}{\kappa}\left(\frac{H+\tau_1\sigma}{1+\tau_1\sigma}\right)W. \qquad \text{... (35)}$$

where $T_A=\dfrac{\Omega^2 d^4}{\nu^2}$ denotes the Taylor number.

Eliminating Θ and Z between eqs. (33)-(35), we obtain

$$\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)\left(D^2-a^2\right)\left(D^2-a^2-Hp_1\sigma\right)\times$$

$$\left[\left\{1-F\left(D^2-a^2\right)\right\}\left(D^2-a^2\right)-\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)\right]W$$

$$-Ra^2\left(\frac{H+\tau_1\sigma}{1+\tau_1\sigma}\right)\left[\left\{1-F\left(D^2-a^2\right)\right\}\left(D^2-a^2\right)-\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)\right]W$$

$$-T_A\left(D^2-a^2-Hp_1\sigma\right)D^2W$$

$$=\left(D^2-a^2-Hp_1\sigma\right)\left[\left\{1-F\left(D^2-a^2\right)\right\}\left(D^2-a^2\right)-\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)\right]\times$$

$$\left\{1-F\left(D^2-a^2\right)\right\}\left(D^2-a^2\right)^2 W. \qquad \text{... (36)}$$

Here also we consider the case of two free boundaries. The dimensionless boundary conditions appropriate for the problem are

$$W=D^2W=0,\ DZ=0,\ \Theta=0 \text{ at } z=0 \text{ and } 1. \qquad \text{... (37)}$$

The proper solution of (36) satisfying (37) is given by

$$W=W_0\sin\pi z, \qquad \text{... (38)}$$

where W_o is a constant. Substituting (38) in (36) and letting $R_1 = \frac{R}{\pi^4}, x = \frac{a^2}{\pi^2}, i\sigma_1 = \frac{\sigma}{\pi^2}$, $F_1 = \pi^2 F$, $T_1 = \frac{T_A}{\pi^4}$, we obtain the dispersion relation

$$R_1 x = \frac{\left[i\sigma_1 \left(1 + \frac{M}{1 + i\tau_1 \sigma_1 \pi^2} \right) + \left\{ 1 + F_1 (1+x) \right\} (1+x) \right] (1+x)(1+x+iHp_1\sigma_1)(1+i\tau_1\sigma_1\pi^2)}{H + i\tau_1 \sigma_1 \pi^2}$$

$$+ \frac{T_1 (1 + x + iHp_1 \sigma_1)(1 + i\tau_1 \sigma_1 \pi^2)}{(H + i\tau_1 \sigma_1 \pi^2) \left[\left\{ 1 + F_1 (1+x) \right\} (1+x) + i\sigma_1 \left(1 + \frac{M}{1 + i\tau_1 \sigma_1 \pi^2} \right) \right]}. \quad \text{... (39)}$$

6.1 The Stationary Convection

Eq. (39) for stationary convection (i.e. $\sigma = 0$) reduces to

$$R_1 = \frac{(1+x)^3 [1 + F_1 (1+x)]^2 + T_1}{Hx [1 + F_1 (1+x)]} \quad \text{... (40)}$$

To study the effect of rotation and suspended particles, we examine the natures of $\frac{dR_1}{dT_1}$ and $\frac{dR_1}{dH}$. Eq. (40) gives

$\frac{dR_1}{dT_1} = \frac{1}{xH [1 + F_1 (1+x)]}$, which is always positive meaning thereby that rotation has a stabilizing effect on the couple-stress fluid with suspended particles, heated from below.

Also eq. (40) yields

$$\frac{dR_1}{dH} = -\frac{1}{H^2} \left[\frac{(1+x)^3 \left\{ 1 + F_1 (1+x) \right\}^2 + T_1}{x \left\{ 1 + F_1 (1+x) \right\}} \right], \quad \text{... (41)}$$

which is always negative meaning thereby that suspended particles have a destabilizing effect on couple-stress fluid with suspended particles, heated from below.

