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OBLIQUE WAVE SCATTERING BY A SEMI-INFINITE RIGID DOCK IN THE PRESENCE OF BOTTOM UNDULATIONS

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The problem of oblique wave scattering by a semi-infinite rigid dock in the presence of varying bottom topography is investigated here using linear water wave theory. Employing a simplified perturbation analysis together with appropriate use of Green's integral theorem, the reflection coefficient up to first order is obtained in terms of an integral involving the shape function representing the bottom topography. The zero-order reflection coefficient is obtained by using the residue calculus method of complex variable. The bottom undulations are described by sinusoidal and an exponentially decaying profile. The first order correction to the reflection coefficient is depicted graphically in a number of figures for the two shape functions characterizing the bottom undulations and appropriate conclusions are drawn.

Key words : Rigid dock; oblique water wave scattering; reflection coefficient.

1. Introduction

The problems of free surface flow over an obstacle or a geometrical disturbance at the bottom of an ocean are important for their possible applications in the areas of coastal and marine engineering, and as such these are being studied by scientists and engineers for a long time. The problem of reflection of surface waves by patches of bottom undulations has received an increasing amount of attention as its mechanism is important in the development of shore parallel bars. These problems are, in general, somewhat difficult to solve analytically although there exist various approximate mathematical techniques by which quantities of physical interests, namely the reflection and the transmission coefficients, can be estimated numerically. There exists only one explicit solution for the two-dimensional problem of wave propagation over a particular bottom topography considered by Roseau [11]. He used a mapping function to transform the fluid region with smoothly varying bottom undulations into a strip of constant width. The solution obtained by Roseau [11] provides an explicit representation of velocity potential produced due to incidence of a time harmonic plane wave upon a smoothly varying ocean floor. For general bed forms, a variety of approximate numerical methods have been devised in the literature.

There exists a class of mostly naturally occurring bottom standing obstacles such as sand ripples, which can be assumed to be small in some sense, for which some sort of perturbation technique can be employed in obtaining the first order corrections to the reflection and transmission coefficients. Heins [3] used the Wiener-Hopf technique to solve the interaction of oblique waves (generated by a line source along the edge of the plate) with a semi-infinite dock in water of finite depth. The reflection and transmission coefficients up to the first order were approximately obtained by Miles [9] in terms of integrals involving a small cylindrical deformation of the bottom by using small perturbation theory when the wave train was obliquely incident. In a major work, Davies [1] investigated the reflection of normally incident surface waves by a patch of sinusoidal undulations on the sea bed in a finite region. Mandal and Basu [5] generalized the problem of Miles [9] to include the effect of surface tension at the free surface. Martha and

Bora [8] solved the problem of oblique water-wave diffraction by small undulation on the sea-bed obtaining detailed results for a patch of sinusoidal ripples, as a specific example. Another class of water wave scattering problems involving a discontinuity in surface boundary condition was considered by Peters [10], Weitz and Keller [12] in connection with the study of propagation of surface waves at an inertial surface composed of a thin but uniform distribution of non-interacting floating materials, e.g. broken ice, floating mat, etc. These problems were solved by using Wiener-Hopf technique. Evans and Linton [2] used the residue calculus techniques to solve the problem considered by Weitz and Keller [12]. Recently Mandal and De [6] considered water wave scattering by bottom undulations in the presence of a discontinuity in the surface boundary condition.

In this paper the problem of wave scattering by a semi-infinite rigid dock over a channel with bottom undulations is considered. This type of situation arises near the mouth of a wide river where the depth of the bottom becomes variable due to deposition of slits. The problem is formulated for the case of an obliquely incident train of surface waves. The dock is located on the upper surface of the channel ($x > 0$) and a train of progressive waves propagating from negative infinity is obliquely incident on the bottom having small undulations. A simplified perturbation method is applied directly to the governing partial differential equation, the boundary and infinity conditions satisfied by the potential function describing the fluid motion. Use of perturbation analysis procedure produces two boundary value problems (BVPs) for the potential functions upto first order. The BVP (BVP-I) for the zero-order potential function is concerned with the problem of scattering of obliquely incident surface water waves by a rigid dock in water of uniform finite depth. This problem has been solved explicitly by employing the residue calculus method of complex variable theory and the zero-order reflection coefficient is determined explicitly. The BVP (BVP-II) is a radiation problem in water of uniform finite depth. Without solving BVP-II, the first-order correction to the reflection coefficient is evaluated by simple application of Green's integral theorem. Analytical expression for the first-order correction R_1 to the reflection coefficient R is obtained in terms of integrals involving the shape function describing the bottom topography and the solution of BVP-I. $|R_1|$ is depicted graphically against the wave number for two different shape functions.

