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A SURVEY OF SYMPLECTIC AND CONTACT TOPOLOGY

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In this article, we give a brief survey of major historical developments in the field of Contact and Symplectic Geometry. This field has grown into an area in its own right due to rapid progress seen in the last five decades. The community of Indian mathematicians working on this field is small but steadily growing. The contribution from Indian mathematicians to this field is noted in the article.

Key words : Contact geometry; symplectic geometry.

1. Introduction

Symplectic geometry has its origin in the Hamiltonian formulation of classical mechanics while contact geometry appears on constant energy surfaces in the phase space of classical mechanics. Contact geometry is also the mathematical language of thermodynamics, geometric optics and fluid dynamics. Symplectic and contact geometry in the simplest terms can be best described as the geometry of differential forms. In recent times, this has grown into an independent subject in its own right. It has also found applications in important problems of low dimensional topology, complex geometry, algebraic geometry, foliation theory and has given new directions in dynamics.

A symplectic form on a manifold M is a non-degenerate 2-form ω which is also closed. The pair (M,ω) is called a symplectic manifold. The non-degeneracy of ω implies that the manifold must be even dimensional and orientable. Indeed, an orientation can be defined by ω^n. The Euclidean space $\mathbb{R}^{2n}$ with the coordinate system $(x_1, y_1, .., x_n, y_n)$ has a canonical symplectic form $\omega_0 = \sum_{i=1}^n dx_i \wedge dy_i$. The cotangent bundle T^*M of any manifold M admits a tautological 1-form λ defined by $\alpha^*\lambda =$

α for any section α of T^*M. The exact form $d\lambda$ defines a symplectic form on the cotangent bundle. These two examples are very important for understanding the structure of symplectic manifolds. Also, every oriented surface with an area form is an example of a symplectic manifold. However, none of the even dimensional spheres $\mathbb{S}^{2n}$, $n > 1$, supports a symplectic form because of Stokes' Theorem. In fact, for the existence of a symplectic form on a closed manifold one requires a cohomology class α in $H^2_{dR}(M)$ for which α^k is non-vanishing for $1 \leq k \leq n$.

The contact manifolds are odd-dimensional counterpart of symplectic manifolds. A manifold M together with a hyperplane distribution ξ, defined as the kernel of a 1-form α satisfying $\alpha \wedge (d\alpha)^n \neq 0$, is called a contact manifold. The distribution ξ is then called a contact structure. It follows that the 2-form $d\alpha$ restricts to a symplectic form on ξ. Every odd dimensional Euclidean space $\mathbb{R}^{2n+1}$ with the coordinate system $(x_1, y_1, .., x_n, y_n, z)$ carries a canonical contact structure ξ_0 given by the 1-form $\alpha_0 = dz + \sum_{i=1}^n x_i\, dy_i$. Every odd-dimensional sphere carries a contact structure given by the the 1-form $i_{\partial_r}\omega_0$, which is obtained by contracting the canonical symplectic form ω_0 by the radial vector field ∂_r on the sphere.

Symplectic and contact manifolds are studied up to *symplectomorphism* and *contactomorphisms* respectively. A diffeomorphism $\varphi : (M, \omega) \to (N, \sigma)$ between two symplectic manifolds satisfying $\varphi^*\sigma = \omega$ is called a *symplectomorphism*. A diffeomophism $\varphi : (M, \xi) \to (M', \xi')$ between two contact manifolds is said to be a contactomorphism if the derivative map $D\varphi$ carries ξ onto ξ'. Darboux's theorem says that any symplectic manifold is locally symplectomorphic to ω_0. On the other hand, every contact structure is locally contactomorphic to ξ_0. Thus, there are no local invariants for symplectic and contact manifolds which makes the classification problems more difficult than that for the Riemannian manifolds where curvature is a local invariant. This hints at the topological nature of symplectic and contact geometry.

Two symplectic forms ω_0 and ω_1 on M are said to be isotopic if there is an isotopy $\{\varphi_t\}$ with $\varphi_0 = id_M$, such that $\varphi_1^*\omega_1 = \omega_0$. The de-Rham cohomology class of a symplectic form on a given manifold is clearly an isotopy invariant. It is a result due to Moser which says that two symplectic forms on a *closed* manifold are isotopic if they are connected by a homotopy of symplectic forms in the same cohomology class. Thus, the symplectic forms exhibit certain rigidity through cohomology classes. Similarly, any two contact structures on a *closed* manifold are isotopic if they are connected by a homotopy of contact structures. This is a result of Gray [32].

