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ANALYTIC NUMBER THEORY IN INDIA DURING 2001-2010

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In this article we summarize the contribution of Indian mathematicians to analytic number theory during 2001-2010.

Key words : Zeta function; Diophantine equations; additive combinatorics; transcendental number theory.

1. INTRODUCTION

In this short note I will try to summarise the contribution of the Indian mathematicians to the analytic theory of numbers in the first decade of the new millennium. The choice of the time period may seem completely arbitrary. Indeed my original goal was to cover all works in the new millennium till date, but I realised that it was impossible to report on all of it in limited space. The choice was either to hand-pick what I would consider important or to limit the time period. I chose the latter to eliminate the possibility of any bias from my side. The year 2010 was an important year in the history of Indian mathematics, as for the first time India became the host of the International Congress of Mathematicians (ICM 2010). Whether this gala event had any influence on research in India is a question to ponder over, but it seems there has been a rise in the number of papers in analytic number theory since then. At a more personal level, K. Ramachandra who spearheaded research in analytic number theory in India, passed away in January 2011 bringing to an end an era.

Analytic number theory is a broad subject without any well-defined boundary. So the choice of topics may also seem arbitrary to the reader. I have picked those which, to me, have certain analytic flavour. Of course the theory of the Riemann zeta function and that of the related L-functions, form the core of the subject. So we will begin this article by first focusing on works directly related to the zeta function, and then discuss the large volume of work done on Diophantine equations, additive combinatorics and transcendental number theory.

Since this is a report about work done in the country, works of several very prominent mathematicians from India who worked outside India has been omitted. In particular, I do not speak about the work of K. Soundararajan who has contributed so much to analytic number theory. Also I will not mention works of M. Bhargava and A. Venkatesh. My focus is on research generated from this soil, especially from the three strong seats of learning - Harish-Chandra Research Institute, Allahabad (HRI), Institute of Mathematical Sciences, Chennai (IMSc) and Tata Institute of Fundamental Research, Mumbai (TIFR).

2. Riemann Zeta Function and Related Objects

The theory of the Riemann zeta function is central in analytic number theory. The zeta function which is defined by the Dirichlet series

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

in the half plane $\mathrm{Re}(s) = \sigma > 1$, has analytic continuation to whole of the complex plane except $s = 1$, and satisfies a functional equation relating the value of ζ at s with that at $1 - s$. Of course the most important problem in this theory is the Riemann Hypothesis - the assertion that all the non-trivial solutions of $\zeta(s) = 0$ lie on the critical line $\sigma = 1/2$. This is the so called ‘Holy Grail’ of analytic number theory. Riemann showed that this is intrinsically related with the distribution of prime numbers.

A recurring motif in analytic number theory is to attach functions to arithmetic sequences and then use the analytic properties of the function to understand the sequence. One standard way to define such a function is via Dirichlet series

$$D(s) = \sum_{n=1}^{\infty} \frac{a_n}{n^s}.$$

The summatory function $A(x) = \sum_{n<x} a_n$ of the sequence (a_n) can be expressed as an integral of this series $D(s)$, and this can be used to derive results regarding the sequence. For example the prime number theorem

$$\pi(x) = \sum_{\substack{p<x \\ p \text{ prime}}} 1 \sim \frac{x}{\log x}$$

can be proved using this idea. Dirichlet was the pioneer in this field and he used this idea to prove infinitude of primes in arithmetic progression. As another example let us take the sequence $a_n = d(n)$ where $d(n)$ denotes the number of divisors of the non-negative integer n. In this case the associated Dirichlet series is just given by

$$\sum_{n=1}^{\infty} \frac{d(n)}{n^s} = \zeta^2(s).$$

The summatory function $\sum_{n<x} d(n)$ was studied by Dirichlet, who established an asymptotic formula for this sum with an error term of size $O(x^{1/2+\varepsilon})$. A deep conjecture, known as the Dirichlet divisor problem, says that the error term in this asymptotic is actually of the size $O(x^{1/4+\varepsilon})$. Another related problem is Gauss circle problem, which gives an asymptotic for the number of lattice points inside a circle centred at the origin and with area $x \to \infty$.