2. Statement and Formulation

We assume that the fluid under our consideration is incompressible, inviscid and the motion is two dimensional and irrotational. We choose a rectangular cartesian co-ordinate system in which y-axis is taken vertically downwards and x-axis is along the mean free surface of the ocean. The position of the rigid dock is given by $y = 0,\ x > 0$. The bottom of the ocean with small undulations is described by $y = h + \varepsilon\ c(x)$. Here ε is a non-dimensional small positive number which gives a measure of smallness of the bottom undulations. The function $c(x)$ is a continuous bounded function which has a compact support so that far away from the undulations, the bottom of ocean is of uniform finite depth h below the mean free surface. We consider a train of progressive surface wave represented by $Re\{\phi(x,y)e^{i\nu z - i\sigma t}\}$, is obliquely incident upon the dock from negative infinity making an angle θ with the mean free surface of ocean. Then $\phi(x,y)$ satisfies the following boundary value problem (BVP) (cf [7]):

$$(\nabla^2 - \nu^2)\phi = 0 \quad \text{in the fluid region.} \tag{2.1}$$

The free surface condition is given by

$$K\phi + \phi_y = 0 \quad \text{on } y = 0,\ \ x < 0, \tag{2.2}$$

where $K = \sigma^2/g$ and σ is the angular frequency of the incoming wave train and g being the acceleration due to gravity.

The conditions of no motion of the rigid dock and bottom of ocean respectively are given by

$$\frac{\partial \phi}{\partial y} = 0 \quad \text{on } y = 0,\ \ x > 0, \tag{2.3}$$

and

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on } y = h + \varepsilon\ c(x), \tag{2.4}$$

$\frac{\partial}{\partial n}$ being the normal derivative.

The condition of boundedness of the velocity at the edge of the dock is given by

$$r\frac{\partial\phi}{\partial r} = 0 \quad as \;\; r = (x^2+y^2)^{1/2} \to 0. \tag{2.5}$$

The condition at infinity is given by

$$\phi(x,y) = \begin{cases} (e^{i\mu x} + Re^{-i\mu x})\psi_0(y) & \text{as } x \to -\infty, \\ 0 & \text{as } x \to \infty \end{cases} \tag{2.6}$$

Here R is the reflection coefficient, i.e., the amplitude of the waves reflected by the dock and the bottom undulations. Also,

$$\left.\begin{array}{l} \nu = k_0 \sin\theta, \;\; \mu = k_0\cos\theta, \\ \psi_0(y) = N_0 \cosh k_0(y-h) \\ \text{with} \\ N_0 = \frac{2(k_0)^{\frac{1}{2}}}{(2k_0h + \sinh 2k_0h)^{\frac{1}{2}}} \end{array}\right\} \tag{2.7}$$

and k_0 is the unique real positive root of the dispersion relation

$$k\tanh kh = K. \tag{2.8}$$

The bottom condition (2.4) can be approximated up to the first order of ε as

$$-\frac{\partial\phi}{\partial y} + \varepsilon\left[c'(x)\frac{\partial\phi}{\partial x} - c(x)\frac{\partial^2\phi}{\partial y^2}\right] = 0 \quad \text{on} \;\; y = h.$$

This suggests that a perturbation technique can be employed to solve the BVP described by (2.1)-(2.6) approximately. This is described in the next section.

3. Method of Solution

The approximate boundary condition (2.8) suggests that ϕ and R can be expanded in terms of the perturbation parameter ε as

$$\begin{aligned} \phi(x,y;\varepsilon) &= \phi_0(x,y) + \varepsilon\phi_1(x,y) + O(\varepsilon^2), \\ R(\varepsilon) &= R_0 + \varepsilon R_1 + O(\varepsilon^2). \end{aligned} \tag{3.1}$$

Substituting the expansions (3.1) in (2.1)-(2.3), (2.5), (2.6) and (2.8) we find after equating the coefficients of ε^0 and ε^1 from both sides, that the functions $\phi_0(x,y)$ and $\phi_1(x,y)$ satisfy the following BVPs:

BVP-I: The function $\phi_0(x,y)$ satisfies

$$
\begin{aligned}
(\nabla^2-\nu^2)\phi_0(x,y) &= 0 \ \text{ in } \ 0<y<h,\ -\infty<x<\infty,\\
K\phi_0(x,y)+\phi_{0y}(x,y) &= 0 \ \text{ on } \ y=0, x<0,\\
\phi_{0y}(x,y) &= 0 \ \text{ on } \ y=0, x>0,\\
r\frac{\partial\phi_0}{\partial r} &= 0 \ \text{ as } \ r=(x^2+y^2)^{1/2}\to 0,\\
\phi_{0y}(x,y) &= 0 \ \text{ on } \ y=h,\\
\phi_0(x,y) &= \begin{cases} (e^{i\mu x}+R_0e^{-i\mu x})\psi_0(y) & as\ \ x\to-\infty,\\ 0 & as\ \ x\to\infty. \end{cases}
\end{aligned} \tag{3.2}
$$

