EFFECTS OF CHEMICAL REACTION AND RADIATION ON AN UNSTEADY MHD FLOW PAST AN ACCELERATED INFINITE VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS TRANSFER

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This paper deals with the study of the effects of first order chemical reaction and radiation on an unsteady MHD flow of an incompressible viscous electrically conducting fluid past an accelerated infinite vertical plate with variable temperature and mass transfer. The resulting approximate dimensionless system of governing partial differential equations are integrated in closed form by the Laplace transform technique. A uniform magnetic field is assumed to be applied transversely to the direction of the flow. Rosseland model of radiation has been chosen in the investigation, the expressions for the velocity field, temperature field and concentration field and skin-friction in the direction of the flow, coefficient of heat transfer and mass flux at the plate have been obtained in non-dimensional form and these are illustrated graphically for various physical parameters involved in the study. Investigation reveals that the fluid velocity is decelerated in the region adjacent to the plate, due to the effect of first order chemical reaction and the rate of
heat transfer (from plate to the fluid) decreases due to the absorption of thermal radiation. The results obtained in this work are consistent with physical situation of the problem.

**Key words**: Heat transfer; mass transfer; MHD; first order chemical reaction; radiation; infinite plate.

1. **Introduction**

The study of the problems of the MHD free convection flow with heat and mass transfer have attracted the attention of a number of scholars due to the importance of such problems in many branches of science and technology. Magneto hydrodynamics is the study of the interaction of magnetic fields and electrically conducting fluids in motion. This subject is also known as MHD (in short). The idea of MHD is that magnetic fields can induce currents in a moving conductive (i.e. electrically conducting) fluid, which create forces on the fluid, and also modifies the magnetic field itself. The higher the electrical conductivity of the liquid or gas, the greater will be the MHD effect. While framing the MHD equations for the flow of charged particles, the entire assemblage may be treated as a single fluid which can carry an electric current. By single fluid, it is implied that there is only one density (the mass density) and temperature, and there is also only one velocity (i.e. there is no need to treat the electrons and/or ions separately). From technological point of view, MHD convection flow problems are also very significant in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. For natural convection, the existence of the large temperature difference between the surface and ambient causes the radiation effect to be important. At high temperatures thermal radiation can significantly affect the heat transfer and temperature distribution in the boundary layer flow of participating fluid. The study of radiative heat transfer flow is very important in manufacturing industry for the design of reliable equipments, nuclear power plants, gas turbines and various propulsion devices for air craft, missiles, satellites and space vehicle.

In view of the importance of the thermal radiation effect, several authors have carried out their research works to investigate the effect of it on some heat and mass
transfer problems. Some of them are Hosain and Takhar [1], Sattar and Kalim [2] and Chandrakala and Antony [3]. The effect of radiation on free convection from a porous vertical plate was discussed by Hosain et al. [4]. Muthucumaraswamy and Kumar [5] presented the thermal radiation effects on moving infinite vertical plate, taking in to account of variable temperature and mass diffusion. The joint effect of free convection and thermal radiation on MHD unsteady flow of a viscous incompressible fluid past an impulsively started vertical plate with uniform heat and mass flux was analysed by Prasad et al. [6]. Recently, Ahmed and Sarmah [7] have studied the thermal radiation effect on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate.

In many times it is observed that the foreign mass reacts with the fluid and in such a situation chemical reaction plays an important role in chemical industry. Theoretical descriptions of non-linear chemical dynamics have been presented by Epstein and Pojman [8] and Gray and Scott [9]. The effects of chemical reaction and mass transfer on MHD flow past a semi-infinite plate was analysed by Devi and Kandasamy [10]. Jaiswal and Soundalgekar [11] have presented an oscillatory flow of viscous incompressible fluid past an infinite vertical porous plate with variable suction, mass transfer and chemical reaction. The effects of mass transfer, Soret effect and chemical reaction on an oscillatory MHD free convective flow through a porous medium have been investigated by Ahmed and Kalita [12]. Recently, Muthucumaraswamy et al. [13] have analyzed an unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. Recent works involving chemical reaction and thermal radiation are by Muthucumaraswamy and Shankar [14] and Ziya Uddin and Manoj Kumar [15].