6.2 Stability of the System and Oscillatory Modes

In this section, we consider the possibility of oscillatory modes, if any, on the stability problem due to the presence of rotation.

Multiplying (33) by W^* and making use of eqs. (34) and (35), together with boundary conditions (37), we obtain

$$-\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)I_1+\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{1+\tau_1\sigma^*}{H+\tau_1\sigma^*}\right)(I_2+Hp_1\sigma^* I_3)$$

$$-d^2(I_4+FI_5)-d^2\sigma^*\left(1+\frac{M}{1+\tau_1\sigma^*}\right)I_6=I_7+FI_8 \qquad \ldots (42)$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,$$

$$I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,$$

$$I_3=\int_0^1 |\Theta|^2\,dz,$$

$$I_4=\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz,$$

$$I_5=\int_0^1\left(|D^2Z|^2+2a^2|DZ|^2+a^4|Z|^2\right)dz,$$

$$I_6=\int_0^1 |Z|^2\,dz,$$

$$I_7=\int_0^1\left(|D^2W|^2+2a^2|DW|^2+a^4|W|^2\right)dz,$$

$$I_8=\int_0^1\left(|D^3W|^2+3a^2|D^2W|^2+3a^4|DW|^2+a^6|W|^2\right)dz, \qquad \ldots (43)$$

which are all positive definite. Putting $\sigma=i\sigma_i$ in (42), and equating imaginary parts, we obtain

$$\sigma_i\left[\left(1+\frac{M}{1+\tau_1\sigma}\right)I_1+\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{\tau_1(H-1)}{H^2+\tau_1^2\sigma_i^2}I_2+\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2\sigma_i^2}Hp_1I_3\right)-d^2\left(1+\frac{M}{1+\tau_1\sigma^*}\right)I_6\right]=0. \qquad \ldots (44)$$

In the absence of rotation, eq. (44) reduces to

$$\sigma_i\left[\left(1+\frac{M}{1+\tau_1^i\sigma_i^2}\right)I_1 + +\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{\tau_1(H-1)}{H^2+\tau_1^2\sigma_i^2}I_2+\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2\sigma_i^2}Hp_1 I_3\right)\right] = 0. \quad \text{... (45)}$$

The terms in the bracket are positive definite. Thus $\sigma_i = 0$ which means that oscillatory modes are not allowed and that principle of exchange of stabilities is satisfied in the absence of rotation. It is evident from eq. (44) that the presence of rotation brings oscillatory modes (as σ_i may not be zero) which were non-existent in its absence, for a couple-stress fluid layer with suspended particles, heated from below.

6.3. The Overstable Case

Here we discuss the possibility of whether instability may occur as an overstability. When the marginal state is oscillatory, we must have $\sigma_i \neq 0$, $\sigma_r = 0$.

Since for overstability we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it will suffice to find conditions for which eq. (39) will allow for solutions with σ_i real.

Separating the real and imaginary parts of eq. (39) and eliminating R_1 between them, we obtain

$$A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0, \quad \text{... (46)}$$

where we have put $c_1 = \sigma_1^2$, $b = 1 + x$ and

$$A_3 = b^2 \tau_1^4 \pi^8 (1 + Hp_1 (1 + F_1 b)) + Hp_1 \tau_1^3 \pi^6 b (M - h),$$

$$A_2 = b^4 \tau_1^4 \pi^8 (1 + F_1 b)^2 (1 + Hp_1 (1 + F_1 b)) + b^3 \tau_1^3 \pi^6$$

$$\{h(1 + F_1 b) + 2M (1 + F_1 b)(1 + Hp_1 (1 + F_1 b))$$

$$+ Hp_1 (1 + F_1 b)^2 (M - h)\} + b^2 \tau_1^2 \pi^4$$

$$\{(1 + M)(1 + M + H) + Hp_1 (1 + F_1 b)(1 + M)(M - h)$$

$$+ Hp_1 (1 + F_1 b)(1 + M) M + 2H^2 p_1 (1 + F_1 b)\}$$

$$+ b\tau_1 \pi^2 Hp_1 (1 + M)^2 (M - h)$$

$$+ T_1 \left\{ \tau_1^3 \pi^6 Hp_1 (M - h) + b \tau_1^4 \pi^8 ((Hp_1 - 1) + Hp_1 F_1 b) \right\},$$