BVP-II: The function $\phi_1(x,y)$ satisfies

$$
\begin{aligned}
(\nabla^2-\nu^2)\phi_1(x,y) &= 0 \ \text{ in } \ 0<y<h,\ -\infty<x<\infty,\\
K\phi_1(x,y)+\phi_{1y}(x,y) &= 0 \ \text{ on } \ y=0, x<0,\\
\phi_{1y}(x,y) &= 0 \ \text{ on } \ y=0, x>0,\\
\phi_{1y} &= \frac{d}{dx}\left(c(x)\frac{\partial\phi_0(x,h)}{\partial x}\right)-\nu^2c(x)\phi_0(x,h) \ \text{ on } \ y=h,\\
r\frac{\partial\phi_1}{\partial r} &= 0 \ \text{ as } \ r=(x^2+y^2)^{1/2}\to 0,\\
\phi_1(x,y) &= \begin{cases} (R_1e^{-i\mu x})\psi_0(y) & \text{as } \ x\to-\infty,\\ 0 & \text{as } \ x\to\infty. \end{cases}
\end{aligned} \tag{3.3}
$$

BVP-I corresponds to the problem of water wave scattering by a rigid dock present in the surface of ocean with uniform constant depth h. Here the motion is described by zeroth order velocity potential $\phi_0(x,y)$ and the zeroth order reflection coefficient R_0. The BVP-II is a radiation problem in water of uniform finite depth h, in which, the bottom condition involves ϕ_0, the solution of BVP-I. Without solving $\phi_1(x,y)$ explicitly, the first order correction R_1 can be determined in terms of integrals involving the shape function $c(x)$ and $\phi_0(x,h)$. To show this, we apply Green's integral theorem to the functions $\phi_0(x,y)$ and $\phi_1(x,y)$ in the regions bounded by the lines $y=0,\ -X\leq x\leq X;\ x=\pm X,\ 0\leq y\leq h;\ y=h,\ -X\leq x\leq X$ where X is large and positive, and ultimately make X to tend to infinity. This produces

$$2i\mu R_1=\int_{-\infty}^{\infty}c(x)[\phi_{0x}^2(x,h)+\nu^2\phi_0^2(x,h)]dx. \tag{3.4}$$

Therefore, R_1 is derived in terms of integrals involving the shape function $c(x)$ and the zero-order potential function $\phi_0(x,h)$. We now briefly describe the method of solution of BVP-I.

Now, $\phi_0(x,y)$ can be expressed as

$$\phi_0(x,y)=\begin{cases}(e^{i\mu x}+R_0e^{-i\mu x})\psi_0(y)+\sum_{n=1}^{\infty}A_ne^{(k_n^2+\nu^2)^{1/2}x}\psi_n(y) & x<0,\\ \sum_{n=0}^{\infty}\frac{\epsilon_n}{2}B_ne^{-(\frac{n^2\pi^2}{h^2}+\nu^2)^{1/2}x}\cos(\frac{n\pi}{h})y & x>0,\end{cases} \tag{3.5}$$

where
$\epsilon_0=1$ and $\epsilon_n=2$ for $n\geq 1$.

Here, $\pm ik_n$ $(n=1,2,3,.....)$ are purely imaginary roots of the transcendental equation $k\tanh kh=K$ and

$$\psi_n(y)=N_n\cos k_n(y-h) \tag{3.6}$$

with

$$N_n=\frac{2(k_n)^{\frac{1}{2}}}{(2k_nh+\sin 2k_nh)^{\frac{1}{2}}}\ ,n\geq 1.$$

$A_n(n=1,2,3,.....)$ and $B_n(n=0,1,2,3,.....)$ are unknown constants to be determined along with the unknown reflection coefficient R_0.

We define

$$\begin{aligned}\alpha_0 &= -i\mu=-i(k_0^2-\nu^2)^{\frac{1}{2}}\ ,\\ \alpha_n &= (k_n^2+\nu^2)^{\frac{1}{2}}\ ,\ n\geq 1\ ,\\ \gamma_n &= \left(\left(\frac{n\pi}{h}\right)^2+\nu^2\right)^{\frac{1}{2}}.\end{aligned}$$

We now match ϕ_0 and $\frac{\partial \phi_0}{\partial x}$ at $x = 0$ and using the orthogonal properties of eigenfunction $\psi_0(y)$ in $(0, h)$ we obtain

$$(1 + R_0) = N_0 i k_0 \sin(i k_0 h) \sum_{n=0}^{\infty} \frac{\epsilon_n}{2} \frac{B_n}{-\left(k_0^2 + \left(\frac{n\pi}{h}\right)^2\right)}$$

and

$$\alpha_0(R_0 - 1) = N_0 i k_0 \sin(i k_0 h) \sum_{n=0}^{\infty} \frac{\epsilon_n}{2} \frac{B_n \gamma_n}{k_0^2 + \left(\frac{n\pi}{h}\right)^2}.$$