The object of the present work is to investigate the joint effect of chemical reaction (first order) and thermal radiation (Rosseland model) on an unsteady MHD flow past an accelerated infinite vertical plate with variable temperature and mass transfer.

2. Mathematical Formulation

We now consider an unsteady flow of an incompressible, electrically conducting
viscous radiating and chemically reacting Boussinesq fluid past an accelerated infinite vertical plate with variable temperature and mass transfer under the influence of a uniform transverse magnetic field of strength \( B_0 \). We introduce a coordinate system \((x', y', z')\) with the \( X' \)-axis is taken along the plate in the upward vertical direction, \( Y' \)-axis along the normal to the plate directed in to the fluid region and \( Z' \) axis along the width of the plate. Let \( \vec{q} = (u', 0, 0) \) denote the fluid velocity at the point \( P (x', y', z') \) in the fluid at time \( t' > 0 \). Initially (i.e. when \( t' \leq 0 \)), the plate is at rest relative to the fluid (i.e. \( u' = 0 \)) under no-slip condition) and the fluid at the plate’s surface has the same temperature and molar species concentration as those at the edge of the boundary layer, namely \( T'_\infty \) and \( C'_\infty \) respectively. In fact (by our model), initially (i.e. when \( t' \leq 0 \)) the fluid’s temperature and molar species concentration remain constant all throughout the fluid region (i.e. \( \forall y' \)). At time \( t' > 0 \), the plate is accelerated with a velocity \( u' = at' \) in its own plane and the temperature at the plate is raised linearly with respect to time and the concentration level near the plate rises to \( C'_w \). Under the usual boundary layer and Boussinesq approximations, the equations governing the flow and heat and mass transfer of a chemically reacting and radiating fluid under Rosseland model are:

\[
\frac{\partial u'}{\partial x'} = 0. \tag{2.1}
\]

Momentum equation:

\[
\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g \frac{\partial (T' - T'_\infty)}{\partial y'} + g \frac{\partial (C' - C'_\infty)}{\partial y'} - \frac{\sigma B_0^2 u'}{\rho}.
\tag{2.2}
\]

Energy equation:

\[
\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q'}{\partial y'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + \frac{16 \sigma_1 T'_\infty^3 \kappa}{3k_1} \frac{\partial^2 T'}{\partial y'^2}.
\tag{2.3}
\]

Species continuity equation:

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} + (C'_\infty - C') \lambda
\tag{2.4}
\]

The relevant initial and boundary conditions are:

\[
u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \forall y', \quad t' \leq 0
\tag{2.5}
\[ u' = at', \quad T' = T'_\infty + \left( T'_w - T'_\infty \right) At', \quad C' = C'_w \quad \text{at } y' = 0 \]
\[ u' \to 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad \text{at } y' \to \infty \]
\( \forall t' > 0 \)

\[(2.6)\]

where \( A = \left( \frac{a^2}{\nu} \right)^\frac{1}{3} \), \( u' \) is the fluid velocity in the \( X' \)-direction, \( g \) the acceleration due to gravity, \( T' \) the temperature, \( \nu \) the kinematic viscosity, \( \rho \) the fluid density, \( \sigma \) the electrical conductivity, \( \sigma_2 \) is the Stefan-Boltzmann constant, \( \kappa \) the thermal conductivity, \( C_p \) the specific heat at constant pressure, \( \beta \) the coefficient of volume expansion for heat transfer, \( \tilde{\beta} \) the coefficient of volume expansion for mass transfer, \( q'_r \) the radiative flux, \( D \) the coefficient of chemical molecular diffusivity, \( C'_\infty \) the species concentration in the free stream, \( k_1 \) the mean absorption coefficient, \( T'_\infty \) the fluid temperature in the free stream, \( \lambda \) is the coefficient of first order chemical reaction, \( a \) is the acceleration of the plate and the other symbols have their usual meanings.