$$A_1 = b^5 \tau_1^3 \pi^6 h (1 + F_1 b)^3 + b^4 \tau_1^2 \pi^4$$

$$\{2Mh(1+F_1b)^2 + H(1+F_1b)^2(1+M) + (1+F_1b)^2$$

$$+ Hp_1(1+H)(1+F_1b)^3\} + b^3\tau_1\pi^2\{Hp_1(1+F_1b)^2(M-h)$$

$$+ 2MH^2p_1(1+F_1b)^3 + 2Mh(M+1)(1+F_1b) + (1+F_1b)$$

$$(1+M)(M-h)\} + b^2H(1+M)^2\{Hp_1(1+F_1b)+1+M)\}$$

$$+ T_1\Big[\tau_1^2\pi^4\Big\{bMh + Hb(Hp_1 - 1 + Hp_1F_1b) + b(Hp_1 - 1 - M + Hp_1F_1b)\Big\}$$

$$+ b^2\tau_1^3\pi^6 h(1+F_1b) + \tau_1\pi^2 Hp_1(h(M+1)+MH)\Big],$$

$$A_0 = b^5\tau_1\pi^2h(1+F_1b)^3 + b^4H(1+F_1b)^2(1+M+Hp_1(1+F_1b))$$

$$+ T_1(\tau_1\pi^2b^2h(1+F_1b) + Hb(Hp_1 - 1 - M + Hp_1F_1b)).$$

Since σ_1 is real for overstability, the three values of $c_1\ (=\sigma_1^2)$ are positive. Eq. (46) implies that this is clearly impossible if

$$M > h \text{ and } Hp_1 > 1+M, \qquad \text{... (47)}$$

i.e. if

$$c_v > c_{pt},\ \left(1+\frac{mN_0}{\rho_0}\frac{c_{pt}}{c_v}\right)v > \left(1+\frac{mN_0}{\rho_0}\right)\kappa, \qquad \text{... (48)}$$

for then $A_0 - A_3$ are all positive and eq. (46) does not involve any change of sign. Thus, (48) are the conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

7. Effect of Magnetic Field Perturbation Equations and Dispersion Relation

Here instead of a uniform rotation, a uniform vertical magnetic field $\vec{H}$ $(0, 0, H)$ pervades the system and the couple-stress fluid is electrically conducting.

Here the linearized hydromagnetic perturbation equations are

$$\frac{\partial\vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\,\delta p + \left(v - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{q} + \vec{g}\frac{\delta\rho}{\rho_0} + \frac{\mu_e}{4\pi\rho_0}\times$$

$$(\nabla\times\vec{h})\times\vec{H} + \frac{KN_0}{\rho_0}(\vec{q_d} - \vec{q}), \qquad \text{... (49)}$$

together with (10)–(12) and

$$\frac{\partial \vec{q}}{\partial t} = (\vec{H} \cdot \nabla)\, \vec{q} + \eta \nabla^2 \vec{h}, \qquad \ldots (50)$$

$$\nabla \cdot \vec{h} = 0, \qquad \ldots (51)$$

where η stands for the electrical resistivity.

Writing the scalar components of (49) and eliminating $u, v, h_x h_y, \delta p$ between them by using (10) and (51), we obtain

$$\left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left[\frac{\partial}{\partial t}\nabla^2 w - g\alpha\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right) - \frac{\mu_e H}{4\pi \rho_0}\frac{\partial}{\partial z}\nabla^2 h_z\right]$$

$$+\frac{KN_0}{\rho_0}\frac{m}{K}\frac{\partial}{\partial t}\nabla^2 w = \left(\frac{m}{K}\frac{\partial}{\partial t}+1\right)\left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^4 w, \qquad \ldots (52)$$

together with (12) and z-component of (50) yields

$$\left[\frac{\partial}{\partial t} - \eta \nabla^2\right] h_z = H\frac{\partial w}{\partial z}. \qquad \ldots (53)$$