Using the above two equations we get

$$\sum_{n=0}^{\infty} \frac{\epsilon_n}{2} \frac{B_n}{\gamma_n + \alpha_0} = \frac{-2\alpha_0 R_0}{N_0 K \cosh k_0 h}, \tag{3.7}$$

$$\sum_{n=0}^{\infty} \frac{\epsilon_n}{2} \frac{B_n}{\gamma_n - \alpha_0} = \frac{2\alpha_0}{N_0 K \cosh k_0 h}. \tag{3.8}$$

Now, using the orthogonal property of eigenfunctions $\psi_m(y)$ $(m = 1, 2...)$ and eliminating the constants A_m $(m = 1, 2..)$ we obtain,

$$\sum_{n=0}^{\infty} \frac{\epsilon_n}{2} \frac{B_n}{\gamma_n - \alpha_m} = 0, \quad m = 1, 2, ... \tag{3.9}$$

Combining equations (3.8) and (3.9) we get

$$\sum_{n=0}^{\infty} \frac{\epsilon_n}{2} \frac{B_n}{\gamma_n - \alpha_m} = A\delta_{m0}, \quad m = 0, 1, 2, ... \tag{3.10}$$

where,

$$A = \frac{2\alpha_0}{N_0 K \cosh k_0 h}. \tag{3.11}$$

Again, matching ϕ_0 and $\frac{\partial \phi_0}{\partial x}$ at $x = 0$ and using the orthogonal properties of eigenfunctions $\cos \frac{n\pi}{h} y \ \ (n = 0, 1, 2...)$ in $(0, h)$ we obtain

$$2c_{0m} + \sum_{n=0}^{\infty} A_n c_{nm} = \frac{h}{2} B_m, \qquad m \geq 0. \tag{3.12}$$

$$\sum_{n=0}^{\infty} A_n \alpha_n c_{nm} = -\frac{h}{2} \gamma_m B_m, \qquad m \geq 0. \tag{3.13}$$

where,

$$A_0 = R_0 - 1$$

and

$$\begin{aligned} c_{nm} &= \int_0^h \psi_n(y) \cos \frac{m\pi}{h} y \, dy \\ &= \frac{N_n x_n \sin x_n h}{x_n^2 - \left(\frac{m\pi}{h}\right)^2} \end{aligned}$$

with $x_0 = -ik_0$ and $x_n = k_n$ for $n \geq 1$.

Elimination of the constants $B_m \ \ (m = 0, 1, 2..)$ from the equations (3.12) and (3.13) leads to an infinite system of equations

$$-2c_{0m}\gamma_m = \sum_{n=0}^{\infty} A_n(\alpha_n + \gamma_m) c_{nm}, \qquad m \geq 0. \tag{3.14}$$

Using the above equation the unknown constants $A_n \ \ (n = 0, 1, 2..)$ can be obtained numerically. Once $A_n's$ are known, $B_m \ \ (m = 0, 1, 2..)$ are obtained numerically using (3.12).

The zero-order reflection coefficient R_0 can be obtained by using the residue calculus as given in Linton and McIver [4]. For the sake of completeness we describe the method briefly. We consider the integral

$$\frac{1}{2\pi i} \int_{C_N} \frac{f(z)}{z - \alpha_m} dz, \tag{3.15}$$

where $f(z)$ has simple poles at $z = \gamma_n$ $(n = 0, 1, 2, ...)$ and simple zeros at $z = \alpha_n$ $(n = 1, 2, ...)$ and $f(z) = O(|z|^{-1})$ as $z \to \infty$ on C_N, C_N being a sequence of circles whose radius R_N increases without bound as $N \to \infty$ whilst avoiding the zeros of the integrand. The conditions on $f(z)$ are sufficient to ensure that the integral tends to zero as $N \to \infty$. If we further assume that $f(\alpha_0) = -1$, we obtain

$$\sum_{n=0}^{\infty} \frac{Res[f(z); \gamma_n]}{\gamma_n - \alpha_m} = \delta_{m0}, \ (m = 0, 1, 2, ...). \tag{3.16}$$

By comparing (3.10) and (3.16) we find

$$\frac{\epsilon_n}{2} B_n = ARes[f(z); \gamma_n]. \tag{3.17}$$

An appropriate form of the function $f(z)$ is given by

$$f(z) = \frac{\gamma_0 - \alpha_0}{z - \gamma_0} \prod_{n=1}^{\infty} \frac{(1 - \frac{z}{\alpha_n})(1 - \frac{\alpha_0}{\gamma_n})}{(1 - \frac{\alpha_0}{\alpha_n})(1 - \frac{z}{\gamma_n})}. \tag{3.18}$$

We consider again another integral

$$\frac{1}{2\pi i} \int_{C_N} \frac{f(z)}{z + \alpha_m} dz, \tag{3.19}$$

where $f(z)$ is the same as above. This integral also tends to zero as $N \to \infty$. Thus, for $m = 0$, this produces

$$\sum_{n=0}^{\infty} \frac{\epsilon_n}{2} \frac{B_n}{\gamma_n + \alpha_0} = -Af(-\alpha_0). \tag{3.20}$$