We find it convenient to now introduce the following non-dimensional quantities and parameters represented by (2.6a):

\[
\begin{align*}
  u &= \frac{u'}{(\nu a)^{\frac{1}{3}}}, \quad t = t' \left( \frac{a^2}{\nu} \right)^{\frac{1}{3}}, \quad y = y' \left( \frac{a}{\nu^2} \right)^{\frac{1}{3}}, \\
  \theta &= \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\
  P_r &= \frac{\mu C_p}{\kappa}, \quad S_c = \frac{\nu}{D}, \quad G_r = \frac{g \beta (T'_w - T'_\infty)}{a}, \\
  G_m &= \frac{g \tilde{\beta} (C'_w - C'_\infty)}{a}, \quad M = \frac{\sigma B_0^2 \nu}{(\nu a)^\frac{1}{2}}, \\
  N &= \frac{\kappa k_1}{4 \sigma_1 T'_w^{\frac{3}{2}}}, \quad k = \frac{\lambda \nu}{\left( \frac{a^2}{\nu} \right)^{\frac{1}{3}} D}. \tag{2.6(a)}
\end{align*}
\]

where \( G_r \) is the Grashof number for heat transfer, \( G_m \) the Grashof number for mass transfer, \( M \) the Hartmann number, \( N \) the radiation parameter, \( P_r \) the Prandtl number, \( \tilde{\beta} = \frac{1}{2} e^{-\sqrt{\nu}} \) the Schmidt number, \( k \) the first order chemical reaction parameter. Further, \( u \), \( t \), \( \theta \) and \( \phi \) denote respectively the non-dimensional main flow field, time, temperature field and species concentration field.
Using (2.6a), the non-dimensional form of equations (2.2), (2.3), and (2.4) are as follows:

\[
\frac{\partial u}{\partial t} = G_x \theta + G_m \phi + \frac{\partial^2 u}{\partial y^2} - Mu \tag{2.7}
\]

\[
P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + \frac{4}{3N} \frac{\partial^2 \theta}{\partial y^2} = \left(1 + \frac{4}{3N}\right) \frac{\partial^2 \theta}{\partial y^2} = \xi \frac{\partial^2 \theta}{\partial y^2} \tag{2.8}
\]

\[
S_c \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} - K \phi \tag{2.9}
\]

Using (2.6a) in (2.5) and (2.6), the appropriate initial and boundary conditions now become:

\[
u = 0, \theta = 0, \phi = 0 \forall y, \quad t \leq 0 \tag{2.10}
\]

\[
u = t, \quad \theta = t, \quad \phi = 1 \quad \text{at} \quad y = 0
\]

\[
u \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{at} \quad y \to \infty \tag{2.11}
\]

3. Method of Solution

Now we seek the solution of the equations (2.7), (2.8) and (2.9) subject to the conditions (2.10) and (2.11) on using the closed form Laplace Transform Technique. On taking Laplace Transform on equations (2.7), (2.8) and (2.9), the following differential equations are derived.

\[
\frac{d^2 \bar{u}}{dy^2} - (M + s)\bar{u} = -G_x \bar{\theta} - G_m \bar{\phi} \tag{3.1}
\]

\[
\xi \frac{d^2 \bar{\theta}}{dy^2} = P_r s \bar{\theta} \tag{3.2}
\]

\[
\frac{d^2 \bar{\phi}}{dy^2} = (k + S_c s) \bar{\phi} \tag{3.3}
\]

Subject to the boundary conditions:

\[
\bar{u} = \frac{1}{s^2}, \quad \bar{\theta} = \frac{1}{s^2}, \quad \bar{\phi} = \frac{1}{s} \quad \text{at} \quad y = 0 \tag{3.4}
\]

\[
\bar{u} = 0, \quad \bar{\theta} = 0, \quad \bar{\phi} = 0 \quad \text{at} \quad y \to \infty \tag{3.5}
\]
The solutions of the equations (3.1) to (3.3) under the conditions (3.4) and (3.5) are respectively:

\[
\tilde{u} = \left[ \frac{1}{s^2} + \left( \frac{G_r}{s(b^2 - 1)s^2(s - h)} + \frac{G_m}{s(S_c - 1)(s + q)} \right) \right] e^{-\sqrt{(M+\xi)} y} \\
- \frac{G_r}{s(b^2 - 1)s^2(s - h)} e^{-\sqrt{\beta s} y} - \frac{G_m}{(S_c - 1)s(s + q)} e^{-\sqrt{(s+d)} S_c y} \tag{3.6}
\]