Most of the papers written by Ramachandra during the last decade of his life were on this theme. In a series of papers jointly with his students he explored various avenues to prove new results in this field and sometimes provided clever proofs of old results (see [17-20, 116-119]). For example in [17] a new proof was given for the omega result for the generalised divisor function $d_k(n)$, and in [18] new results were proved for quasi L-functions of the form

$$\sum_{n=1}^{\infty} \chi(n)(n+\alpha)^{-s} F(n+\alpha)$$

where χ is a Dirichlet character and F is any complex valued function satisfying $F^j(x) \ll x^{-k+\varepsilon}$. In another joint paper with Balasubramanian [19], Ramachandra studies the Hurwitz zeta function

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s},$$

and proves a new approximate functional equation. In [118] it was shown that the mean value estimate

$$\sum_{n_1,\ldots,n_k \le x} \mu(n_1)\ldots\mu(n_k) \ll x e^{-c(\log x)^{1/(a+1)}}$$

(where μ is the Möbius function) yields the zero free region

$$\zeta(s) \neq 0, \quad \text{for} \quad \mathrm{Re}(s) \geq 1 - \frac{c'}{(\log(|\mathrm{Im}(s)| + 100))^a}$$

for some constant c' depending on the constant c. This is the reverse of the usual way where zero-free region of the zeta function is used to derive asymptotics for several summatory functions. These themes were further explored by many other researchers in India, especially by Balasubramanian and Sankaranarayanan. As examples, let me mention [52] where it is proved that

$$\sum_{|\gamma|\leq T} \frac{1}{|\zeta'(\rho)|} \gg T$$

as $T \to \infty$ where the sum is over the nontrivial zeros $\rho = \beta + i\gamma$ of $\zeta(s)$. The proof cleverly bypasses the Riemann Hypothesis by employing an estimate of Ramachandra and Sankaranarayanan, but one still needs to assume the simplicity of the zeros. In [21] Balasubramanian *et al*. give yet another equivalent statement for the Riemann Hypothesis. For any $x > 1$ let F_x be the increasing sequence of reduced fractions ρ_v in $(0, 1]$ with denominators $\leq x$. Suppose $\Phi(x) = \#F_x$. Let

$$T(y) = \sum_{m,n=0}^{\infty}\sum \frac{\cos(2\pi 2^m(2n+1)u)}{2^m(2n+1)^2}$$

be the Takagi function. Then among other things it is shown in [21] that the RH is equivalent to showing

$$\sum_{v=1}^{\Phi(x)} T(\rho_v) = \frac{\Phi(x)}{2} + O(x^{1/2+\varepsilon}).$$

Estimation of exponential sums is a common component in most works dealing with classical questions like estimation of the size of the zeta function or Dirichlet divisor problem or Gauss circle problem. In general the problem can be framed in the following way. Given f a real valued function estimate the size of the sum

$$\sum_{N\leq n<2N} e(f(n))$$

where $e(x) = e^{2\pi ix}$. One would expect a lot of cancellation in such a sum if f is sufficiently differentiable and has derivatives of large size. The general methods of Weyl, van der Corput and Vinogradov, are often used to derive such results. A harder version of the problem asks for cancellation in the above sum with added restriction on the sum, e.g. when n runs over prime numbers. Vaughan's identity is usually used to deal with such restrictions. In [97] one such problem is considered. Let f be a polynomial with real coefficients of degree k and leading coefficient a which is of type 1. Then the main result of [97] gives

$$\sum_{\substack{p\leq N \\ p \text{ prime}}} (\log p)e(f(p)) \ll N^{1-1/4(2^k+1)+\varepsilon}.$$

The other papers [96, 98, 99, 104], also deal with different types of exponential sums, sometimes with arithmetic weights, but they are more technical.

Large sieve inequalities are often useful when the problem at hand is about estimating an object over a family. As a prototype let us mention the following celebrated result, originally due to Davenport and Halberstam. Let (a_n) be a sequence of complex numbers, and let t_r with $1 \leq r \leq R$ be a collection of distinct fractions such that $\|t_r - t_s\| \geq \delta$ for $r \neq s$. (Here $\|.\|$ denotes the distance from the nearest integer.) Then one has

$$\sum_{r=1}^{R} \left| \sum_{n=1}^{N} a_n e(nt_r) \right|^2 \ll (N - 1 + \delta^{-1}) \sum_{n=1}^{N} |a_n|^2.$$