Comparison with (3.7) then shows that

$$\begin{aligned} R_0 &= f(-\alpha_0), \\ &= \frac{\alpha_0 - \gamma_0}{\alpha_0 + \gamma_0} \prod_{n=1}^{\infty} \frac{(1 + \frac{\alpha_0}{\alpha_n})(1 - \frac{\alpha_0}{\gamma_n})}{(1 - \frac{\alpha_0}{\alpha_n})(1 + \frac{\alpha_0}{\gamma_n})}, \\ &= e^{-2i\theta} e^{2i\Theta}, \\ &= e^{2i(\Theta - \theta)}, \end{aligned} \tag{3.21}$$

where

$$\Theta = \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{\mu}{\gamma_n}\right) - \tan^{-1}\left(\frac{\mu}{\alpha_n}\right). \tag{3.22}$$

It may be noted from equation (3.21), $|R_0| = 1$, which is expected. Since $A_n \ \ (n = 0, 1, 2..)$, $B_n \ \ (n = 0, 1, 2..)$ and R_0 are all known, thus the function $\phi_0(x, y)$ is obtained in principle.

Hence, R_1 can be computed numerically from the equation (3.4) once the shape function $c(x)$ is known. First we consider $c(x)$ to represent a sinusoidal variation for which $c(x)$ is chosen in the form

$$c(x) = \begin{cases} c_0 \sin\lambda(x-a) & a - \frac{m\pi}{\lambda} \le x \le a + \frac{m\pi}{\lambda}, \\ 0 & \text{otherwise}, \end{cases} \tag{3.23}$$

where c_0 is the ripple amplitude and m is a positive integer denoting the number of sinusoidal ripples at the bottom with wave number λ. In this case, with the assumption that $|a| < \frac{m\pi}{\lambda}$, we obtain the first-order reflection coefficient as

$$\begin{aligned} R_1 = \frac{c_0}{2i\mu}\Bigg[&(N_0^2(\nu^2-\mu^2))\Bigg(\frac{1}{\lambda^2-4\mu^2}[-2i\mu\sin\lambda a-\lambda\cos\lambda a+\lambda(-1)^m e^{2i\mu(a-\frac{m\pi}{\lambda})}] \\ &-\frac{R_0^2}{\lambda^2-4\mu^2}[-2i\mu\sin\lambda a+\lambda\cos\lambda a-\lambda(-1)^m e^{-2i\mu(a-\frac{m\pi}{\lambda})}]\Bigg) \\ &+\frac{2R_0N_0^2(\mu^2+\nu^2)}{\lambda}[\cos\lambda a-(-1)^m] \\ &+\nu^2\int_{a-\frac{m\pi}{\lambda}}^{0}\left(\sum_{n=1}^{\infty}A_n e^{\alpha_n x}N_n\right)^2\sin\lambda(x-a)\,dx \\ &+\int_{a-\frac{m\pi}{\lambda}}^{0}\left(\sum_{n=1}^{\infty}A_n\alpha_n e^{\alpha_n x}N_n\right)^2\sin\lambda(x-a)\,dx \\ -2N_0&\sum_{n=1}^{\infty}\frac{A_n(\nu^2+i\mu\alpha_n)N_n}{\lambda^2+(\alpha_n+i\mu)^2}\left[(\alpha_n+i\mu)\sin\lambda a+\lambda\cos\lambda a-\lambda(-1)^m e^{(\alpha_n+i\mu)(a-\frac{m\pi}{\lambda})}\right] \\ -2N_0R_0&\sum_{n=1}^{\infty}\frac{A_n(\nu^2-i\mu\alpha_n)N_n}{\lambda^2+(\alpha_n-i\mu)^2}\left[(\alpha_n-i\mu)\sin\lambda a+\lambda\cos\lambda a-\lambda(-1)^m e^{(\alpha_n-i\mu)(a-\frac{m\pi}{\lambda})}\right] \end{aligned}$$

$$+\int_0^{a+\frac{m\pi}{\lambda}} \sin\lambda(x-a)\left(\sum_{n=0}^{\infty}\frac{\epsilon_n}{2}B_n(-1)^n(-\gamma_n)e^{-\gamma_n x}\right)^2 dx$$

$$+\nu^2\int_0^{a+\frac{m\pi}{\lambda}} \sin\lambda(x-a)\left(\sum_{n=0}^{\infty}\frac{\epsilon_n}{2}B_n(-1)^n e^{-\gamma_n x}\right)^2 dx\Bigg]. \tag{3.24}$$

Here, it may be noted that, $|R_1|$ becomes very large and produces resonance when $\lambda = 2\mu$, i.e., when the wavelength of incident field is twice the wavelength of the ripples of sinusoidal bottom. This type of behaviour of sinusoidal bottom topography was also observed in [1], [5] and [8].