\[
\tilde{\bar{u}} = \frac{1}{s^2} e^{-\sqrt{\beta s} y} \tag{3.7}
\]

\[
\tilde{\varphi} = \frac{1}{s} e^{-\sqrt{S_c(s+d)} y} \tag{3.8}
\]

where

\[
b = \sqrt[3]{\frac{P_r}{\xi}}, \xi = 1 + \frac{4}{3N}, d = \frac{k}{S_c}, h = \frac{M}{b^2 - 1}, q = \frac{S_c d - M}{S_c - 1} \tag{3.9}
\]

Taking inverse Laplace transforms of the equations (3.6) to (3.8), we derive the following:

\[
u = B_0 \left[ (t + \eta \sqrt{\frac{t}{M}}) e^{2\eta \sqrt{\sqrt{M} \xi}} \text{erfc}(\eta + \sqrt{\sqrt{M} \xi}) + (t - \eta \sqrt{\frac{t}{M}}) e^{-2\eta \sqrt{\sqrt{M} \xi}} \text{erfc}(\eta - \sqrt{\sqrt{M} \xi}) \right]
\]

\[+ B_1 e^{ht} \left[ e^{2\eta \sqrt{(M+h) t}} \text{erfc}(\eta + \sqrt{(M+h) t}) + e^{-2\eta \sqrt{(M+h) t}} \text{erfc}(\eta - \sqrt{(M+h) t}) \right] \]

\[+ B_2 \left[ e^{2\eta \sqrt{\sqrt{M} t}} \text{erfc}(\eta + \sqrt{\sqrt{M} t}) + e^{-2\eta \sqrt{\sqrt{M} t}} \text{erfc}(\eta - \sqrt{\sqrt{M} t}) \right] \]

\[+ B_3 e^{-qt} \left[ e^{2\eta \sqrt{(M-q) t}} \text{erfc}(\eta + \sqrt{(M-q) t}) + e^{-2\eta \sqrt{(M-q) t}} \text{erfc}(\eta - \sqrt{(M-q) t}) \right] \]

\[+ B_4 e^{ht} \left[ e^{2\eta h \sqrt{\bar{M}} \text{erfc}(b\eta + \sqrt{h \bar{M}}) + e^{-2\eta h \sqrt{\bar{M}}} \text{erfc}(b\eta - \sqrt{h \bar{M}})} \right] + B_5 \text{erfc}(b\eta) \]

\[+ B_6 t \left[ (1 + 2b^2 \eta^2) \text{erfc}(b\eta) - \frac{2b}{\sqrt{\pi}} \eta e^{-\eta^2 b^2} \right] \]
\[ + B_7 \left[ e^{2\eta \sqrt{S_c t}} \text{erfc} \left( \sqrt{S_c} \eta + \sqrt{dt} \right) + e^{-2\eta \sqrt{S_c t}} \text{erfc} \left( \sqrt{S_c} \eta - \sqrt{dt} \right) \right] \]
\[ + B_8 e^{-q t} \left[ e^{2\eta \sqrt{S_c (d-q)t}} \text{erfc} \left( \sqrt{S_c} \eta + \sqrt{(d-q)t} \right) + e^{-2\eta \sqrt{S_c (d-q)t}} \text{erfc} \left( \sqrt{S_c} \eta - \sqrt{(d-q)t} \right) \right] \]

\[ \theta = t \left[ \left( 1 + 2b^2 \eta^2 \right) \text{erfc}(b\eta) - \frac{2b}{\sqrt{\pi}} \eta e^{-\eta^2 b^2} \right] \quad (3.11) \]

\[ \phi = \frac{1}{2} \left[ e^{2\eta \sqrt{S_c t}} \text{erfc} \left( \sqrt{S_c} \eta + \sqrt{dt} \right) + e^{-2\eta \sqrt{S_c t}} \text{erfc} \left( \sqrt{S_c} \eta - \sqrt{dt} \right) \right] \quad (3.12) \]