Roughly speaking the right hand side has two terms - the term $N \sum_n |a_n|^2$ accounts for the maximum possible size of the trigonometric polynomial $\sum a_n e(nt)$ and the other term $\delta^{-1} \sum_n |a_n|^2$ reflects the expected square-root cancellation on average. These type of inequalities are in great demand, and are often used as a substitute for the Riemann Hypothesis. Ramana and Prakash have proved interesting results in this topic (see [114, 115]). Let P be a polynomial with integer coefficients then in [114] they show that

$$\sum_{\rho \in F_Q} \left| \sum_{n=1}^{N} a_n e(\rho P(n)) \right|^2 \ll Q^{1+\varepsilon}(N + Q) \sum_{n=1}^{N} |a_n|^2.$$

where F_Q is the sequence of Farey fraction of order Q (see above). Other interesting works of Ramana include [38, 122, 123].

Dirichlet related class number of quadratic forms with special values of his L-functions. This interesting relation has opened up a genre in number theory which falls at the intersection of algebraic and analytic number theory. The difficulty in understanding the distribution of class number for number fields is well-known. Landau and Siegel related this difficulty to effectively understanding the non-vanishing issue of the L-function. Chakraborty, and also Mukhopadhyay, have written several papers dealing with class number (see [29, 31-33]). For example in [30] the authors consider $N(d, g)$ - the number of real quadratic function fields $K = \mathbb{F}_q(t, \sqrt{D})$ with $\deg(D) \leq d$ and the ideal class groups containing an element of order g. It is shown that $N(d, g) \gg q^{d/g}/d^2$ for odd q and even g.

The papers [34-36, 46, 57, 59], also fall under the theme of this section.

3. Modular Forms and Their Fourier Coefficients

Modern analytic number theory is synonymous with modular forms and their L-functions. Modular forms are certain holomorphic functions on the upper half plane, which transforms in a special way with respect to the action of $SL(2, \mathbb{Z})$, and satisfies certain conditions at the 'cusps'. The very existence of such objects is a mystery, but there are geometric ways to precisely 'count' them. Suppose f

is a modular cusp form of level q and weight k, then we have the Fourier expansion at infinity given by

$$f(z) = \sum_{n=1}^{\infty} \lambda_f(n) n^{(k-1)/2} e(nz)$$

where $e(x) = e^{2\pi i x}$. The coefficients $\lambda_f(n)$ are called the normalised Fourier coefficients of f. The Dirichlet series attached to this sequence

$$L(s, f) = \sum_{n=1}^{\infty} \frac{\lambda_f(n)}{n^s}$$

converges absolutely in the half plane Re$(s) = \sigma > 1$, has analytic continuation to whole of the complex plane, and satisfies a functional equation with s going to $1 - s$. Moreover when f is a common eigenfunction of all the Hecke operators, then $L(s, f)$ has an Euler product representation for $\sigma > 1$, and one again expects that a version of the Riemann Hypothesis holds. The Riemann zeta function can be obtained from such L-functions by taking the Dirichlet series associated with the Eisenstein series, which is a modular form orthogonal to the cusp forms. In this way we can put the Riemann zeta function, which a priori is an isolated object, in a family of L-functions.

A standard problem in this category is the following. Given a family $\mathcal{F}$ of L-function, show that there exists at least one (or infinitely many) L-functions in this family which is (or are) not vanishing at a prescribed point. For example, let χ_D denote the quadratic character associated with the discriminant D. Then the collection of Dirichlet L-functions

$$\mathcal{F} = \{L(s, \chi_D) : D \text{ fundamental discriminant}\}$$

is a family. A deep conjecture of Chowla states that $L(1/2, \chi_D) \neq 0$ for all members in this family. Major progress has been made towards this conjecture, and it is now known that a positive proportion satisfies the conjecture. Another intriguing problem is to compute the moment of L-functions over a family. This is often used to show that a given family of L-functions satisfy the Lindelöf hypothesis on average. Several works of Sengupta are on this topic (see [28, 65, 66, 68-71]). As an example let us mention the following result. Let H_k be the Hecke basis for the space of cusp forms of full level and weight k, then one has

$$\sum_{f \in H_k} L(1/2, f) \ll k^{1+\varepsilon}, \quad \text{and} \quad \sum_{f \in H_k} L(1/2, \text{Sym}^2 f) \ll k^{1+\varepsilon}.$$

(Incidentally similar problems were also considered in [102].) Also in [65] the following nice result is proved. Given a point s in the critical strip, with Re$(s) \neq 1/2$, there exists a k_0, such that for all

$k \geq k_0$ there exists $f \in H_k$ such that $L(s, f) \neq 0$. The papers [62, 67, 72-74, 142] deal with other interesting problems, like the first sign change in the sequence of Fourier coefficients $(\lambda_f(n))$ of a given modular form f. Some deal with expansions of Siegel modular forms and associated Dirichlet series.