The second shape function is taken as

$$c(x) = b_0 e^{-\xi|x-b|} \quad (\xi > 0), \quad -\infty < x < \infty. \tag{3.25}$$

This corresponds to an exponentially damped undulation. The expression for the first-order correction to the reflection coefficient in this case is given by

$$\begin{aligned}
R_1 \;=\; & \frac{b_0}{2i\mu}\Bigg[e^{-\xi b}\Bigg(N_0^2(\nu^2-\mu^2)\left(\frac{1}{\xi+2i\mu}+\frac{R_0^2}{\xi-2i\mu}\right)+\frac{2R_0N_0^2(\mu^2+\nu^2)}{\xi} \\
& +\int_{-\infty}^{0}\left(\sum_{n=1}^{\infty}A_n\alpha_n e^{\alpha_n x}N_n\right)^2 e^{\xi x}dx+\nu^2\int_{-\infty}^{0}\left(\sum_{n=1}^{\infty}A_n e^{\alpha_n x}N_n\right)^2 e^{\xi x}dx \\
& +2N_0\sum_{n=1}^{\infty}\frac{A_n(\nu^2+i\mu\alpha_n)N_n}{\xi+(\alpha_n+i\mu)}+2N_0R_0\sum_{n=1}^{\infty}\frac{A_n(\nu^2-i\mu\alpha_n)N_n}{\xi+(\alpha_n-i\mu)}\Bigg) \\
& +\int_0^b e^{-\xi(b-x)}\left(\sum_{n=0}^{\infty}\frac{\epsilon_n}{2}B_n(-1)^n(-\gamma_n)e^{-\gamma_n x}\right)^2 dx+\nu^2\int_0^b e^{-\xi(b-x)} \\
& \left(\sum_{n=0}^{\infty}\frac{\epsilon_n}{2}B_n(-1)^n e^{-\gamma_n x}\right)^2 dx+\int_b^{\infty} e^{-\xi(x-b)} \\
& \left(\sum_{n=0}^{\infty}\frac{\epsilon_n}{2}B_n(-1)^n(-\gamma_n)e^{-\gamma_n x}\right)^2 dx \\
& +\nu^2\int_b^{\infty} e^{-\xi(x-b)}\left(\sum_{n=0}^{\infty}\frac{\epsilon_n}{2}B_n(-1)^n e^{-\gamma_n x}\right)^2 dx\Bigg].
\end{aligned} \tag{3.26}$$

Unlike the sinusoidal bottom topography,in this case $|R_1|$ does not exhibit any resonating effect.

4. Numerical Results

We have computed $|R_1|$ for different values of wave number Kh, for two types of shape functions $c(x)$ characterizing the unevenness of the bottom as mentioned earlier. The actual reflection coefficient R as given in equation (2.6) is $|R_1|$ = $|R_0 + \varepsilon R_1|$ upto first order where $|R_0 = 1$.

For numerical computation the value of the non-dimensional parameter $\frac{c_0}{h}$ is taken as 0.01 and λh as 1 for the figures (1) to (4) which depicts $|R_1|$ against Kh for $c(x)$ given by (3.23).

In Figs. 1(a) and 1(b), $|R_1|$ is plotted against Kh for $\frac{a}{h}$=3 with number of ripples m=2,3,5 and 7 and θ=$\frac{\pi}{9}$, and $\frac{\pi}{3}$ respectively. In both the figures it is observed that $|R_1|$ is oscillatory in nature and when the number of ripples increases, the general feature of $|R_1|$ remains the same but with the observation that the overall value of $|R_1|$ increases and the oscillatory nature of $|R_1|$ against Kh is more noticeable with the number of zeros of $|R_1|$ increasing. As the angle of incidence increases from $\frac{\pi}{9}$ to $\frac{\pi}{3}$, it is seen that the oscillatory nature of $|R_1|$ decreases and the peak value increases. In Fig. 1(c), θ has been taken as 89.994^o (i.e. almost grazing incidence) with all other parameters fixed.We expect that $|R_1|$ should be very small which is indeed the case as is evident from the numerical computation. It is observed $|R_1|$ increases (but remains small), reaches a peak and then decreases to zero. The peak value increases as m increases from 2 to 7.

For Figs. 2(a), 2(b) and 2(c), $\frac{a}{h}$ has been chosen to be zero, that is, the shape function is symmetric about $x = 0$, keeping other parameters fixed. Similar nature of $|R_1|$ is observed as seen in the above figures. Fig. 2(c) again shows that the first-order reflection coefficient vanishes when the angle of incidence is taken very close to $\frac{\pi}{2}$. The value of $|R_1|$ is greater when $\frac{a}{h}$=0 as compared to when $\frac{a}{h}$=3. The oscillatory nature of $|R_1|$ in all the figures may be attributed due to multiple interactions of the incident wave train with sinusoidal bottom and the edge of the dock.

Occurrence of zeros of $|R_1|$ for certain values of Kh implies that the sinusoidal bottom does not affect the incident waves at first order for certain frequencies. Also, a Bragg resonance interaction is observed to occur in all the cases.