Analyzing the disturbances into normal modes, we assume that h_z is of the form

$$[h_z] = [K(z)]\ \exp\ (ik_x x + ik_y y + nt), \qquad \ldots (54)$$

then non-dimensional froms of (52) and (53) yield

$$\left(D^2 - a^2\right)\left[\sigma\left(1+\frac{M}{1+\tau_1 \sigma}\right) + F(D^2-a^2)^2 - (D^2-a^2)\right] W$$

$$= -\frac{g\alpha d^2 a^2 \Theta}{\nu} + \frac{\mu_e H d}{4\pi \rho_0 \nu}(D^2-a^2) DK \qquad \ldots (55)$$

$$\left(D^2 - a^2 - p_2 \sigma\right) K = -\left(\frac{Hd}{\eta}\right) DW. \qquad \ldots (56)$$

Following the same procedure as before, we have

$$\left(D^2-a^2\right)\left(D^2-a^2-Hp_1\sigma\right)\left(D^2-a^2-p_2\sigma\right)$$

$$\left[\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right) - \left(D^2-a^2\right) + F\left(D^2-a^2\right)^2\right] W$$

$$= Ra^2\left(\frac{H+\tau_1\sigma}{1+\tau_1\sigma}\right)\left(D^2-a^2-p_2\sigma\right)W - Q\left(D^2-a^2\right)\left(D^2-a^2-Hp_1\sigma\right)D^2W, \qquad \ldots (57)$$

where $R = \frac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number and $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta}$ is the Chandrasekhar number.

The additional boundary condition is

$$DK = 0 \text{ or } z = 0 \text{ and } 1. \qquad \ldots (58)$$

Following the same lines, we obtain the dispersion relation

$$R_1 = \frac{(1+x)(1+x+iHp_1\sigma_1)\left[(1+x+ip_2\sigma_1)\left(i\sigma_1\left(1+\frac{M}{1+i\tau_1\sigma_1\pi^2}\right)+(1+x)(1+F_1(1+x))\right)+Q_1\right]\times(1+i\tau_1\sigma_1\pi^2)}{x(1+x+ip_2\sigma_1)(H+i\tau_1\sigma_1\pi^2)} \qquad \ldots (59)$$

where

$$R_1 = \frac{R}{\pi^4},\ Q_1 = \frac{Q}{\pi^2},\ a^2 = \pi^2 x,\ i\sigma_1 = \frac{\sigma}{\pi^2},\ F_1 = \pi^2 F.$$

7.1. The Stationary Convection

Equation (58), for the stationary convection (i.e. $\sigma = 0$), reduces to

$$R_1 = \frac{(1+x)}{xH}\left[(1+x)^2 + F_1(1+x)^3 + Q_1\right]. \qquad \ldots (60)$$

It is evident from (60) that

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{xH},$$

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{xH},$$

$$\frac{dR_1}{dH} = -\frac{(1+x)}{xH^2}\left[(1+x)^2 + F_1(1+x)^3 + Q_1\right].$$

Thus the couple-stress and magnetic field have stabilizing effects whereas suspended particles have a destabilizing effect on the thermal instability of a couple-stress fluid with suspended particles, for stationary convection.