Symplectic manifolds (M^{2n}, ω) have some special submanifolds L of dimension n, called Lagrangians on which ω restricts to the zero form. If L is a lagrangian submanifold of a symplectic manifold (M, ω) then a neighbourhood of L in M is symplectomorphic to a C^1-neighbourhood of the zero section of the cotangent bundle T^*L, where the symplectomorphism carries the submanifold L canonically onto the zero section. Legendrian submanifolds are the contact counterpart of Lagrangians. Legendrians in a contact manifold of dimension $2n+1$ are n-dimensional submanifolds tangent to the contact distribution.

The theories of symplectic and contact manifolds are intimately related and each one complement the other. Every contact form α on a manifold M defines an exact symplectic form $d(e^t\alpha)$ on $M \times \mathbb{R}$, called its symplectization. On the other hand, every symplectic form on a closed manifold M representing an integral cohomology class gives rise to a contact manifold which is a principal S^1-bundle over M [5]. Moreover, certain codimension 1 submanifolds of a symplectic manifold inherit contact structures which are compatible with the symplectic form. In particular, boundaries of some symplectic manifolds are contact. Such contact structures are referred as symplectically fillable.

We refer to [44] for a detailed exposition on the basics of Symplectic Topology and [28] for Contact Topology.

2. DEVELOPMENT DURING 1965-2005

The question of existence and classification of symplectic and contact structures can be expressed in terms of first order differential relations for 2-forms and 1-forms respectively. Partial differential relations, in short PDR, include equations as well as inequalities. Every PDR for functions/sections defined on a manifold M can be associated with an algebraic relation by substituting the derivatives $D^k f$ by independent functions. A solution of the algebraic relation is called a formal solution of the PDR. The space of genuine solutions of a PDR can be viewed as a subspace of the space of formal solutions.

$$\text{Genuine solutions of a PDR} \hookrightarrow \text{Formal solutions of a PDR}$$

If every formal solution can be homotoped to a genuine solution in the space of formal solutions then we say that the solutions of the relation satisfy the (existence) *h-principle* [34]. A result in Differential Geometry is classified as a flexibility result if it only depends on the algebraic (topological) data. Therefore, any h-principle result is a flexibility type result. On the other hand, existence of any symplectic invariant is a manifestation of rigidity. Symplectic and contact geometry is an area where both rigidity and flexibility phenomena are equally prominent and frequently observed.

First important result in symplectic and contact topology came in the form of an h-principle as Gromov proved the following results on open manifolds [34]:

- Every non-degenerate 2-form on an *open* manifold is homotopic to a symplectic form (through non-degenerate forms). Moreover, the symplectic form can be chosen to represent a prescribed de Rham cohomology class. The space of non-degenerate 2-forms on a manifold has the same homotopy type as the space of all almost complex structures on it. In particular, every symplectic manifold admits an almost complex structure. The h-principle further implies that *open* almost complex manifolds are symplectic.

- Every almost contact structure (α, β), (a pair consisting of a 1-form α and a 2-form β such that $\alpha \wedge \beta^n$ is non-vanishing), on an *open* manifold is homotopic through almost contact structures to a pair $(\eta, d\eta)$. Note that η is a contact form.

This shows that the obstruction to the existence of symplectic and contact structures on open manifolds is purely topological. On the other hand, symplectic forms on closed manifolds do not satisfy the h-principle even when the cohomology condition is taken into account. Using Seiberg-Witten theory, Taubes [49, 50] proved that the connected sum of odd number of copies of $\mathbb{C}P^2$ does not support a symplectic form compatible with its almost complex structure.

Maps of a symplectic manifold into another which preserve the symplectic structures are called iso-symplectic maps. Similarly, iso-contact maps are maps between contact manifolds which preserve the contact structures. Among various other flexibility results, one has an h-principle for iso-symplectic embeddings in codimension 4 and also in codimension 2 when the domain is an open manifold [34]. Iso-symplectic embedding in codimension 2 is beyond h-principle. In [12] Donaldson proved the existence of codimension 2 symplectic submanifolds in *closed* symplectic manifolds using almost Kähler geometry. Iso-contact embeddings in codimension 2, Lagrangian immersions in a symplectic manifold and Legendrian immersions in a contact manifold also satisfy the h-principle [14, 34, 40].