Now, for the second shape function given by (3.25) which represents an exponentially decaying bottom topography. $|R_1|$ is depicted against Kh for two different values of ξh (Figs. 3-4) when $\frac{b_0}{h}$=0.01. It is seen that for for each value of ξh, $|R_1|$ first increases with Kh, attains a maximum and then decreases as Kh is further increased. For all the figures it is observed that the peak value of $|R_1|$ decreases as ξh increases.

The figures 3(a) and 3(b) shows the variation of $|R_1|$ against Kh for θ=$\frac{\pi}{9}$ and $\frac{\pi}{3}$ respectively with $\frac{b}{h}$=3. It is observed that $|R_1|$ increases with the increase of the incident angle. Similar behaviour is observed in the figures 4(a) and 4(b) where $\frac{b}{h}$ is taken as zero, that is, symmetric about $x = 0$ and keeping all the other parameters fixed. Also, the value of $|R_1|$ is much greater when $\frac{b}{h}$=0 compared to when $\frac{b}{h}$=3.

For the figures 3(c) and 4(c), θ has been chosen as 89.994^o with $\frac{b}{h}$=3 and $\frac{b}{h}$=0 respectively. The peak values of $|R_1|$ is observed to decrease with the increase of $\frac{b}{h}$ and $|R_1|$ becomes negligible when θ is chosen close to $\frac{\pi}{2}$.