where \( \eta = \frac{x}{2 \sqrt{t}} \), \( h = \frac{M}{b^2 - 1} \), \( A_1 = \frac{1}{b^2} \), \( A_2 = -\frac{1}{b^2} \), \( A_3 = -\frac{1}{b^2} \), \( b = \frac{\sqrt{t}}{\xi} \),
\[ \zeta = 1 + \frac{4}{4c}, d = \frac{K}{S_c}, q = \frac{S_c d - M}{S_c - 1}, B_0 = \frac{1}{2} + \frac{G \xi A_3}{2(\xi^2 - 1)}, B_2 = \frac{G \xi A_2}{2(\xi^2 - 1)} + \frac{G_m}{2q(S_c - 1)} = B_5 - B_3, B_3 = -\frac{G_m}{2q(S_c - 1)}, B_4 = -\frac{G_m A_1}{2(\xi^2 - 1)} = -B_1, B_5 = -\frac{G_m A_2}{2(\xi^2 - 1)}, B_6 = -\frac{G_m A_4}{(b^2 - 1)}, B_7 = \frac{G_m}{2q(S_c - 1)}, B_8 = \frac{G_m}{2q(S_c - 1)}. \]

4. Skin-Friction and Coefficient of Rate of Heat Transfer and Mass Transfer

The non-dimensional skin-friction coefficient at the plate in the direction of motion of the plate is given by
\[ \tau = -\frac{1}{2 \sqrt{t}} \left( \frac{\partial u}{\partial \eta} \right) \text{evaluated at } \eta = 0 \quad (4.1) \]
\[ = B_0 \left[ \frac{1}{\sqrt{M}} \text{erf} \left( \sqrt{Mt} \right) + 2t \sqrt{M} \text{erf} \left( \sqrt{Mt} \right) + \frac{2t}{\sqrt{\pi}} e^{-Mt} \right] + 2B_2 e^{ht} \left[ \sqrt{M + h} \text{erf} \left( \sqrt{(M + h)t} \right) + \frac{1}{\sqrt{\pi}} e^{-(M+h)t} \right] + 2B_2 \left[ \sqrt{M} \text{erf} \left( \sqrt{Mt} \right) + \frac{1}{\sqrt{\pi}} e^{-Mt} \right] \]
\[ +2B_3 e^{-qt} \left[ \sqrt{M - q} \text{erf} \left( \sqrt{(M - q)t} \right) + \frac{1}{\sqrt{\pi}} e^{-(M - q)t} \right] \]

\[ + 2B_4 e^{ht} \left[ \sqrt{h} \text{erf} \left( \sqrt{ht} \right) + \frac{1}{\sqrt{\pi}} e^{-ht} \right] \]

\[ + \frac{bB_5}{\sqrt{t\pi}} + 2bB_6 \sqrt{\frac{t}{\pi}} + 2B_7 \left[ \sqrt{S_c} \text{erf} \left( \sqrt{dt} \right) + \sqrt{\frac{S_c}{\pi}} e^{-dt} \right] \]

\[ + 2B_8 e^{-qt} \left[ \sqrt{S_c (d - q)} \text{erf} \left( \sqrt{(d - q)t} \right) + \sqrt{\frac{S_c}{\pi}} e^{-(d - q)t} \right] \]

The coefficient of heat transfer at the plate in terms of Nusselt number is given by:

\[ \text{Nu} = -\frac{1}{2\sqrt{t}} \left( \frac{\partial \theta}{\partial \eta} \right)_{\eta=0} = 2b \sqrt{\frac{t}{\pi}} \]  (4.2)

The mass flux in terms of Sherwood number is given below:

\[ \text{Sh} = -\frac{1}{2\sqrt{t}} \left( \frac{\partial \phi}{\partial \eta} \right)_{\eta=0} = \frac{2}{\sqrt{t}} \left[ \sqrt{S_c} \text{erf} \left( \sqrt{dt} \right) + \sqrt{\frac{S_c}{\pi}} e^{-dt} \right] \]  (4.3)