In the mid-sixties, Arnold formulated a number of conjectures [1] in symplectic topology which became the main driving force in the years to come and shaped the theory of symplectic topology as we see it today. The most important one of these is the conjecture on the number of fixed points of certain symplectomorphisms, called Hamiltonian diffeomorphisms.

The contraction of the 1-form ω by a vector field X, $X \mapsto i_X\omega$, defines a bijection between the set of vector fields and 1-forms on a symplectic manifold (M, ω). A vector field X_H that corresponds to an exact form dH, for some smooth function H on M, is called a Hamiltonian vector field. A time dependent *Hamiltonian vector field* X_{H_t} generates an isotopy consisting of symplectomorphisms of M, where H_t, $0 \leq t \leq 1$, is a smooth 1-parameter family of functions. The time 1 map of this isotopy is called a *Hamiltonian diffeomorphism*.

Arnold conjectured that for every Hamiltonian diffeomorphism φ of a symplectic manifold (M,ω) one must have

$$\#\text{ Fixed points of }\varphi \;\; \geq \min_{f\in C^\infty(M)} \#\text{ critical points of } f$$

The conjecture was motivated by Poincaré-Birkoff Theorem proved in early 1900 [4, 48] which states that an area preserving diffeomorphism of an annulus which moves the two components of the boundary in opposite directions must have at least two fixed points. points.

The fixed points of a Hamiltonian diffeomorphism φ are in 1-1 correspondence with the 1-peridic orbits of the associated Hamiltonian flow. This observation links Arnold's conjecture to Hamiltonian dynamics. Arnolds' conjecture is also related to the question of existence of a non-trivial symplectic invariant. A symplectic manifold (M,ω) has a natural volume form ω^n and its volume is defined as $\int_M \omega^n$ provided M is *closed*. Every symplectomorphism f necessarily preserves the associated volume form ω^n and hence, is a volume preserving diffeomorphism. It is known that the volume is the only invariant for manifolds with volume form [45]. In early 70's, Gromov proved the following striking result which is referred as *Gromov's alternative*.

***Theorem* 2.1** — *The group of symplectomorphisms is either C^0-closed or C^0-dense in the group of volume preserving diffeomorphisms. Furthermore, the resolution is the same for all symplectic manifolds.*

Eliashberg, in 1980 [17], resolved this in favour of the former thereby exhibiting a rigidity in symplectic topology. If the resolution were in favour of C^0-denseness then there would have been no special fixed point properties of hamiltonian diffeomorphisms and Arnold's conjecture would have been false. As a passing note, we record here that the set $Ham(M,\omega)$ of Hamiltonian diffeomorphism is a subgroup of the symplectomorphism group $Symp(M,\omega)$, and there is a bi-invariant metric, constructed by Hofer [35], on this group which shows further rigidity in symplectic topology.

There is no preferred volume form on a contact manifold (M,ξ) since a defining 1-form α for ξ is not unique. The corresponding rigidity result for contact manifolds says that the subgroup of contactomorphisms is C^0-closed in the space of *all* diffeomorphisms of the manifold M.

In 1985, Gromov revolutionized the theory of symplectic topology by introducing the notion of J-holomorphic curves in [33]. Every symplectic manifold (M,ω) admits an almost complex structure J (that is, a bundle morphism $J : TM \to TM$ satisfying $J^2 = -I$). A J-holomorphic curve is a smooth map ϕ from a Riemann surface (Σ, j) into (M,J) such that the differential $d\phi$ commutes with the complex structures, i.e., $d\phi \circ j = J \circ d\phi$. Gromov's insight was to use almost complex structures J *tamed* by a symplectic form ω, meaning that $\omega(v, Jv) > 0$ for all $v \neq 0$, to study the global properties of the symplectic manifold (M, ω). He showed that such almost complex structures exist on every symplectic manifold and J-holomorphic curves are abundant. Using J-holomorphic curves, Gromov proved a number of interesting results including the famous non-squeezing theorem.

***Theorem* 2.2** — *[Non-squeezing Theorem [33]]. Let $\mathbb{B}^{2n}_r$ denote the ball of radius r in $(\mathbb{R}^{2n}, \omega_0)$ centered at origin. If there is a symplectic embedding $\varphi : \mathbb{B}^{2n}_r \to \mathbb{B}^2_R \times \mathbb{R}^{2n-2} \subset \mathbb{R}^2 \times \mathbb{R}^{2n-2}$, then $r \leq R$.*

The non-squeezing theorem gave rise to the first non-trivial symplectic invariant, known as symplectic area or symplectic width w_G:

$$w_G(M, \omega) = \sup\{\pi r^2 \mid B^2_r \text{ symplectically embeds in } (M, \omega)\}$$

This result illustrates a geometric behaviour of symplectic structures as opposed to flexibility.