The Fourier coefficients uniquely determine the modular form and hence the associated L-values. (For an interesting result in this direction see [14]). What can one say about the reverse? Suppose we are given a collection of matching L-values, can we say that they are generated by same forms? In [44] it is proved that if $f_i \in H_{\ell_i}$ for $i = 1, 2$ are two forms such that

$$L(1/2, f_1 \otimes g) = L(1/2, f_2 \otimes g)$$

for all $g \in H_k$ for infinitely many k, then $f_1 = f_2$. This is an extension of a celebrated result of Luo and Ramakrishnan. Ganguly has also established interesting results regarding the dimension of cusp forms of weight 1. More precisely, let $S_1(N, \chi)$ denote the vector space of weight one modular forms of level N and nebentypus χ. A result by Serre and Deligne shows that one can attach to a Hecke eigenform $f \in S_1(N, \chi)$ an irreducible, continuous Galois representation $\rho : G_{\mathbb{Q}} \to GL_2(\mathbb{C})$ to the absolute Galois group of $\mathbb{Q}$. These representations have very special properties, for example the image of ρ in PGL_2 should be either of D_{2n}, A_4, S_4, A_5. The dihedral case is well understood due to the work of Hecke. For S_4, in [45], it is shown that the dimension of $S_1(q, \chi)$ is bounded by $q^{3/4+\varepsilon}$ for $q \equiv 3(4)$ a prime.

Gun has considered the interesting problem of understanding the zeros of the modular forms (see [53, 54]). Let E_k denote the Eisenstein series of weight k for the group $SL(2, \mathbb{Z})$, and let $\mathcal{F}$ be the standard fundamental domain for the action of $SL(2, \mathbb{Z})$ on the upper half plane. A classic result of Rankin and Swinnerton-Dyer says that all the zeros of E_k in $\mathcal{F}$ lie on the arc

$$\{e^{i\theta} : \pi/2 \leq \theta \leq 2\pi/3\}.$$

Kohnen showed that any such zero if not a twelfth root of unity, is necessarily transcendental. The result of Rankin and Swinnerton-Dyer was later extended to cover a special class of non-cusp forms by Getz. In [54] the author proves the analogue of Kohnen's result for this special class of non-cusp forms. In the papers [56, 57], Gun and Ramakrishnan study the number of representations of integers by sum of square, and establish remarkable identities relating them with special values of Dirichlet L-functions.

Ramakrishnan together with Manickam, have written several papers on the theory of Jacobi forms (see [100, 101, 103]). In the paper [101] the authors derive the Saito-Kurokawa correspondence for the spaces of Eisenstein series for arbitrary level and trivial nebentypus. In the other paper [100] the authors study the newform theory in the space of Jacobi forms and prove that the respective newform spaces are isomorphic under the Eichler-Zagier map. As an application, the authors derive the explicit Waldspurger result for Jacobi cusp newforms. They also study the Shimura and Shintani correspondences between Jacobi forms and modular forms of integral weight, which is a generalization of similar correspondences studied by Gross, Kohnen and Zagier. Ramakrishnan has also worked on other interesting problems (see [120, 121]). For other works of Manickam see [55, 63]. Das has also proved important results in the theory of Jacobi forms (see [40-42]). In [41] he constructs an explicit differential operator on the space of Hermitian Jacobi forms (HJF). This operator commutes with $SL_2(\mathbb{R})$ and the Hecke actions. It induces a map from elliptic cusp forms to HJF. Using this, a map from a certain subspace of HJF to a finite direct sum of elliptic modular forms is obtained. Whether this map extends to the full space HJF is an interesting open problem.