5. RESULTS AND DISCUSSION

In order to have a physical view of the problem, we have computed the numerical calculations for non-dimensional velocity field, temperature field, concentration field and skin-friction, Nusselt number and Sherwood number at the plate for different values of physical parameters involved and these values have been demonstrated in graphs. Our investigation is restricted to \( P_r \) (Prandtl number) equal to 0.71 which corresponds to air. The value of Grashof number \( G_r \) for heat transfer is considered to be 10. In the present investigation, the values of Schmidt numbers \( S_c \) are chosen as 0.22, 0.60 and 0.78 respectively which correspond to Hydrogen, steam and Ammonia at 25\(^\circ\) C and 1 atmospheric pressure. The values of the other parameters namely Hartmann number \( M \), Grashof number \( G_m \) for mass transfer,
first order chemical reaction parameter \( k \) and radiation parameter \( N \) are chosen arbitrarily.

The behavior of velocity field \( u \) versus normal coordinate \( \eta \) under the influence of Schmidt number \( S_c \), first order chemical reaction parameter \( k \), radiation \( N \), Grashof number for mass transfer \( G_m \), and time \( t \) are depicted respectively in figures 1, 2, 3, 4 and 5. It is marked that the fluid velocity increases from \( u = t \) (at the plate) in a very thin layer of the fluid adjacent to the plate and after this fluid layer the fluid velocity asymptotically decreases to its zero value as \( \eta \to \infty \). This phenomenon is clearly supported by the physical reality, since the buoyancy effects are significant near the heated plate surface, which results in a sudden rise of fluid velocity adjacent to the plate surface. Further, it is also observed that the velocity \( u \) falls due to the increasing values of Schmidt number \( S_c \) and chemical reaction parameter \( k \), whereas it rises under the effects of radiation parameter \( N \), Grashof number for mass transfer \( G_m \) and time \( t \). In other words, the velocity field is accelerated under the effects of the radiation, Grashof number for mass transfer and time whereas, this motion is retarded due to the effect of Schmidt number and first order chemical reaction. An increase in absorption of thermal radiation i.e. an increase in \( |N| \) causes temperature \( \theta \) to rise (figure 6), and in turn which leads to an increase in kinetic energy of the fluid. Consequently, the main flow velocity increases.

It is inferred from figures 6 and 7 that the fluid temperature decreases from its value \( \theta = t \) (at the plate) to its zero value as \( \eta \to \infty \). Further, it is noticed that the fluid temperature rises due to the increasing values of radiation \( N \) and time \( t \). This phenomenon clearly agrees with the physical reality.

Figures 8, 9 and 10 exhibit the variation of the species concentration \( \phi \) against normal coordinate \( \eta \) under the influence of Schmidt number \( S_c \), first order chemical reaction \( k \) and time \( t \). These figures clearly show that a rise in the value of Schmidt number or chemical reaction or time causes a fall in the concentration of the fluid. Moreover, it is seen that the concentration field \( \phi \) asymptotically decreases from its maximum value \( \phi = 1 \) to its minimum value \( \phi = 0 \) as \( \eta \to \infty \).
The effects of Hartmann number $M$ on skin-friction $\tau$ against time $t$ is presented in figure 11. It is observed that initially, the magnitude of the skin-friction $\tau$ increases under the effect of Hartmann number $M$. However, for larger values of $t$, the above trend is reversed i.e. $|\tau|$ decreases due to the increasing values of Hartmann number $M$. In other words, the imposition of the applied magnetic field causes the magnitude of the drag force to increase initially and this behavior takes a reverse trend for higher values of $t$. Thus the application of the magnetic field is fruitful in stabilizing the flow over larger time intervals.

Figures 12, 13, 14 and 15 demonstrate the behavior of skin-friction $\tau$ versus time $t$ under the influence of the Grashof number for mass transfer $G_m$, radiation $N$, chemical reaction $k$ and Schmidt number $S_c$ respectively. It is clear from these figures that the magnitude of the drag force $\tau$ rises under the effect of Grashof number for mass transfer $G_m$, radiation $N$ and time $t$, whereas it falls due to the chemical reaction $k$ as well as Schmidt number $S_c$.

Figure 16 demonstrates the behavior of Nusselt number $\text{Nu}$ at the plate against time $t$ under the influence of radiation $N$. Here it is seen that $\text{Nu}$ increases due to the increasing values of time $t$ and it falls under radiation effect.