J-holomorphic curves have played a central role in symplectic and contact topology. Powerful invariants such as Floer homology theories, Gromov-Witten invariants, Fukaya category and Symplectic Field Theory are defined by using Moduli spaces of pseudoholomorphic curves.

Floer theory, in general, is an infinite dimensional Morse theory for a specific functional defined on an appropriate space. The chain groups of Floer homology are abelian groups generated by a collection of critical points of the functional and the boundary maps are defined by counting the gradient flow lines which join two critical points and satisfy a differential equation. These theories have played an important role in understanding the relations between low-dimensional topology, symplectic topology and contact topology.

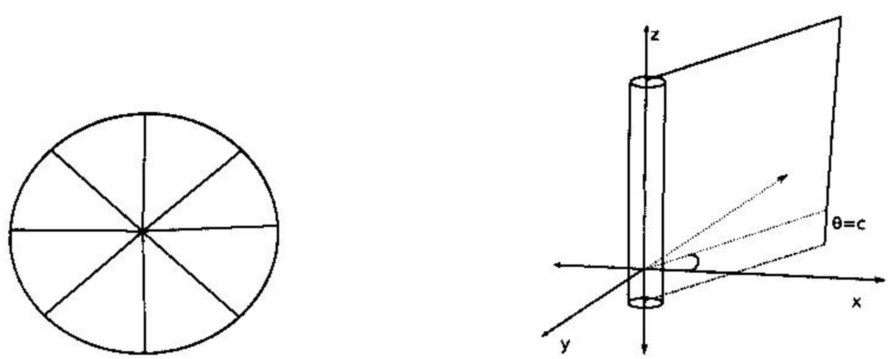
To prove Arnold's conjecture for a symplectomorphism φ, Floer defined a functional on the free loop space of the symplectic manifold whose critical points are the 1-periodic orbits of the Hamiltonian flow defining φ [24]. An arbitrary path in the loop space can be viewed as a cylinder in the symplectic manifold. The geometric equation governing the gradient flow for the path (for a suitable metric) is a perturbed Cauchy-Riemann equation for this cylinder. The differential counts the number of such J-holomorphic cylinders and Gromov's compactness theorem is then used to show that the differential is a boundary operator of the Floer chain complex.

Gromov invariants were defined in the mid 90's which counts the number of *stable* J-holomorphic curves satisfying some constraints. Taubes discovered the equivalence of Gromov invariants and Seiberg-Witten invariants (of closed oriented 4-manifolds) thereby establishing a link between symplectic and the low dimensional topology. Around 2000, Eliashberg, Givental and Hofer proposed Symplectic Field Theory (SFT) in [16] as a unifying theory for J-holomorphic curves. SFT is an invariant of contact manifolds and symplectic cobordism (with contact boundary). Here one associates to each contact manifold a Differential algebra generated by closed Reeb orbits. The differential counts the number of holomorphic curves in the cylinder over the contact manifolds. Moreover, SFT gives a recursive algorithm for computing the number of of algebraic curves of various degrees in $\mathbb{CP}^q$.

The growth of contact topology remained relatively slow in these years compared to that of the symplectic topology. However, there were some very significant results in 3-dimensional contact topology during this period. Around 1970, Martinet [42] and Lutz [41] independently showed that every *closed* 3-manifold admits a contact structure. Lutz's result was the first flexibility result for contact structures on *closed* 3-manifolds.

Apart from the standard one, there is another very important contact structure on $\mathbb{R}^3$ defined by the 1-form $\alpha_{ot} = \cos r\, dz + r \sin r\, d\theta$ in cylindrical polar coordinates (r, θ, z) on $\mathbb{R}^3$. Bennequin [3] proved that $\xi_{ot} = \ker \alpha_{ot}$ is not isotopic to the standard contact structure ξ_{std} on $\mathbb{R}^3$.