In 1989 Selberg extracted the essential properties of L-functions, and gave an axiomatic definition of a class of L-functions. The members of the class are Dirichlet series which obey four axioms - Euler product, analytic continuation, Ramanujan conjecture, functional equation of prescribed form. These seem to capture the essential properties satisfied by most functions that are commonly called L-functions or zeta functions. Although the exact nature of the Selberg class is conjectural, one hopes to classify its contents. Such a classification is based on a real-valued invariant d called degree, and the degree conjecture asserts that $d \in \mathbb{N}$ for every L-function in the Selberg class. A recent deep result of Kaczorowski and Perelli settles the degree conjecture for $d < 2$. Several interesting results regarding the Selberg class have been proved by Srinivas and his collaborators, especially Mukhopadhyay (see [108, 109, 145]). For example in [145] Srinivas considers two functions $F \neq G$ in the Selberg class with deg$F \geq$ deg$G$, and shows that for T sufficiently large, there is a zero $\rho = \beta + i\gamma$ of F/G with γ in the interval $[T - O(\log\log T), T + O(\log\log T)]$. From this, he deduces that the number of zeros of F/G with imaginary part in $[0, T]$ and counted with multiplicity is $\gg T(\log\log T)$. Also in [109] an analogue of Hardy's theorem is proved for degree L-functions in Selberg class. Other interesting works of Srinivas include [64, 88, 110, 118].

The other interesting papers broadly falling under the category of this section are [39, 120].

4. ADDITIVE COMBINATORICS

The 'zero sum problem' in additive combinatorics has been the focus of many researchers in India, especially Adhikari and his collaborators (see [1, 2, 4-6, 10, 16, 47, 49-51, 144, 147]). This is a combinatorial problem about the structure of a finite abelian group. More precisely, given a finite abelian group G and an integer $n > 0$, one asks for the least k such that every sequence of k elements of G contains n terms that sum to 0. In the special case where n is taken to be the cardinality of G, i.e. $n = |G|$, then this number is denoted by $E(G)$. The classic result in this field is about cyclic groups. In 1961, Erdös, Ginzburg and Ziv proved that for $G = \mathbb{Z}/n\mathbb{Z}$ one has $E(G) = 2n-1$. This means that if $A = (a_1, a_2, \dots, a_{2n-1})$ is a sequence integers then one can pick $1 \le i_1 < i_2 < \cdots < i_n \le 2n-1$ such that $a_{i_1} + \cdots + a_{i_n}$ is a multiple of n. Note that the sequence $A = (0, \dots, 0, 1, \dots, 1)$ with $n-1$ copies of 0 and $n-1$ copies of 1 contains no such subsequence.

Another interesting number that one can attach to a finite abelian group G is the Davenport constant $D(G)$. This is defined as the smallest n, such that every sequence of n elements, contains a non-empty sub-sequence with sum 0. The most basic result in this topic is $D(\mathbb{Z}/n\mathbb{Z}) = n$. In fact the following relation exists

$$E(G) = D(G) + n - 1.$$

In [11] a generalization of both $E(G)$ and $D(G)$ was introduced. Suppose we have a non-empty set $A \subset \mathbb{Z}$, the Davenport constant of G with weight A, denoted by $D_A(G)$, is defined to be the least natural number n such that for any sequence $\{x_1, \dots, x_n\}$ of n (again, not necessarily distinct) elements in G, there exists a non-empty subsequence $\{x_{j_1}, \dots, x_{j_\ell}\}$ and $a_1, \dots, a_\ell \in A$ such that $\sum_i a_i x_{j_i} = 0$. Similarly one defines $E_A(G)$.

The following basic set up is already interesting. For natural numbers n and d, consider the additive group $G = (\mathbb{Z}/n\mathbb{Z})^d$. Let $A \subset \mathbb{Z}/n\mathbb{Z} - \{0\}$. Let $D_A(n, d)$ denote $D_A(G)$ in this case. Much is known about these numbers, but a lot still remains unanswered. The weighted version of the previous problem can also be formed. Indeed one defines the constant $E_A(n, d)$ to be the least natural number k such that for any sequence $\{x_1, \dots, x_k\}$ of k elements of $(\mathbb{Z}/n\mathbb{Z})^d$ there exists a subsequence $\{x_{j_1}, \dots, x_{j_n}\}$ of length n and $a_1, \dots, a_n \in A$ such that $\sum_i a_i x_{j_i} = 0$. The relation between $D_A(n, d)$ and $E_A(n, d)$ still remains a mystery in general. However for $d = 1$, the conjectured relation $E_A(n, 1) = D_A(n, 1) + n - 1$ has been proved.