Figures 17 and 18 indicate that a rise in the first order chemical reaction $k$ and Schmidt number $S_c$ leads to a rise the Sherwood number $Sh$. It is noted that initially $Sh$ steadily decreases; however for larger values of $t$ it remains constant. Moreover, for larger values of time $t$, the effect of Schmidt number is insignificant on Sherwood number $Sh$.

It is inferred from table 1 that fluid velocity is retarded due to the increasing values of Hartmann number $M$ i.e. the imposition of the transverse magnetic field is helpful in stabilizing the flow.

**Conclusions**

Based on our problem set-up and the ensuing solutions, we arrive at the following conclusions for our transport model:
Table 1: Velocity $u$ versus $\eta$, under the effect of Hartmann number $M$ for $t = 0.2$, $P_r = 0.71, S_c = 0.78, K = 4.0, N = -4.0, G_r = 10, G_m = 6$

<table>
<thead>
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<th>$\eta$</th>
<th>$u(M = 2)$</th>
<th>$u(M = 5)$</th>
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</thead>
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<tr>
<td>0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2312607</td>
<td>0.223563</td>
</tr>
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<td>1</td>
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<td>0.091033</td>
</tr>
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<td>1.5</td>
<td>0.0275354</td>
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</tr>
<tr>
<td>3</td>
<td>0.0000462</td>
<td>0.000038</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 1: Velocity $u$ versus $\eta$, under the effect of Schmidt number $S_c$, for $t = 0.2$, $P_r = 0.71, k = 4.0, N = -4.0, G_r = 10, G_m = 6, M = 5$
Figure 2: Velocity $u$ versus $\eta$, under the effect of Chemical Reaction Parameter $k$, for $t = 0.2, P_r = 0.71, S_e = 0.78, N = -4.0, G_r = 10, G_m = 6, M = 5$

Figure 3: Velocity $u$ versus $\eta$, under the effect of Radiation Parameter $N$, for $t = 0.2, P_r = 0.71, S_e = 0.78, k = 4.0, G_r = 10, G_m = 6, M = 5$
Figure 4: Velocity $u$ versus $\eta$, under the effect of $G_m$ for $t = 0.2, M = 5, N = -4.0, P_r = 0.71, S_c = 0.78, k = 4.0, G_r = 10$

Figure 5: Velocity $u$ versus $\eta$, under the effect of $t$, for $M = 5, N = -4.0, P_r = 0.71, S_c = 0.78, k = 4.0, G_r = 10, G_m = 6$
Figure 6: Temperature $\theta$ versus $\eta$, under the effect of Radiation Parameter $N$,
for $t = 0.2$, $P_r = 0.71$, $M = 5$

Figure 7: Temperature $\theta$ versus $\eta$, under the effect of $t$,
for $P_r = 0.71$, $M = 5$, $N = 4$
Figure 8: Species concentration $\phi$ versus $\eta$, under the effect of Schmidt number $S_c$, for $t = 0.2$, $k = 4.0$, $M = 5$

Figure 9: Species concentration $\phi$ versus $\eta$, under the effect of Chemical Reaction parameter $k$, for $t = 0.2$, $S_c = 0.78$, $M = 5$
Figure 10: Species concentration $\phi$ versus $\eta$, under the effect of $t$,
$S_{c} = 0.78, M = 5, k = 4$

Figure 11: Skin friction $\tau$ versus $t$, under the effect of Hartmann number $M$, for
$N = -4.0, P_r = 0.71, S_{c} = 0.78, k = 4.0, G_r = 10, G_{m} = 6$
Figure 12: Skin friction $\tau$ versus $t$, under the effect of $G_m$, for $M = 5$, $N = -4.0$, $P_r = 0.71$, $S_e = 0.78$, $k = 4.0$, $G_r = 10$

Figure 13: Skin friction $\tau$ versus $t$, under the effect of Radiation Parameter $N$, for $M = 5$, $P_r = 0.71$, $S_e = 0.78$, $k = 4.0$, $G_r = 10$, $G_m = 6$
Figure 14: Skin friction $\tau$ versus $t$, under the effect of Chemical Reaction Parameter $k$, for $M = 5, P_r = 0.71, S_c = 0.78, G_r = 10, G_m = 6, N = -4.0$.