Building on the work of Bennequin, Eliashberg defined [15] overtwisted contact structures on *closed* 3-manifolds and gave a complete classification of them. A surface Σ in a contact 3-manifold (M, ξ) admits a singular 1-dimensional foliation defined by the line field $T\Sigma \cap \xi$. This foliation is called the *characteristic foliation* of ξ on Σ. The disc $D^2_\pi \subset \mathbb{R}^2 \times 0$ with radius π and centre at origin, in the contact manifold $(\mathbb{R}^3, \xi_{ot})$ is called the *overtwisted disc* (Figure 1(a)). Eliashberg calls a contact structure ξ on a 3-manifold *overtwisted* if it admits an embedded overtwisted disc. If it is not overtwisted then it is called *tight*.



(a) Overtwisted disk (b) Open book

Figure 1 :

***Theorem* 2.3** — *(Eliashberg). Every oriented 2-plane field on a 3-manifold is homotopic to an overtwisted contact structure. Moreover, any two contact structures which are homotopic as 2-plane fields are isotopic.*

This was an important breakthrough as he introduced the dichotomy overtwisted vs tight (non-overtwisted). It was observed [18, 33] that overtwisted contact structures are not symplectically fillable.

Now, we turn our attention to topological description of contact structures in dimension 3 by open book decomposition. An open book decomposition of a 3-manifold M is a fibration $\pi : M - B \to \mathbb{S}^1$, where $B \subset M$ is a 1-dimensional submanifold. For instance, $(r, \theta, z) \to \theta$ gives an open book decomposition of $\mathbb{R}^3$ with z-axis removed from it (Figure 1(b)). In 2002 Giroux [30] showed that there is a one-one correspondence between contact structures up to isotopy on a closed oriented 3-manifold M and open book decompositions of M up to "positive stabilizations". This was an adaptation of Donaldson's work [13] on Lefshetz pencil structures on symplectic manifolds to dimension 3. Further, Giroux's work completed the story of Thurston-Winkelnkemper [53] by observing that every open book supports a contact structure and that any two contact structures, supported by the same open book, are isotopic.

3. Major Breakthroughs in The Last 10 Years

3.1 The last decade has seen many landmark developments in contact topology. Giroux's open book characterization of contact structures on 3-manifolds paved the way for the use of topological techniques in understanding the contact structures. Properties of contact structures - such as Stein fillability, tightness - can be detected from the associated open-book decompositions. We refer to the works of Giroux, Goodman [31], Honda-Kazez-Matic [38]. In [55], Wand characterized tightness in terms of a given open book and using this characterization settled the Legendrian surgery conjecture which states that contact (-1)-surgery preserves tightness.

A legendrian knot in a contact 3-manifold admits a natural framing induced from the contact structure, known as the contact framing. One can use this as a reference framing to describe Dehn surgeries on the manifold. A notion of contact surgery along Legendrian links, analogous to Dehn surgery, can be defined. A classical theorem of Lickorich and Wallace says that every closed oriented connected 3-manifold is obtained by of '± 1-surgeries' along some link in $\mathbb{S}^3$. Ding and Geiges proved a contact version of this theorem, that any contact connected 3-manifold can be obtained by ± 1-contact surgeries along some Legendrian link in $(\mathbb{S}^3, \xi_{std})$.

Besides the above development, Ozsvath and Szábo [47], using Giroux's correspondence, defined a contact invarinat on a 3-manifold that lives in Heegard-Floer homology groups. This invariant vanishes for all overtwisted contact structures. Hence, the non-triviality of the invariant detects tightness. The invariant is known to be non-trivial for Stein fillable contact structures, which are necessarily tight [47]. However, this is not true, in general, as Massot [43] produced examples of tight contact structures on trivial circle bundles over surfaces with vanishing contact invariants. Baldwin-Etnyre [2] demonstrated another family of tight contact manifolds for which the contact invariants vanish. Hence the invariant fails to detect overtwisted contact structures.

Starting from an arbitrary overtwisted contact manifold, Conway-Kaloti-Kulkarni [11] constructed infinite families of universally tight contact structures, for which the contact invariants vanish. This work showed that such examples are not sporadic and any refinement of the invariant must be sensitive to these examples.

3.2 Another important development happened with respect to Weinstein conjecture on Reeb dynamics. The Reeb vector field R on a contact manifold $(M, \xi = \ker \alpha)$. is defined by the properties $i_R\alpha = 1$ and $i_R d\alpha = 0$.

Conjecture 3.1 : (Weinstein, 1978). The flow of the Reeb vector field must have a periodic orbit on any closed contact manifold.

The above statement is not true for general vector fields on closed manifolds [39]. Weinstein conjecture, exhibiting a rigidity phenomenon, played an important role in the development of contact topology that is similar to the role of Arnold's conjecture in symplectic topology.