Adhikari has worked extensively on the zero sum problem. In [10] for the case, $A = 1, -1$, it is shown that $E_A(n, 1) = n + [\log 2n]$. The case '$n$ is even' yields to a relatively simple argument, the case where n is odd is taken care of by a sequence of clever combinatorial arguments. Together with Rath in [9], Adhikari considers the special case in which $n = p$ is a prime, and determines the values of $D_A(p)$ and $E_A(p)$ where A is either $\{1, 2, \dots, r\}$ or the set of quadratic residues modulo p. The d dimensional case was considered in [6] and [8]. Adhikari has also contributed to the 'visibility problem' (see [11]).

Among other works on this theme, we note that in [23] it is shown that for G an abelian group of order n and exponent m, and let $k \leq 7$ be such that $n/m \geq k$, then the Davenport constant $D(G) \leq n/k + (k-1)$. Sury and Thangadurai [144] consider the Gao's conjecture which says that for $G = (\mathbb{Z}/n\mathbb{Z})^2$ if a sequence S of $4n-4$ elements has no subsequence of length n with sum $(0,0)$, then S is of the form $a^{n-1}b^{n-1}c^{n-1}d^{n-1}$ with some $a, b, c, d \in G$. In [144] this conjecture is settled for $n = 7$. For further works of Thangadurai on additive combinatorics see [47, 48, 50, 147, 149].

5. Diophantine Equations and Transcendental Numbers

In this section we will focus on the work done on Diophantine problems, i.e. finding integer solutions of polynomial equations. Laishram, Sharadha and Shorey have written several papers on this topic (see [76-78, 80-83, 85, 91, 92, 131-141, 143]).

In [135] Saradha and Shorey completely solve the equation

$$n(n+1)\dots(n+i-1)(n+i+1)\dots(n+k-1) = by^2$$

in positive integers n, k, i, b, y, where $k \geq 4$, $n > k^2$, $i < k-1$ and b is square-free. As a consequence, they solve a question of Erdös and Selfridge by showing that, for $k \geq 3$, there is no square other than 12^2 and 720^2 such that it can be written as a product of $k-1$ integers out of k consecutive positive integers. Another Diophantine equation which has attracted a lot of attention is

$$n(n+d)\dots(n+(k-1)d) = by^\ell,$$

where the left hand side is a product of consecutive members in an arithmetic progression. Erdös conjectured that this equation has no solutions unless $k = \ell = 3$. In [136] significant progress was made towards this conjecture. Variations of this theme were considered in [138, 141, 143]. Saradha has also worked on other Diophantine problems, e.g. see [137, 139, 140]. Shorey has studied Diophantine problems involving products of consecutive numbers from Lucas sequence (see [91, 92]).

Laishram, partly jointly with Shorey, has written a series of papers studying the arithmetic nature of a product of consecutive terms in an arithmetic progression, and sometimes with one term omitted in between. For example in [82] it is shown that there are integral solution of

$$n(n+d)\dots(n+(k-1)d) = y^2,$$

with $k \geq 5$, and $\omega(d) \leq 5$. Here $\omega(d)$ denotes the number of distinct prime divisors of d. In another work [61] Laishram *et al.* extends a theorem of Euler, which states that the product of four terms in an arithmetic progression is never a square. The paper [61] shows that the product of up to 110 terms in an arithmetic progression can never be a square.

Finally let me mention some results in the realm of transcendental number theory (see [13, 110, 111, 133]). In [13] the authors investigate the transcendence of the series

$$S = \sum_{n=0}^{\infty} \frac{f(n)}{Q(n)}$$

where Q is a polynomial with simple rational roots and f is polynomial with coefficients in $\bar{\mathbb{Q}}$. Roughly speaking the authors prove that either S is transcendental or is a 'computable' algebraic number. In particular it follows from their main theorem that

$$\sum_{n=0}^{\infty} \frac{1}{(3n+1)(3n+2)(3n+3)}$$

is a transcendental number. The results of this paper were extended in [133]. Various transcendence results are proved for the digamma function $\psi(x) = \Gamma(x)/\Gamma'(x)$ in [110] and [111]. In particular in [110] the authors prove that the $\phi(q)$ many numbers $\gamma + \psi(a/q)$, with $(a, q) = 1$ are algebraically independent over any number field without a non-trivial q-th root of unity. Here γ is the Euler constant.

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