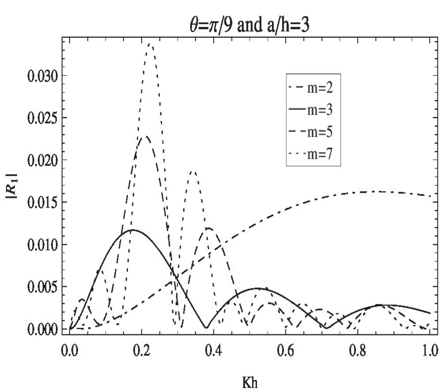

Fig. 1(a): First order reflection coefficient

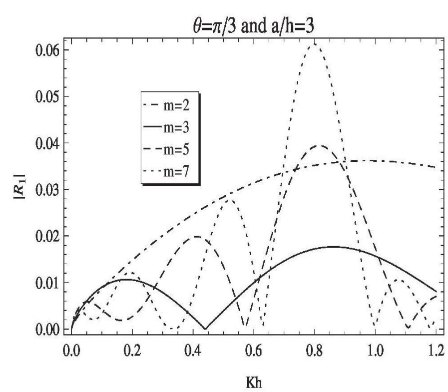

Fig. 1(b): First order reflection coefficient

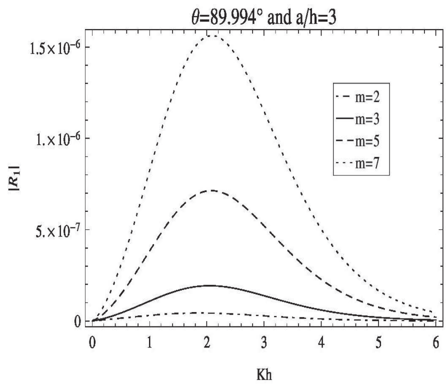

Fig. 1(c): First order reflection coefficient

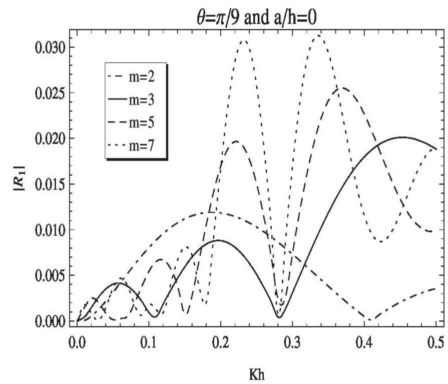

Fig. 2(a): First order reflection coefficient

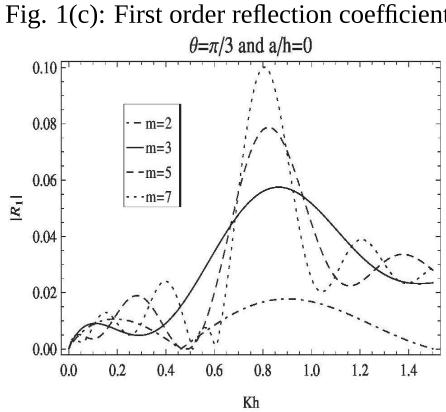

Fig. 2(b): First order reflection coefficient

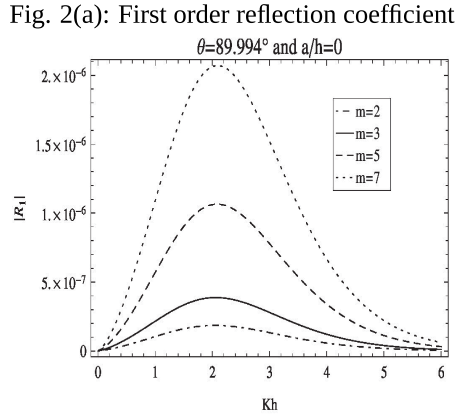

Fig. 2(c): First order reflection coefficient

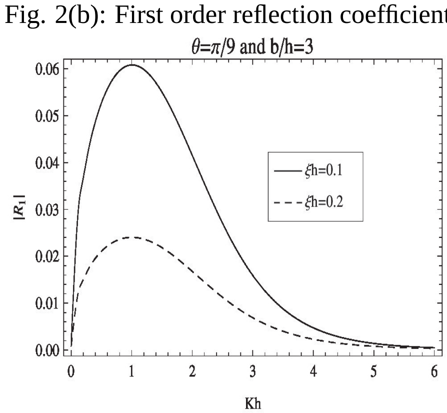

Fig. 3(a): First order reflection coefficient

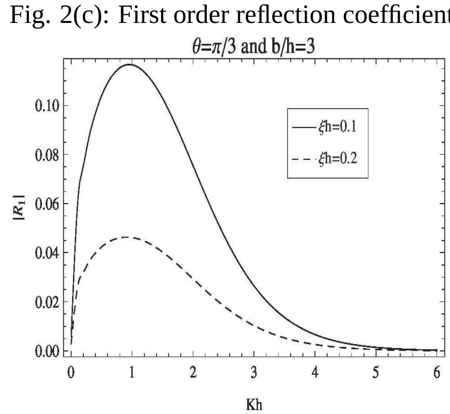

Fig. 3(b): First order reflection coefficient

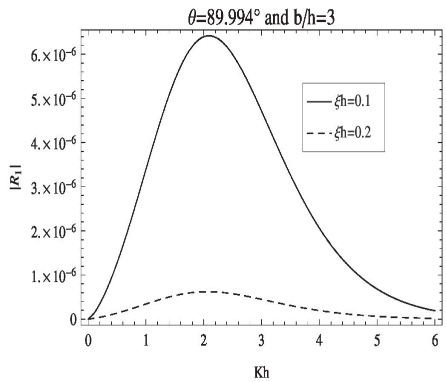

Fig. 3(c): First order reflection coefficient

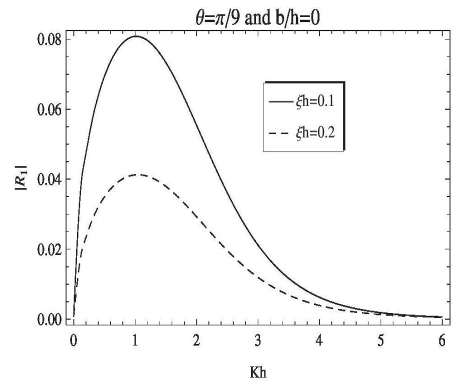

Fig. 4(a): First order reflection coefficient

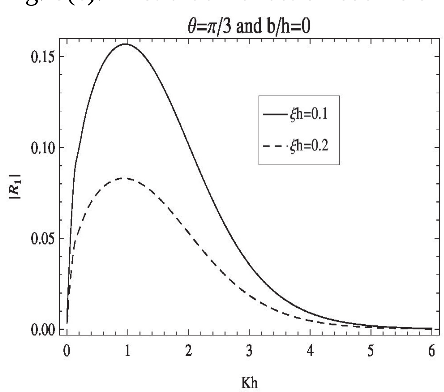

Fig. 4(b): First order reflection coefficient

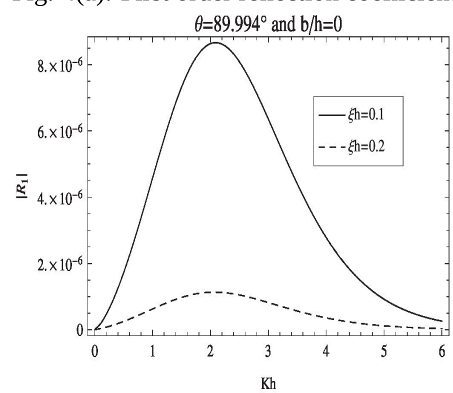

Fig. 4(c): First order reflection coefficient

5. Conclusion

A simplified perturbation analysis together with appropriate use of Green's integral theorem is employed to obtain first-order correction to the reflection coefficient. The zero-order reflection coefficient is obtained by using the residue calculus method of complex variable theory. The bottom topography is described by sinusoidal curve and an exponentially decaying curve. For the case of a patch of sinusoidal ripple bed with wave number equal to twice the component of the wave number of the incident wave field along the x-axis, the first-order reflection coefficient is found to increase with the number of undulations which suggests that comparatively large reflection of the incident wave energy is possible by making the number of undulations somewhat larger. For the case of an exponentially decaying form, the peak values of $|R_1|$ are seen to decrease with the increase of ξh.

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