During 1987-1993, Floer, Hofer, Viterbo and Zehnder proved Weinstein conjecture in a series of articles [23, 37, 54] for specific contact manifolds which were all symplectically fillable. The conjecture was also settled for overtwisted contact 3-manifolds in a work of Hofer in [36]. Finally, in 2007 Taubes proved Weinstein conjecture in full generality for dimension 3 using the Seiberg-Witten Floer cohomology [51]. Michael Hutchings had earlier introduced Embedded Contact Homology (ECH) which is a Floer type homology generated by certain collections of closed Reeb orbits. Taubes demonstrated in a series of articles, starting with [52], that this homology is equivalent to the Seiberg-Witten Floer cohomology.

3.3 There has been several important breakthroughs in the high dimensional symplectic and contact topology in recent years.

Eliashberg and Cieliebak [8-10] introduced a class of symplectic manifolds, called Weinstein manifolds, which lie in the juncture of Symplectic and Stein manifolds (affine complex manifolds) and proved several flexibility results for them. Earlier Eliashberg proved an h-principle for Stein manifolds [19]. Weinstein manifolds are *open symplectic manifolds* with a compatible triple (ω, ϕ, λ), where ω is a symplectic form, ϕ is a generalised Morse function and λ is a vector field which is Liouville for ω and gradient-like for ϕ. Each Stein manifold comes with a complex structure J and a plurisubharmonic function ϕ. A Weinstein structure on it can be defined by the triple (ω, ϕ, λ) where $\lambda = \nabla\phi$ and $\omega = d(d\phi \circ J)$.

***Theorem* 3.2** — *Every Weinstein structure $\mathcal{W}_0 = (\omega, X, \phi)$ on a manifold M of dimension ≥ 6 admits a homotopy of Weinstein structures $\mathcal{W}_t = (\omega_t, X_t, \phi)$, $0 \leq t \leq 1$, such that $\mathcal{W}_1$ is induced by a Stein structure.*

Further, if two Stein structures $\mathcal{S}_0$ and $\mathcal{S}_1$ on M are such that the associated Weinstein structures are homotopic through such structures then $\mathcal{S}_0$ and $\mathcal{S}_1$ are homotopic through Stein structures.

Equivalently, the inclusion of Stein within Weinstein induces a bijection at the level of π_0 if the underlying manifold is of dimension ≥ 6.

Eliashberg and Cieliebak also established an *existence* h-principle for a class of Weinstein cobordism in dimension $2n > 4$.

3.4 It is known that Lagrangians and Legendrians do not satisfy the h-principle, in general. However, in dimension > 3, Murphy discovered that the h-principle holds for a certain class of Legendrians, which she calls as *loose* Legendrians. Using this result, Eliashberg and Murphy [21, 46] proved an h-principle for Lagrangian embeddings with loose Legendrian boundaries.

Very recently, a high-dimensional analogue of overtwisted contact structures is discovered by Borman, Eliashberg and Murphy in [6] for which the parametric version of h-principle works. It is beyond the scope of this article to delve into technical definition of overtwisted contact structures. Before the advent of this comprehensive result, there were number of results establishing flexibility of contact structures in dimension 5. These were due to Geiges [26, 27], Geiges and Thomas [25, 29] and most notably followed by the work of Casals, Pancholi and Presas [7], where the authors use a generalized Lutz twist developed in the work of Etnyre and Pancholi [22]. In the full generality, the existence and classification of overtwisted contact structures on a closed manifold of odd-dimensions is achieved by Borman, Eliashberg and Murphy in [6]. This extends the dichotomy, overtwisted vs tight, for higher dimensional contact structures.

4. Concluding Remarks

Symplectic and Contact topology is a rapidly growing area and the literature on this subject is vast.

We have presented, in Section 2, only a few major works which are turning points in the history of the subject, leaving out a large number of significant contributions. We have consulted various expository and survey articles while writing this survey but most importantly the article of Eliashberg [20] and a number of reviews on Mathscinet.

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Symplectic and Contact topology is an active area of research for the last 30 years. However, it has just started to emerge as one of the main research area in India. To highlight Indian contribution in Symplectic and Contact topology, we have included a separate bibliography and we have tried to be exhaustive within our limitation. Indian contributions listed here are authored by Datta [4-7], Gadgil [9-11], Islam [6], Kulkarni [3, 11], Pancholi [1, 8], Poddar [2, 12], Mukherjee [7], Sarkar [13], Venugopalan [14].

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