Figure 15: Skin friction $\tau$ versus $t$, under the effect of Schmidt number $S_c$, for $M = 5, P_r = 0.71, G_r = 10, G_m = 6, N = -4.0, k = 4$. 
Figure 16: Nusselt number Nu versus t, under the effect of Radiation Parameter N, for $M = 5$, $P = 0.71$

Figure 17: Sherwood number Sh versus t, under the effect of Chemical Reaction Parameter $k$, for $M = 5$, $S_r = 0.78$
Figure 18: Sherwood number $Sh$ versus $t$, under the effect of Schmidt number $S_c$, for $M = 5, k = 4$

i) Application of the transverse magnetic field retards the fluid motion near the plate. Thus, the applied magnetic field will aid in maintaining laminar flow control as well as minimizing the drag on the plate surface.

ii) The fluid velocity is decelerated in the region adjacent to the plate, due to the effects of Schmidt number as well as first order chemical reaction. Hence, the phenomena of convective mass transfer accompanied by chemical reaction induced by diffusion of chemical species can help in achieving laminar flow control and a reduction of drag on the plate.

iii) The fluid velocity near the plate increases due to the absorption of thermal radiation, Grashof number for mass transfer and for increasing time. This indicates that the main flow field exhibits an augment due to the absorption of radiative heat, rise in buoyancy effects owing to mass transfer and increasing time.

iv) The effect of the magnetic field, first order chemical reaction, absorption of thermal radiation and mass diffusion are nullified at the fluid region far away
from the plate. In other words, the benefits of the applied magnetic field, radiative heat transfer, chemical reaction and convective mass transfer are insignificant at the edge of the boundary layer (i.e. at the quiescent region).

v) The fluid temperature rises as the absorption of thermal radiation increases. That is, the radiative heat transfer from the plate to the fluid will lead to a rise in the temperature of the fluid.

vi) The concentration of the fluid falls as each of Schmidt number, first order chemical reaction parameter and time increases. Evidently the convective mass transfer and the chemical reaction lead to a decrease in the species concentration of the fluid, with increasing time.

vii) The viscous drag is reduced due to the application of transverse magnetic field for larger time. Clearly then, the advantage of the applied magnetic field in minimizing shear stress at the plate surface is observed when the magnetic field is imposed for long extents of time.

viii) An increase in Grashof number for mass transfer, absorption of thermal radiation results in a growth in the magnitude of the drag force and it falls under the effects of first order chemical reaction and Schmidt number. This implies that the shear stress at the plate registers a rise in magnitude with an increase in buoyancy effects owing to species concentration gradients. The absorption of radiative heat causes an escalation in the magnitude of shear stress at the plate. Further, the convective mass transfer accompanied by chemical reaction is beneficial towards reducing the drag at the plate’s surface.

ix) The rate of heat transfer (from plate to the fluid) decreases due to the absorption of thermal radiation. That is, the absorption of radiative heat aids in controlling the heat flux at the plate’s surface.

x) The rate of mass transfer increases due to the first order chemical reaction as well as Schmidt number. Obviously, the mass flux at the plate’s surface can be enhanced via diffusion of foreign chemical species followed by first order chemical reaction.
xi) The aim of our paper (as indicated in the 'Introduction' section) is to examine the combined effect of chemical reaction (first order) and thermal radiation (Rosseland model) on the present (modeled) unsteady MHD flow past an accelerated infinite vertical plate with variable temperature and mass transfer. As anticipated (hypothesized), we finally observe from our results and conclusions that the imposed transverse magnetic field together with radiation and convective mass (chemical species) diffusion accompanied by first order chemical reaction has a significant influence on this transport model involving laminar boundary layer flow, radiative heat and convective mass transfer.

xii) The present problem concerns with a first order homogeneous chemical reaction where the rate of reaction is proportional to the local concentration. However, it is also possible to formulate a similar problem for the zeroth order chemical reaction. In such case, the chemical reaction will occur at a constant rate i.e. the rate of reaction is independent of the local concentration.

REFERENCES