7.2 Stability of the System and Oscillatory Modes

Multiplying (55) by W^*, using (15), (56) and boundary conditions (37) and (58), we obtain

$$\sigma\left(1+\frac{M}{1+\tau_1\sigma}\right)I_1-\frac{g\alpha a^2\kappa}{\nu\beta}\frac{(1+\tau_1\sigma^*)}{(H+\tau_1\sigma^*)}(I_2+Hp_1\sigma^* I_3)$$

$$+\frac{\mu_e\eta}{4\pi\rho_0\nu}(I_4+\sigma^* p_2 I_5)+I_6+FI_7=0, \quad \text{... (61)}$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,$$

$$I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,$$

$$I_3=\int_0^1|\Theta|^2dz, \quad \text{... (62)}$$

$$I_4=\int_0^1\left(|D^2K|^2+2a^2|DK|^2+a^4|K|^2\right)dz,$$

$$I_5=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$

$$I_6=\int_0^1\left(|D^2W|^2+2a^2|DW|^2+a^4|W|^2\right)dz,$$

$$I_7=\int_0^1\left(|D^3W|^2+3a^2|D^2W|^2+3a^4|DW|^2+a^6|W|^2\right)dz,$$

and σ^* is complex conjugate of σ. The integrals are all positive definite. Putting $\sigma=i\sigma_i$ and equating imaginary parts of (61), we obtain

$$\sigma_i\left[\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)I_1+\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{\tau_1(H-1)}{H^2+\tau_1^2\sigma_i^2}I_2+\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2\sigma_i^2}Hp_1I_3\right)-\frac{\mu_e\eta}{4\pi\rho_0\nu}p_2I_5\right]=0.$$

(63)

In the absence of magnetic field, eq. (63) reduces

$$\sigma_i\left[\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)I_1++\frac{g\alpha a^2\kappa}{\nu\beta}\left(\frac{\tau_1(H-1)}{H^2+\tau_1^2\sigma_i^2}I_2+\frac{H+\tau_1^2\sigma_i^2}{H^2+\tau_1^2\sigma_i^2}Hp_1 I_3\right)\right]=0. \quad \dots (64)$$

and the terms in the brackets are positive definite. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed and principle of exchange of stabilities is satisfied in the absence of magnetic field. Thus the presence of magnetic field introduces oscillatory modes (as σ_i may not be zero) which were non-existent in its absence, for a couple-stress fluid layer with suspended particles, heated from below.

7.3. The Case of Overstability

Separating the real and imaginary parts of (59) and eliminating R_1 between them, we obtain

$$A_2 c_1^2 + A_1 c_1 + A_0 = 0, \quad \dots (65)$$

where we have put $c_1 = \sigma_1^2$, $b = 1 + x$ and

$$A_2 = b^2\tau_1^2 p_2^2\pi^4 + b\left\{\tau_1^2 p_2^2 Hp_1\pi^4 + Hp_1 p_2^2\tau_1\pi^2(M-h)\right\},$$

$$A_1 = b^3\left\{Hp_1\tau_1\pi^2(M-h) + \tau_1^2\pi^4(1+F_1 b)(Hp_1 - p_2) + Hp_1\tau_1^2\pi^4\right\}$$

$$+ b^4\tau_1^2\pi^4 + b^2\left\{Hp_1 p_2\tau_1\pi^2(1+F_1 b)h + p_2^2\tau_1\pi^2 h + Hp_2^2(1+M)\right\}$$

$$+ bH^2 p_1 p_2^2 + Q_1\left\{bHp_1 p_2\tau_1\pi^2 h + b^2\tau_1^2\pi^4(Hp_1 - p_2)\right\}$$

$$A_0 = b^4\left\{\tau_1\pi^2 h(2+F_1 b) + H(1+M)\right\} + b^3\left\{H^2 p_2 + H(1+F_1 b)(Hp_1 - p_2)\right\}$$

$$+ Q_1\left\{b^2 H(Hp_1 - p_2) + Q_1 b^3\tau_1\pi^2 h\right\}.$$

Since σ_1 is real for overstability, the two values of $c_1\ (= \sigma_1^2)$ are positive. Equation (65) implies that this is clearly impossible if

$$M > h \text{ and } Hp_1 > p_2, \quad \dots (66)$$

i.e.,

$$c_v > c_{pt} \text{ and } H\eta > \kappa, \quad \dots (67)$$

for then $A_0 - A_2$ are all positive definite and eq. (65) does not involve any change of sign. Thus $c_v > c_{pt}$ and $H\eta > \kappa$ are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply occurrence of overstability.

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